The impact of the parallel Reynolds stress on the prediction of zonal flows

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Introduction

In a toroidal geometry a stationary Zonal Flow (ZF) consists of a poloidal $E \times B$-flow with zero poloidal and toroidal mode numbers and a return-flow along the magnetic field-lines necessary to cancel perpendicular plasma compression effects due to the magnetic inhomogeneities. It is commonly recognized that ZF shearing-action strongly affects the turbulent radial transport of heat and particles in confinement plasmas because the flows disrupt the formation of persistent radial structures otherwise observable in ion-temperature-gradient driven turbulence. At the same time, the turbulence generated perpendicular and parallel Reynolds stresses govern the ZF evolution. In numerical ITG-turbulence studies the stresses exhibit large deterministic features clearly correlated to the flows indicating that the construction of a stress response functional from observations in turbulence studies is possible. The following observations reveal that an adequate treatment of the parallel Reynolds stress is essential to obtain a functional that correctly describes the ZF evolution.

Stress Response Functional

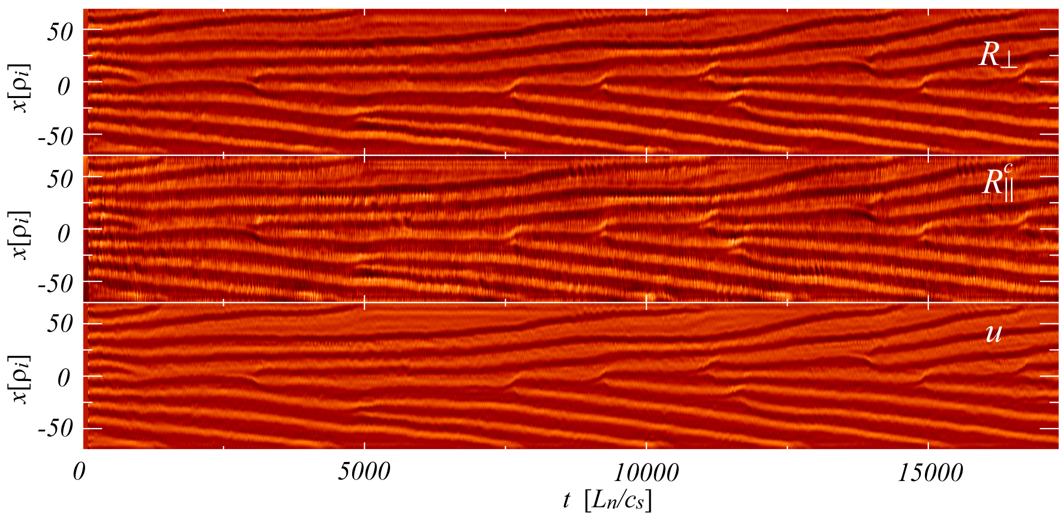


Figure 1: Time-evolution of the perpendicular Reynolds stress $R_\perp$ (top), the parallel Reynolds stress $R_\parallel$ (middle), the shearing-rate u (bottom).

The ZF ITG-turbulence interaction is studied in an electrostatic, large aspect ratio, circular geometry, computationally more practical, two-fluid model with adiabatic electrons [1] which qualitatively reproduces the ZF behavior observed in gyro-kinetic turbulence studies. Figure

1 shows the time-evolution of the ZF shearing-rate $u \equiv \partial_x \overline{\upsilon}_\theta$ for the parameters $2L_n/R = 1$, $L_n/L_{T_i} = 2.4$, $q = 1.5$ and $s = 1$ (L_n, L_{T_i} density/temperature gradient lengths, R major radius, q safety factor and s local shear) which are typical core region parameters. The boundary conditions in this case allowed the ZFs to drift radially thereby changing their radial scale length. But the scale length may only change within highly constrained limits otherwise new flows grow to restore the characteristic pattern which has a scale length different from the initially fastest growing mode. This indicates an intrinsic scale generation mechanism for the ZF-turbulence equilibrium.

The ZF-evolution is governed by the Reynolds stresses:

$$\partial_t \overline{\upsilon}_\theta = -\partial_x \left(R_\perp - 2qR_\|\right) / \left(1 + 2q^2\right) \tag{1}$$

$$R_\perp \equiv \langle \upsilon_x \upsilon_\theta \rangle \tag{2}$$

$$R_\| \equiv \left\langle \cos(\theta)\, \upsilon_\| \upsilon_x \right\rangle \tag{3}$$

where $\langle \ldots \rangle$ and the overbar denote flux-surface-averages and $\upsilon_x, \upsilon_\theta, \upsilon_\|$ are the radial, poloidal, parallel velocity fluctuations respectively. Comparison of the stresses and the shearing rate (Fig. 1) shows that the highly deterministic, nearly stationary (compared to turbulence time scales) stress patterns are correlated to the shearing rate. This suggested that the construction of a stress response functional from observations in turbulence studies is possible.

Using artificial shear-flows in the turbulence studies the following functional dependence of the perpendicular stress response was observed [2]

$$R_\perp = Q\left[\alpha_0 u + \alpha_2 {\partial_x}^2 u + \beta u^3 + \gamma \partial_x \ln Q\right] \tag{4}$$

where $\alpha_0, \alpha_2 > 0$ and $\beta, \gamma < 0$ are coefficients and $Q \equiv \langle \upsilon_x T_i \rangle$ is the radial heat-flux. Disregarding the heat-flux effects this functional dependence on u of the formula was also proposed by a wave-kinetic derivation of a total stress functional [4] which accounted for the parallel stress contribution by a viscous modification of $\alpha_o \to \alpha_0 - 2q\mu$ (μ parallel viscosity). However, numerical solutions of (1) using (4) (constant Q) only have the largest scale fitting into the system as a stable solution. This is contrary to the finite, characteristic scale length observed in self-consistent studies indicating that one ingredient for the intrinsic scale generation mechanism is missing in (4).

The self-consistent ZF pattern shows that the largest scales are damped while intermediate continue to grow. Using a mean-field approximation of $\partial_x u^3 \approx \left\langle u^2 \right\rangle_x \partial_x u$ ($\langle \ldots \rangle_x$ radial average) a growth-rate Γ that reflects this observation can be derived from (4) by adding one additional term and choosing adequate signs for the coefficients

$$\Gamma = K_x^2\left(\alpha_0 + 3\beta \left\langle u^2 \right\rangle_x - \alpha_2 K_x^2 + \alpha_4 K_x^4\right) \langle Q \rangle_x \tag{5}$$

with $\alpha_0 > 0$ and $\beta, \alpha_2, \alpha_4 < 0$.

Figure 2 shows the behavior of Γ for different values of $\langle u^2 \rangle_x$. The region of ZF growth $K_{x,l} < K_x < K_{x,h}$ is confined by the roots of $\Gamma = 0$ with $0 < K_{x,l} \leq K_{x,h}$ for sufficiently high shearing-rates. Since $K_{x,l}$ increases and $K_{x,h}$ decreases with u, the system saturates at $K_{x,s} = K_{x,l} = K_{x,h} = \sqrt{\alpha_2/2\alpha_4}$. That the ZFs indeed show this damping behavior for large scales is

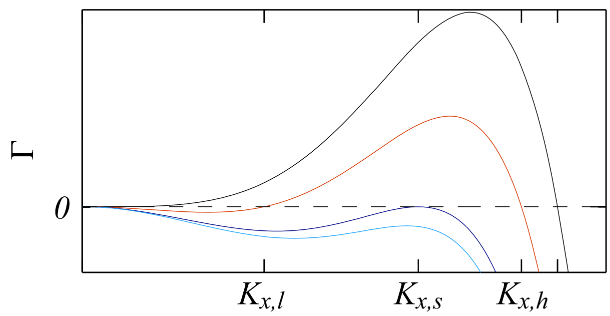
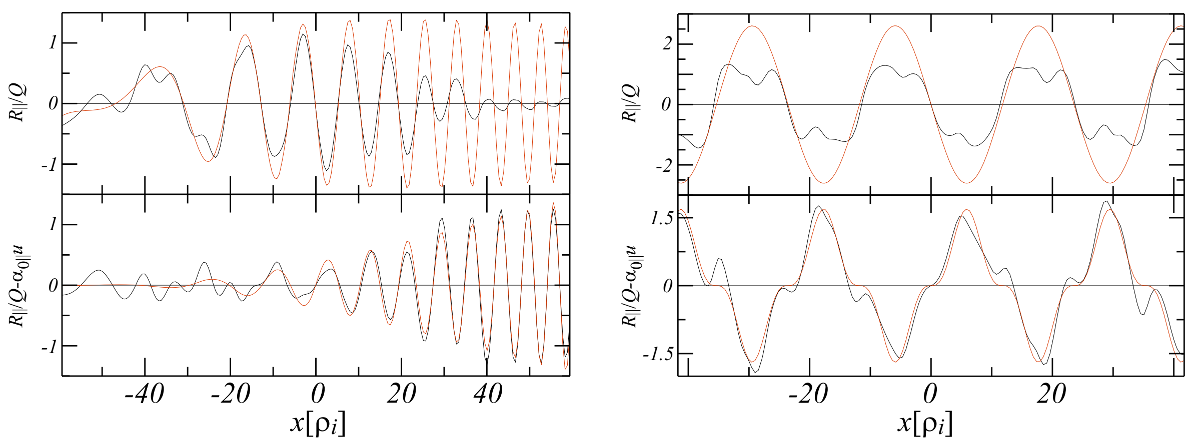


Figure 2: ZF growth-rates Γ for $\langle u^2 \rangle_x = 0/0.1/0.11/0.12$ (black/red/dark blue/light blue).

verified in [2] and the total stress functional $R_t \equiv R_\perp - 2qR_\parallel$ is

$$R_t = Q\left[\alpha_0 u + \alpha_2 \partial_x{}^2 u + \alpha_4 \partial_x{}^4 u + \beta u^3 + \gamma \partial_x \ln Q\right] \tag{6}$$

with $\alpha_0 > 0$ and $\alpha_2, \alpha_4, \beta, \gamma < 0$. The measurement of the coefficients for the perpendicular stress yield positive contribution values for α_2, α_4 as does the wave-kinetic derivation in [4]. However, they are negative for the total stress. This suggests that the parallel stress $R_\parallel$ contributes significantly to all coefficients, a fact that was largely neglected in contemporary ZF theories [3].



(a) Wave-length dependence of $R_\parallel$

(b) Saturation of $R_\parallel$

Figure 3: $R_\parallel$ response to $u \sim \sin\left(2\pi x\rho_i\left(5 + 15x\rho_i/L_x\right)/L_x\right)$ (a). $R_\parallel$ response to $u \sim \sin\left(2\pi x\rho_i/12L_x\right)$ (b).

The parallel stress response to an artificial shearing-rate $u \sim \sin\left(2\pi x\rho_i\left(5+15x\rho_i/L_x\right)/L_x\right)$ is shown in Fig. 3a. The response exhibits a strong wave-length dependence and apparently $\alpha_{2,\|}\partial_x{}^2 u$ fits the residual $R_\|/Q-\alpha_{0,\|}u$ quite well, with $\alpha_{2,\|}=0.84$. Inclusion of the parallel stress contribution now results in the necessary negative coefficient $\alpha_2 \equiv \alpha_{2,\perp}-2q\alpha_{2,\|}=-1.2$ ($\alpha_{2,\perp}=1.3$). The parallel stress response to an artificial shearing-rate $u \sim \sin\left(2\pi x\rho_i/12L_x\right)$ is shown in Fig. 3b. The response exhibits a saturation for large shearing-rates just like the perpendicular stress response does. A fit of u^3 to the residual $R_\|/Q-\alpha_{0,\|}u$ yields the coefficient $\beta_\|=-1.0$ for the parallel response. Overall, investigation of the parallel stress response revealed significant contributions to all coefficients in (6).

Conclusions

It was demonstrated that, in order to describe the self-consistent ZF evolution, the Reynolds stress response functional (6) must describe a damping of largest scales for sufficiently high shearing-rates while intermediate scales continue to grow until the flow saturates with a characteristic radial scale length. This determines the relation of signs between the coefficients in the functional. By taking only contributions of the perpendicular stress to the total stress into account, as done by many contemporary ZF theories, one obtains coefficients with the wrong sign relation such that the resulting response does not describe the ZF-evolution adequately. It was shown that the contributions by the parallel stress to all coefficients of the total stress are significant and are essential in order to obtain a response functional that describes ZF growth, saturation and characteristic radial scale length. The functional obtained using all contributions to the coefficients permits a reliable charting of ZF-turbulence equilibria.

References

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