Analysis of asymmetric zonal flow states observed in self-consistent 3-D drift wave turbulence simulations

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Introduction

Self-consistent simulations of resistive drift wave turbulence have yielded the first finding of transport bifurcations in such a system. These transport states are linked to an asymmetry in the shape of the poloidal zonal flows - with more pronounced, deeper flows in the ion diamagnetic drift direction.

The potential relevance of drift wave turbulence lies in the high gradient tokamak edge (and thus at internal transport barriers) as well as in geostrophic modes, the drift wave analogon in planetary turbulence.

Apart from a qualitative analysis of this flow asymmetry and its associated density corrugations, studies of newly found radial drift wave avalanches (moving downhill the shear flow gradient contrary to naive expectations based on $\partial_t \vec{v}_{gr,x,DW} \propto \partial_x v_{y,shear}$) have been conducted.

Numerical simulations

Using the Braginskii-based two-fluid code NLET [3], a turbulent sheared-slab cold-ion resistive drift-wave system consisting of the following Hasegawa-Wakatani equations has been examined:

$$d_t n = d_t \nabla_\perp^2 \phi \tag{1}$$

$$\hat{\rho}_s^{-3} d_t \nabla_\perp^2 \phi = -\partial_\parallel^2 (\phi - n) \tag{2}$$

where $d_t = \partial_t + \vec{z} \times \nabla_\perp \phi \cdot \nabla_\perp$, $\quad \partial_\parallel = \partial_z - 2\pi s x \partial_y$ and $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ as well as $L_s = 1/2\pi s$ and $L_z(= 2\pi qR)$ as the parallel length scale.

Here, $\hat{\rho}_s = \rho_s/L_\perp$ - the single relevant parameter for the Hasegawa-Wakatani equations - is the dimensionless ratio of the 'ion sound Larmor radius' $\rho_s = m v_{th}/eB = m\sqrt{T_e}/eBm_i$ to the orthogonal length scale $L_\perp = R/L_n (\pi q/s)^{2/3} \left[c_s t_0 n e \eta_\parallel/2B \right]^{1/3}$ (the scale of maximal drift wave growth where the relaxation frequency equals the diamagnetic drift frequency) where $\eta_\parallel$ marks the parallel resistivity and $L_n = -n(dx/dn)$.

Time is normalized to $t_0 = \rho_s/v_{dia,e^-}$ with $v_{dia,e^-} = \alpha(1 + \eta_i)(1 + \tau) t_0 L_\nabla/2L_0 L_n$.

Typical run parameters in the previously defined units are $n_x = n_y = 512$, $n_z = 32$, $L_x = L_y = 104.5\rho_s$, $L_z = 6.3qR$, grid step size $\approx 7.7 \cdot 10^{-3}$, time step $\approx 3.4 \cdot 10^{-4}$ and run time $\approx 8.8 \cdot 10^1$.

Extensive consistency and convergence scans have been performed prior to the parameter scans for $\hat{\rho}_s^{-3}$.

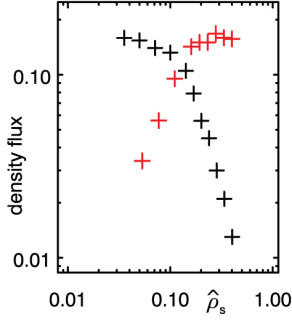## Parameter studies

transport regimes ($L_\perp$ vs. ρ_s)

Figure 1: *Density fluxes in terms of $L_\perp$ and ρ_s*

As is well-known, the linear properties of the flow states are best characterised by the eigenvalue of the unsheared system, since the sheared eigensystem cannot easily reproduce the development of the states. There is no feasible decomposition for this non-orthogonal, nearly collinear eigensystem, thus developing single eigenvectors on their own is rendered impossible. Strictly speaking, there are no growing eigenmodes for $s \neq 0$, hence the general growth rate of modes of the shearless, non-adiabatic case, derived from eqns. (1) & (2), is used

$$\gamma = \Im(\omega) \propto \left[k_\perp^2 + k_\parallel^2 \left(\frac{1}{k_\perp k_y} + \frac{k_\perp}{k_y} \right)^2 \right]^{-1} \tag{3}$$

which is approximated by $\gamma = \omega *^2 / \omega_\parallel = k_\perp^2 / \left(k_\parallel^2 / (\hat{\rho}_s^{-3} k_\perp^2) \right) = \hat{\rho}_s^{-3} k_\perp^4 / k_\parallel^2$.

The mixing length anomalous heat diffusion coefficient $D = \gamma / \vec{k}_\perp^2$ depends on the orthogonal wavenumber, which is determined by one of two scales with a transition at approximately $\hat{\rho}_s \approx 0.12 - 0.20$ (coinciding with the onset of zonal flow formation). For the two regimes we find:

- relaxation scale $L_\perp$ dominant for $\hat{\rho}_s < 0.12$: $\quad \hat{D} = \hat{\gamma}/\hat{k}_\perp^2|_{k_{phys}=L_\perp^{-1}} = \hat{\gamma}/\hat{k}_\perp^2|_{k_{units}=\hat{\rho}_s} \propto \hat{\rho}_s^{-1}$

- diam. drift scale ρ_s dominant for $\hat{\rho}_s > 0.2$: $\quad D_\rho = \gamma_\rho / k_{\rho\perp}^2|_{k_{phys}=\rho_s^{-1}} = \gamma_\rho / k_{\rho\perp}^2|_{k_{units}=1} \propto \hat{\rho}_s^{-3}$

$$\Rightarrow \frac{D_\rho}{\hat{D}} = \hat{\rho}_s^{-2} \text{ (analytically)} \iff \frac{D_\rho}{\hat{D}} = \hat{\rho}_s^{-2\pm0.1} \text{ (numerically)}$$

It has been verified thoroughly by a set of numerical parameter scans over $\hat{\rho}_s$ that D/D_ρ is asymptotically constant for small $\hat{\rho}_s$ and, vice versa, $D/\hat{D}$ for large $\hat{\rho}_s$.

Transport bifurcations

To our knowledge, for the first time in self-consistent simulations, transport bifurcations containing two stable gradients have been found.

These density corrugations represent stationary transport states with regions of high diffusivity and low gradients at the location of the flows pointing in the electron diamagnetic drift direction (the positive flows) while low diffusivity and high gradients can be observed at the

more sharply concentrated, radially tightened negative flows - the bifurcations are accompanied by an asymmetric flow pattern.

This flow structure emerges on time scales which are $\sim O(10^1)$ for a typical parameter $\hat{\rho}_s \approx 0.28$ (and gain approximately one order of magnitude for every doubling of $\hat{\rho}_s$) - this, in addition to a higher resolution, might indicate why they have not been observed in earlier studies [1].

Figure 2: *Flow and density profiles*

Bifurcation mechanism

Using the drift wave action invariant N [2] for the wave packet intensity

$$\partial_t N_{\vec{k}} = -\nabla_{\vec{x}} \left(N_{\vec{k}} \cdot \vec{v}_{gr,\vec{k}} \right) - \nabla_{\vec{k}} \left(N_{\vec{k}}(x) \cdot \dot{\vec{k}}(\vec{x},\vec{k}) \right) \tag{4}$$

(where the second term stems from a shear flow which can change the wave number locally [5], with $\dot{\vec{k}} = -\vec{\nabla}_x \vec{v} \cdot \vec{k}_0$), negative flows are found to repulse the turbulence, while positive flows are attractive - the flows can change the radial wavenumbers of the radially propgating drift waves, acting like forcefields on them. Transport, in concurrence with turbulence levels, is thus reduced at the negative flows.

Since the transport balance $\partial_x \Gamma(x) = 0$ is maintained in equilibrium, higher gradients at the location of the negative flows are required to counterbalance this reduction. This causes density corrugations to form, the steepened gradients of which lead to an increased rate of drift wave generation at the flow minima. These drift waves are then repelled by the negative flows, causing radial movement and thereby Reynolds stresses (via Poynting's theorem) which, in turn, fuel the flow up to its equilibrium level. The associated carry-off of drift waves leads to a deepening of the negative flows and a broadening of the positive flows, resulting in flow asymmetry.

Radial avalanches

In the case of a constant shear flow, drift wave eddies form radial avalanches, indicating apparent outward density transport for negative flow shear - and inward transport for positive flow shear. The opposite would be expected, based on

$$v_{gr,x,cold} = \frac{\partial \omega}{\partial k_x} = \frac{-2k_x k_y \hat{\rho}_s^2}{\left[1 + \hat{\rho}_s^2 (k_x^2 + k_y^2) \right]^2} \quad \text{where} \quad k_x = k_{x_0} - \frac{\partial v_y}{\partial x} t |k_y| \tag{5}$$

yielding $\frac{\partial v_{gr,x}}{\partial t} = 2v_y' k_y^2 \hat{\rho}_s^2$ (negative for negative flow shear) and thus uphill movement.

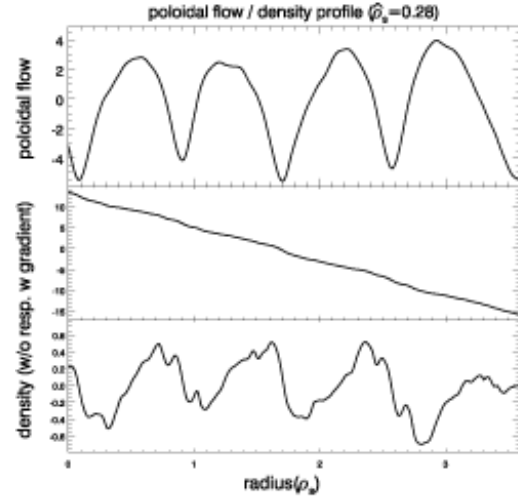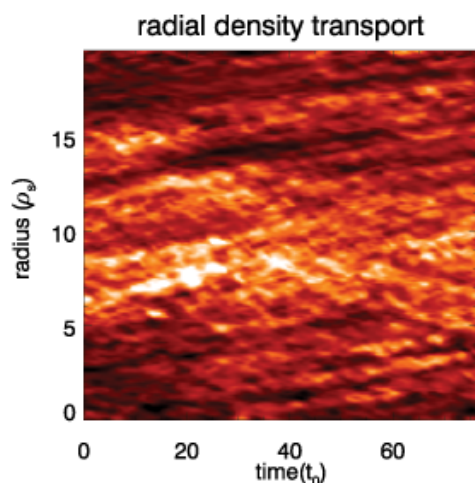radial density transport

Figure 3: *Drift wave avalanches*

In order to be the dominant species despite $v_{gr,x}$, outward-moving eddies must thus experience some form of amplification, conceivably via the drift wave growth rate or simple acceleration. However, the drift wave growth rate (only noticeable for $|k_x| \approx 0$) proves to be too small, with $k_x = k_{x_0} - \frac{\partial v_y}{\partial x} t |k_y|$ leading to $-k_x \gg 0$ very soon. Also, acceleration up the hill can only take place for a short time due to finite $max(v_{gr,x})$ for high $|k_x|$.

However, if the vorticity term - the Laplacian - in the Hasegawa-Wakatani equations becomes significant (meaning $k\rho_s \approx 1$), the (adiabatic) drift waves with short wavelengths experience enough scattering from high $-k_x$ to high k_x in order not to perish.

They are found to thereby extract energy from the flow shear via downhill avalanches, keeping v'_y in check. This effect only becomes significant once the shear flows get close to their self-consistent equilibrium, as evidenced by examinations of the antipodal case with artificially reduced zonal flows, which result in *uphill* avalanches (with the Reynolds stresses being in phase with the flow shear), thus *reinforcing* the flow shear.

Summary

Our sheared slab drift wave turbulence runs yield the first example of transport bifurcations in self-consistent simulations.

These transport states and the associated flow asymmetry pose a robust phenomenon in a considerable parameter range and have been affirmed by means of an ansatz for an asymmetry mechanism.

In addition, and contrary to naive expectation, downhill drift wave avalanches have been found, keeping the flow shear in check.

References

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