Theory of external geodesic acoustic mode excitation

K. Hallatschek[1], G.R. McKee[2]

[1] *Max-Planck-Institut für Plasmaphysik, Garching, Germany*

[2] *University of Wisconsin-Madison, Madison, Wisconsin, USA*

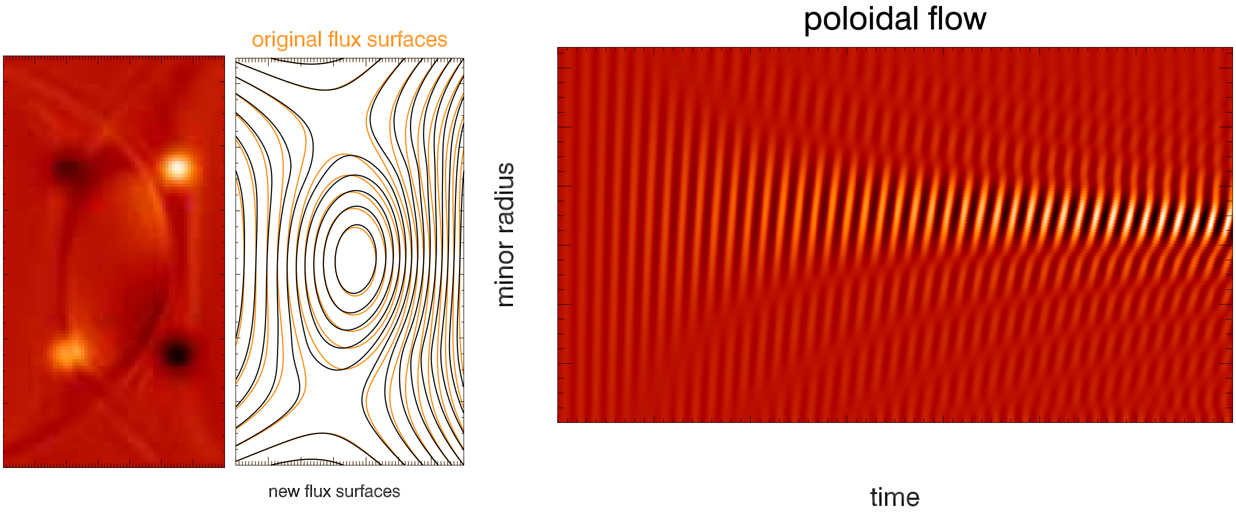
Introduction

It would be extremely appealing to be able to externally excite geodesic acoustic modes in a tokamak, either for diagnostic purposes, since the GAM frequency is dependent on the ion and electron temperature as well as the flux surface shapes, or, provided sufficiently large amplitude is achievable, to artificially reduce the turbulent transport, since GAMs are theoretically expected to impact the transport [1] and recent measurements seem to point in that direction [2].

As for symmetry, it should be possible to generate GAMs by applying an external magnetic field at the GAM frequency with a $(m,n) = (2,0)$ component to the plasma column. (In principle, this could already be done at present, e.g., in the DIII-D tokamak with the in-vessel RWM stabilization coils [I-coils]). (A perturbation with $m = 1$ corresponds to a pure shift of the plasma column, assuming high aspect ratio.)

The action of external coil currents on the interior flux surfaces of the plasma has been studied by means of a novel dynamic equilibrium code as well as analytically, deriving a surprisingly simple transfer function. The driving strength depends on the local flux surface properties (ellipticity, differential Shafranov-shift, etc.) but also more subtly on properties of the magnetic equilibrium such as the second radial derivatives of the pressure. The results offer several control knobs to influence the drive effectivity and aid in designing a GAM drive antenna. On a theoretical plane, the magnetic drive of GAMs is possible despite their well known electrostatic nature, because a GAM localized at a flux surface exhibits nevertheless a small ($O(\rho^*)$), intrinsically nonlocal magnetic perturbation of large radial range ($\sim$ minor radius). (This nonlocal component is justly neglected in most turbulence codes.)

The response of the turbulent plasma to the external driving has been studied using nonlocal NLET code runs [3], exposing a competition between the driving, the radial gradient of the GAM frequency, and the nonlinear wavenumber dispersion [4]. At least in the limit of weak radial dispersion, the external drive always necessarily becomes stronger than the intrinsic turbulent drive/damping terms, eventually overpowering the turbulence.



(a) (b) (c)

Figure 1: Exemplary results from dynamic equilibrium code: (a) Toroidal perturbation current distribution; (b) resulting shift of flux surfaces. (c) Linear GAM excitation in nonlocal scenario, flux surface averaged poloidal flow

Linear excitation

Fig. 1(a,b) shows the static linear response of an eliptic equilibrium to an $m = 2$ perturbation current. The employed external current is visible in fig. 1(b) from the localized feature. In addition it becomes clear that the plasma is not screening (as one might expect) but rather amplifying the perturbation current. This is due to the attraction/repulsion of the toroidal plasma currents by the external current in the same/opposite direction. The toroidal plasma currents in turn are controlled by the gradients of pressure and toroidal magnetic field through the Grad Shafranov equation. The equilibrium displacement can be converted into a drive term useful for numerical turbulence studies by computing the inertial forces for oscillating external currents.

Adding the sketched driving terms to a *local* turbulence computer run would only result in a radially homogenous poloidal motion due to the GAMs. Because of Gallilean invariance of the turbulence equations, this type of flow is inconsequential for the turbulence and also not affected by turbulence. Therefore we study right away a nonlocal scenario, where the GAM frequency varies throughout the computational domain owing to the temperature contrast. As a consequence, resonance with the excitation occurs only on one specific flux surface. This is shown in fig. 1(c), first without turbulence.

After a specific number of oscillations, according to the quality factor of the most weakly damped resonant eigenmode, the flow pattern becomes stationary with a strong resonance peaking as determined by the radial dispersion relationship of the GAMs [5, 6]. There is also a

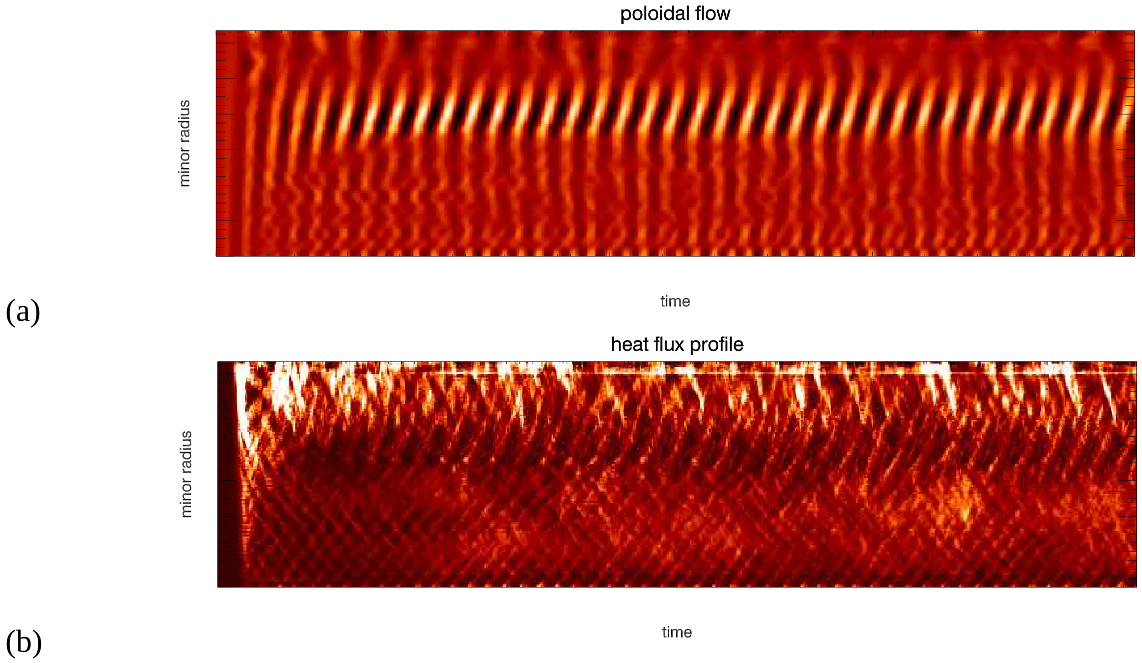


Figure 2: GAM excitation in nonlocal scenario including turbulence; flux surface averaged poloidal flow velocity (a), turbulent heat flux (b)

characteristic phase shift for the flux surfaces further inside/outside whose GAM frequency is higher/lower than the excitation frequency. The radial amplitude and phase variation together cause a significant shearing rate, which may interact with the ambient turbulence.

Interaction with turbulence

Admitting the turbulence in the computation, the quality factor of the resonance clearly decreases and the resonance becomes less pronounced (fig. 2). However, the turbulence modulation due to the flows becomes very obvious in a display of the turbulent heat flux (fig. 2b).

Increasing the amplitude of the coil currents results eventually in turbulence suppression and a concomittant steepening of the temperature profile (fig. 3). Since the steepening makes the resonance peak sharper, the shearing rate of the GAMs is a stronger than linear function of the coil currents, with a potential for nonlinear switching phenomena.

Summary

The magnetic excitation of internal GAMs is in principle possible. The primary agent is thereby the $m = 2$ component of the external field. The excitation is a global effect, which is absent in (quasi) local simulations, proportional to the resonance width over the minor radius. It can be incorporated into nonlinear simulations as the inertial forces resulting from the changing equilibria.

In a nonlocal setup, which is natural for the discussed problem, a localized resonance with

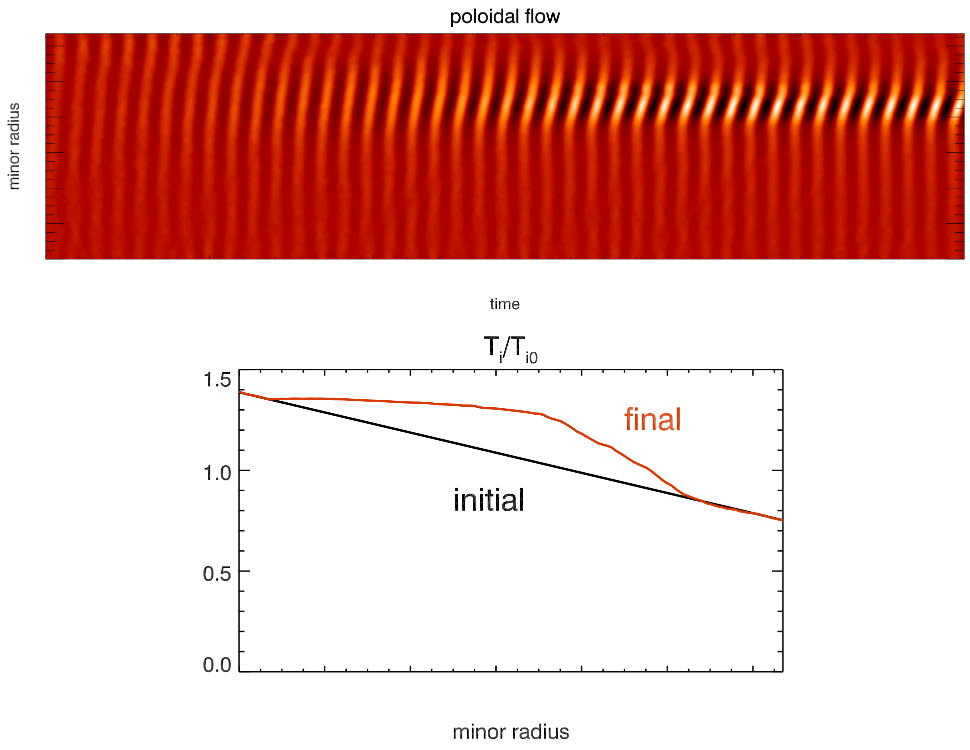


Figure 3: Strong GAM excitation in nonlocal scenario (top), concomittant steepening of ion temperature profile (bottom).

characteristic radial phase dependence results from the external excitation. Depending on the effectivity of the external perturbation it may cause high enough shearing rate to suppress the turbulence.

The effectivity of the external perturbation depends on the magnetic geometry and the pressure gradient, which controls the balance between the screening due to the plasma conductivity and the amplification due to the attraction of parallel currents.

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