

Magnetic geometry effects on geodesic acoustic modes

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Introduction

Radial propagation of geodesic acoustic modes (GAMs), and GAM eigenmodes have been discussed in recent literature [1, 2, 3]. The existence and properties of global GAM eigenmodes, which are influenced by the GAM group velocity, might be relevant for the efficiency of GAM excitation [3, 4]. This, in turn, could affect turbulent transport due to the potential role of GAMs in nonlinear turbulence saturation [5].

In this context we have presented a flexible and robust method to estimate the radial group velocity of a GAM by comparing its energy flux to its total free energy in Ref. [4], however, restricted to up-down symmetric magnetic geometries. In Ref. [6], we generalized the method to also cover up-down asymmetric geometries, which – in contrast to symmetric configurations – can exhibit large group velocities of the order of the magnetic inhomogeneity drift and above. Additionally, the generalized approach allows of interesting conjectures on possible interrelations between the confinement and the group velocity of GAMs.

Group velocity of GAMs in arbitrary toroidal geometry

For simplicity, we discuss the effect of up-down asymmetry using a two-fluid framework in the limit of cold ions and infinite safety factor. The units are chosen such that the magnetic drift velocity equals unity at the outboard midplane. Density n , ion and electron temperature T_i and T_e , and electric potential perturbations ϕ are normalized to $\rho^* n_0$, $\rho^* T_{0,i/e}$, $\rho^* T_{0,e}/e$, respectively, where the subscript 0 indicates the corresponding background value and ρ^* is given by ρ_{se}/R_0 with the major torus radius at the outboard midplane R_0 , $c_{se} \equiv (T_{0,e}/m_i)^{1/2}$, and $\rho_{se} \equiv (m_i c_{se}) / (eB_0)$. The time scale is $t_0 \equiv R_0 / (2c_{se})$. The minor radius of a particular flux-surface is defined as r , which makes r a flux-surface label. Thus, k_r is the wavenumber and v_r the velocity with respect to the coordinate r .

Applying the method presented in Ref. [4], one can evaluate the time derivative of the flux-surface averaged free energy within this framework for general geometry [6] to

$$\partial_t \langle E \rangle = \left\langle -\nabla \cdot \left(\frac{v_d n^2}{2} \right) + \nabla \cdot \frac{(n \nabla \dot{n})}{B_{rel}^2} + \nabla \cdot \frac{(n \nabla \dot{\phi}_0)}{B_{rel}^2} \right\rangle, \quad (1)$$

where v_d is the sum of curvature and ∇B -drift, and $B_{rel} \equiv B/B_0$. The factor $1/B_{rel}$ appears because the polarization related terms implicitly contain a factor ρ_i^2 (normalized to

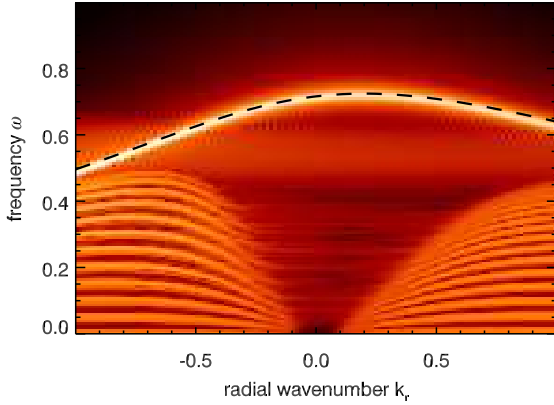


Figure 1: NLET computed log-color-coded GAM spectrum for $\tau = 0$, $q = 100$, and $s_z = 0.3$ with direct numerical solution of the GAM equations (dashed). The extremum of the GAM dispersion is shifted along the k_r -axis due to the up-down asymmetric magnetic geometry.

radial wavenumbers $k_r \ll 1$ can be approximated by

$$n^2 \approx \frac{4v_{d,r}^2 v_E^2}{\omega^2} \left(1 + \frac{2v_{d,r}}{\omega} k_r \right) \quad (2)$$

with the $E \times B$ -drift velocity v_E , the radial component of the magnetic inhomogeneity drift $v_{d,r}$, the major radius R and the GAM phase velocity v_p . Substituting (2) into (1), the leading terms of the first and the third term in Eq. (1) are of order $v_d E_{fluc}$, whereas the second term is only of order $k_r \rho_{se} v_d E_{fluc}$. In case of circular high aspect ratio flux-surfaces, the flux-surface averages of the leading order terms vanish and only terms of order $k_r \rho_{se} v_d E_{fluc}$ remain.

Coming back to our initial statement, the calculation proves on the one hand that higher GAM group velocities are possible, if the leading order parts of the magnetic inhomogeneity and the polarization energy-fluxes do not cancel each other. On the other hand, it shows that up-down asymmetry gives rise to a non-vanishing group velocity of GAMs at $k_r = 0$.

Circular flux-surfaces augmented with an r -dependent vertical shift $Z_0(r)$, i.e. $R(r, \theta) = R_0 + r \cos(\theta)$ and $Z(r, \theta) = Z_0(r) - r \sin(\theta)$, may serve as the most straightforward test of up-down asymmetric geometry in numerical studies. The Z -shifted geometry is the simplest modification of the circular equilibrium which shows the basic effects of up-down asymmetry while avoiding the complexity of force-free asymmetric configurations. Lacking complete consistency, it can be thought of as being maintained by a conductor inside the considered flux-surface.

We have studied the dependence of $v_g(k_r = 0)$ on the differential Z -shift $s_z \equiv \partial_r Z_0$ (which can

its value at the outboard midplane). The flux-surface average is defined by $A_0 \equiv \langle A \rangle \equiv (\oint B^{-1} dl_{\parallel})^{-1} \oint AB^{-1} dl_{\parallel}$, where dl_{\parallel} denotes the line element parallel to the magnetic field. With adiabatic electrons, $\phi = \phi_0 + n$ and $n_0 = 0$.

The first term on the right hand side of Eq. (1) is the flow of the energy of the electron pressure perturbations in the ion magnetic drift direction. The second and third term are polarization energy-fluxes, the latter of which is caused by the *flux-surface averaged* potential ϕ_0 , and vanishes for up-down symmetric magnetic geometries. A detailed discussion of the individual terms can be found in Refs. [4, 6]. Analogous to Ref. [4], the squared density perturbation for ra-

take values between -1 and 1) for cold ions and infinite safety factor using the two-fluid code NLET [7], the gyrokinetic codes GS2 [8] and GYRO [9], and direct numerical solutions of the GAM equation. A GAM spectrum computed with NLET, in which v_d is parallel to the Z -axis, for $s_z = 0.3$ is shown in Fig. 1. As an effect of the additional k_r -independent terms in the group velocity, the extremum of the GAM dispersion is shifted away from $k_r = 0$. The group velocity at $k_r = 0$ is not zero any more as conjectured in Ref. [4]. The group velocity at $k_r = 0$ turns out to be rather accurately linear in $(1 + |s_z|)/(1 - |s_z|)$, which implies $v_g(k_r = 0) \rightarrow \infty$ for $s_z \rightarrow 1$. However, the divergence is an artifact of the shifted-circle configuration which can be removed by a more sophisticated magnetic geometry [6].

An estimate of the GAM group velocity at $k_r = 0$

In Ref. [6], we derived an estimate of the group velocity of GAMs at $k_r = 0$ for warm ions in the infinite safety factor limit, which only depends on two geometry factors and the ratio of ion to electron temperature τ :

$$v_g(\tau) \approx \frac{(1 + \frac{5\tau}{3})^2 + \frac{10\tau^2}{9}}{(1 + \frac{5\tau}{3})} \langle v_{d,r}^3 \rangle_g + \frac{11\tau}{12} \left\langle \frac{\gamma_{rel}^2 v_{d,r}}{B_{rel}^2} \right\rangle_g - \left(1 + \frac{5\tau}{3}\right) \left\langle \frac{\gamma_{rel}^2 v_{d,r}}{B_{rel}^2} \right\rangle_g, \quad (3)$$

where $\langle \dots \rangle_g \equiv \langle \gamma_{rel}^2 / B_{rel}^2 \rangle^{-1} \langle \dots \rangle$. The radial component of the magnetic drift velocity is defined as $v_{d,r} \equiv \gamma_{rel} v_d \cdot \nabla \Psi / |\nabla \Psi|$. Ψ is the poloidal flux, $\gamma \equiv |\nabla \Psi|$, γ_0 is the value of γ at the outboard midplane, and $\gamma_{rel} \equiv \gamma / \gamma_0$. The first term on the right hand side of Eq. (3) corresponds to the first term in Eq. (1), the second term is a finite Larmor radius (FLR) correction to the first one. The third term corresponds to the polarization energy-flux $n \nabla \phi_0 / B_{rel}^2$ in Eq. (1).

Interestingly, in case of large aspect ratio and $q \gg 1$ the flux-surface average $\langle \gamma_{rel}^2 v_{d,r} / B_{rel}^2 \rangle_g$ can be reduced to the condition that the vertical magnetic forces on the central plasma current (i.e. the flux-surfaces) vanish [6]. Thus, in consistent plasma equilibria, $\langle \gamma_{rel}^2 v_{d,r} / B_{rel}^2 \rangle_g \approx 0$ implying that for $k_r = 0$ the energy of GAMs is transported essentially by the magnetic drift energy-flux $v_d E_{fluc}$. In case of vertical force-balance this result agrees with the conjecture in Ref. [4], that in single-null geometry $v_g(k_r = 0)$ has the sign of $v_{d,r}$ at the position opposite to the X-point. However, with low aspect ratio, when the variation of B across the flux-surface cannot be neglected any longer, the remaining two terms in Eq. (3) might be significant.

Figure 2 shows a comparison between the group velocities obtained with Eq. (3) for the shifted-circle geometry discussed above and the gyrokinetic codes GYRO and GS2. For the geometry factors in Eq. (3) one can estimate for large aspect ratio and small s_z , $v_{d,r} \approx -\sin(\theta)(1 + s_z \sin(\theta))$ and $\gamma_{rel} \approx (1 - s_z \sin(\theta))^{-1}$, such that to first order in s_z one obtains $\langle v_{d,r}^3 \rangle_g \approx -3s_z/4$ and $\langle \gamma_{rel}^2 v_{d,r} / B_{rel}^2 \rangle_g \approx -s_z$.

The leading order polarization energy-flux exceeds the energy flux due to the magnetic inhomogeneity drift in the cold ion case leading to a group velocity which is opposite to the magnetic inhomogeneity drift. When the ion temperature is increased ($\tau \gtrsim 0.3$) gyroradius effects overcompensate this effect such that the group velocity changes sign. This behavior and the order of magnitude of v_g is quite well reproduced by our two-fluid approximation.

Conclusions

The two-fluid expression for the Poynting flux of GAMs in the cold ion and infinite safety factor limit with large aspect ratio circular flux-surfaces derived in Ref. [4] has been generalized to arbitrary toroidal geometries, for which the energy flux of the GAM can be of order $v_d E_{fluc}$ instead of $k_r \rho_{se} v_d E_{fluc}$ in the up-down symmetric case. For sufficiently high aspect ratio, the free energy of the GAM can be assumed to be transported mainly by the magnetic inhomogeneity drift. Thus, one could manipulate the direction and magnitude of the GAM group velocity by adequate plasma shaping, e.g. to examine whether the dependence of the H-mode-power-threshold on the magnetic drift direction is really directly related to the X-point itself or possibly rather to the geometry dependence of the GAM propagation direction.

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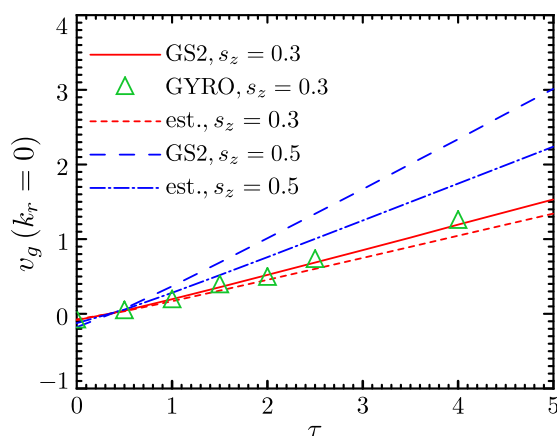


Figure 2: GAM group velocity – computed with GS2, GYRO and with the estimate (3) – at $k_r = 0$ for $s_z = 0.3$ and $s_z = 0.5$ vs. the ratio of ion to electron temperature τ . In contrast to NLET, GS2 and GYRO assume v_d to be in negative Z-direction, one has to compare the results with Eq. (3) for $s_z \rightarrow -s_z$.