A wavelet based method for detecting transient plasma waves and determining their spatial structure

G.I. Pokol[1], N. Lazanyi[1], G. Por[1], A. Magyarkuti[1], G. Papp[1], A. Gude[2], V. Igochine[2], M. Maraschek[2], ASDEX-Upgrade Team[2]

[1] *Institute of Nuclear Techniques, Budapest University of Technology and Economics, Association EURATOM-HAS, Budapest, Hungary*

[2] *Max-Planck-Institut für Plasmaphysik, EURATOM Association, Garching, Germany*

Introduction Continuous wavelet transform using analytical wavelets have been used in analysing transient signals from fusion devices for quite some time [1]. Instantaneous frequencies, amplitudes and mode numbers have been calculated by these methods with a reasonably good time resolution often based on only one or two signals [1, 2]. This paper presents wavelet based methods for detecting short-lived plasma eigenmodes and determining their spatial structure by synthesizing information from several signals.

Wavelet minimum coherence The wavelet minimum coherence method is the generalization of the ordinary wavelet coherence [3], and can safely detect low amplitude coherent modes at the expense of only a small loss in temporal resolution by synthetising information of all available signals. Wavelet coherence is calculated by the formula:

$$COH_{x,y}(u,\xi) = \frac{\left|\overline{Wx(u,\xi)Wy^*(u,\xi)}\right|}{\sqrt{\overline{Wx(u,\xi)Wx^*(u,\xi)}\,\overline{Wy(u,\xi)Wy^*(u,\xi)}}},$$

where, $Wx(u,\xi)$ and $Wy(u,\xi)$ are the continuous wavelet transforms of signals x and y using analytical wavelets (Morlet wavelet in the present paper), * denotes the complex conjugate, u and ξ are the time and frequency parameters respectively, and overbar denotes the averaging implemented by convolution smoothing with an affine invariant rectangular kernel. This way, invariance properties of the continuous wavelet transform – essential for processing transient signals – are preserved in wavelet coherence [4].

Spatially extended coherent structures in the plasma are in most cases picked up by several probes. Coherence can be calculated for each pair, and coherence information can be combined by calculating the wavelet average coherence or the wavelet minimum coherence. Comparison of the two coherence estimates on simulated signals (sin function plus independent additive white noise with varying strength) is shown on Figure 1.

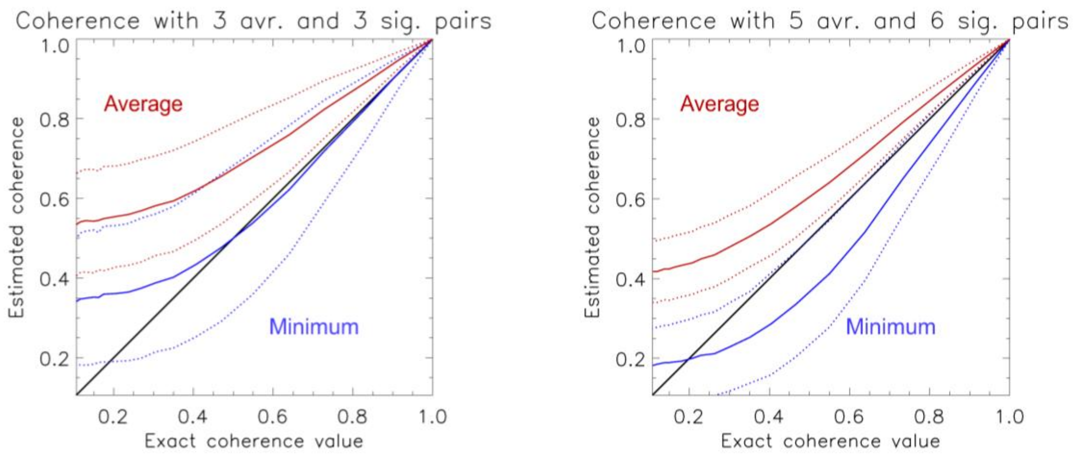

Figure 1 Expected values and standard deviations of minimum and average coherence estimates from 3 and 6 signal pairs using averaging of 3 and 5 measurements respectively.

Figure 1 shows that taking the minimum of 3 coherence values calculated by averaging 3 measurements approximates the real coherence value quite well, whereas for higher number of averaged measurements and higher number of signal pairs, the minimum coherence curve decays steeper than the exact coherence.

Mode number determination The mode number determination method presented in this paper is based on the phase of the continuous analytical wavelet transform:

$$\Theta_{x,y}(u,\xi) = \arg\left(\overline{Wx(u,\xi)Wy^*(u,\xi)}\right)$$

For each (u,ξ) point of the time-frequency plane, $\Theta_{x,y}$ relative phases between all pairs of signals are calculated. For a pure sinusoidal structure, these relative phases would lie on a straight line as a function of the $\varphi_{x,y}$ relative probe position.

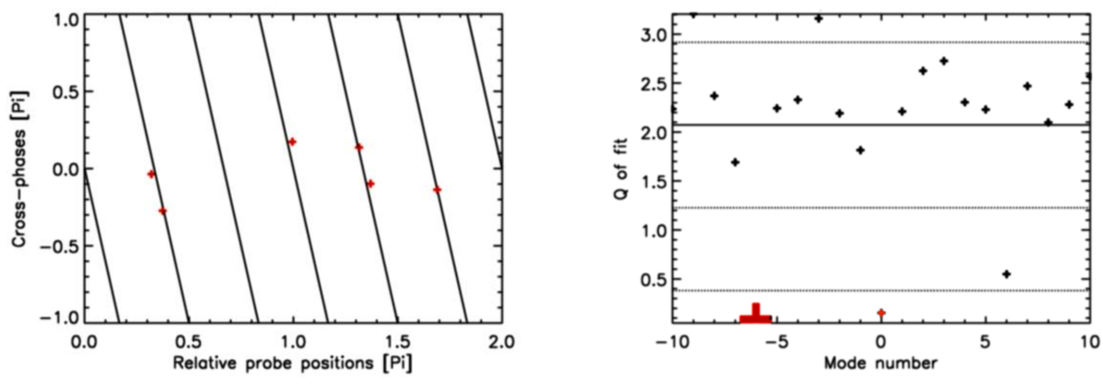

Figure 2 Example of best fitting line of n=-6 and Q_n using 4 magnetic pick-up coils at the (2.024 s, 150 kHz) time-frequency point of discharge #20040 [5].

The method has been tested for toroidal mode numbers, but with the right choice of $\varphi_{x,y}$ (i.e. use a straight field line coordinate system and given radial location) it can be adapted to poloidal mode numbers. The slope of the best fitting straight line gives the (*n*) mode number with the residual defined as:

$$Q_n(u,\xi) = \sum_{x,y} \left\| \Theta_{x,y}(u,\xi) - n\varphi_{x,y} \right\|_{2\pi}^2 ,$$

where $\|\cdot\|_{2\pi}$ is the norm by taking the optimum shift of $\Theta_{x,y}$ by $z2\pi$, $z \in \mathbf{Z}$.

Smoothing in this method is optional, but largely improves the accuracy, and also enables the calculation of wavelet minimum coherence [4].

This method gives a most fitting mode number for each point on the time-frequency plane. However, mode numbers are a relevant quantity only in limited regions, where coherent modes exist. We can find these regions based on a criterion for the $\min_n Q_n(u,\xi)$ values, or on wm-coherence, or on the combination of both, as in this paper.

Application to edge modes Methods presented above were applied to an ASDEX-Upgrade toroidal magnetic pick-up coil array with the aim of determining the toroidal structure of transient edge modes in the proximity of pellet injections and ELMs [5].

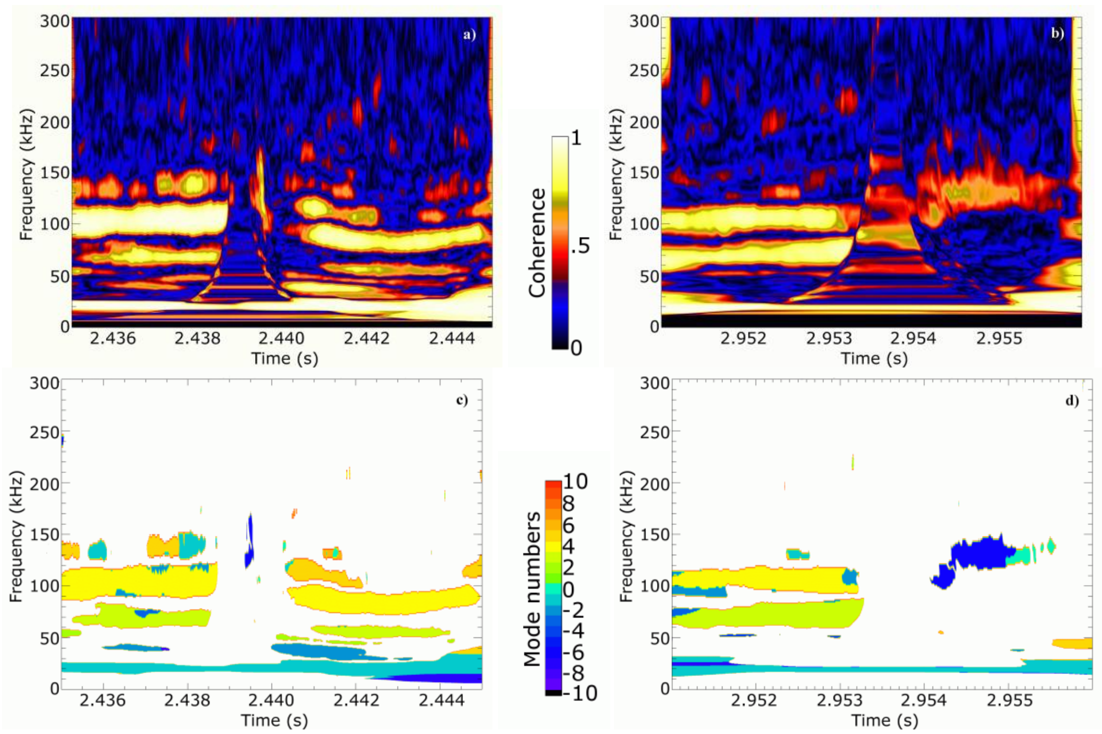

Figure 3 Wavelet minimum coherence (a, b) and toroidal mode numbers (c, d) for the spontaneous (a, c) and pellet triggered (b, d) ELM case in discharge #20040 [5].

Figure 3 shows wavelet minimum coherence and mode numbers for a spontaneous and a pellet triggered ELM. Analysis was performed using 4 coils giving 6 signal pairs, and a smoothing of 5 consecutive measurements. Mode structure during the ELM was not resolved and no global coherence was detected, due to rapid changes and limited spatial coherence, but modes just before and after the ELM are nicely shown. It has been concluded that – besides the washboard-like modes with positive mode numbers and the central n=1 mode – a mode (likely TAE) appears just after the ELM and during pellet ablation with *n*=-6 mode number, which corresponds to rotation in the ion diamagnetic drift direction [5]. The $\pm 6z$, $z \in \mathbf{Z}$ ambiguity of the mode numbers, also illustrated on Figure 2., is due to the approximate 60° symmetry of coil positions.

Application to core modes The method was applied to ASDEX-Upgrade SXR signals [6] with the purpose of studying toroidal mode numbers of sawtooth precursor modes appearing besides the basic (1,1) mode [7]. Toroidal mode numbers were determined using two SXR cameras (F and G) placed 135° apart toroidaly but having the same lines of sight in the poloidal cross-section [6]. The 3 central channel pairs inside the sawtooth inversion radius were used for mode number determination with the averaging of only 3 consecutive measurements.

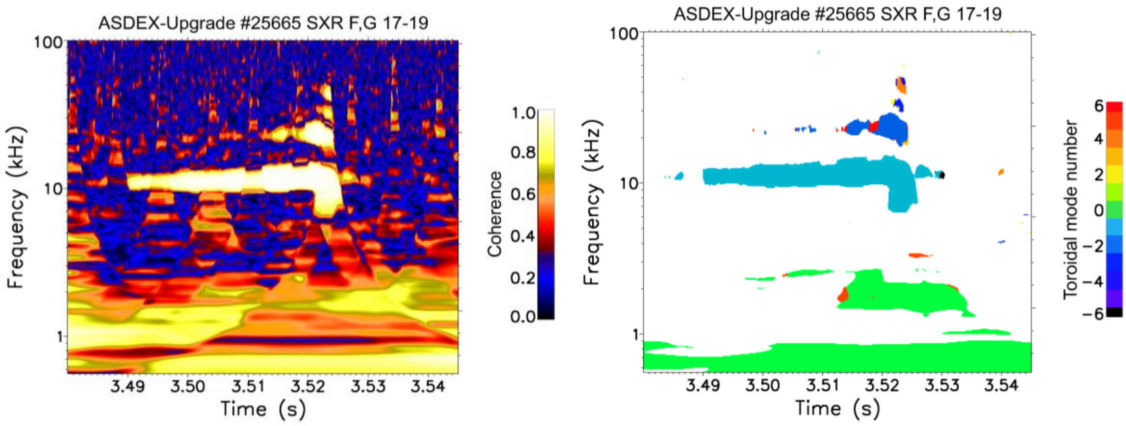

Figure 4 Wavelet minimum coherence and toroidal mode numbers for sawtooth precursors in discharge #25665.

Besides the *n*=-1 basic mode showing up strongly, Figure 4 shows precursors of a sawtooth crash of higher harmonics having toroidal mode numbers -2, -3 and -4 appearing just before the crash. A $\pm 8z$, $z \in \mathbf{Z}$ ambiguity of the mode numbers can be observed, – most prominently at the mode with *n*=4 on Figure 4 – which is due to the relative positions of the lines of sight being 3/8 of a full rotation. Similarly, mode number of the activity in the ~2 kHz region is shown to have *n*=0, but sometimes *n*=5 is indicated likely because 135° being nearly 2/5 of a full rotation.

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