Bayesian Inference applied to Magnetic Equilibrium on MAST

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Recently, a novel probabilistic data assimilation model based on Bayesian principles has been developed. The model is founded on the Bayesian analysis framework MINERVA[1], and incorporates the uncertainties and interdependencies of the diagnostic data and signal-forward functions to yield predictions of internal plasma state, including magnetic topology. Conventional approaches to equilibrium determination in tokamaks are based on least-squared fits of experimental data to the Grad-Shafranov equation[2] . Such methods cannot handle non-Gaussian probabilistic distributions of signals due for example to systematic signal bias. In the Bayesian approach to inference, the posterior distribution of free parameters, *H*, given a set of measurements D, *P(H|D)*, is expressed in the following way:

$$P(H \mid D) = \frac{P(D \mid H)P(H)}{P(D)}$$

This relation, known as the Bayes' formula separates prior information, *P(H)*, the diagnostic model, *P(D|H)* and a normalization factor, *P(D)*. In many cases only the maximum posterior (MAP) solution and its variance are required for which the latter term is not required.

We build on results of current tomography[3] in the Mega Amp Spherical Tokamak (MAST). To date, these have combined measurements of discrete magnetic signals, flux loops, MSE measurements with a toroidal current beam model[4.] The effect of poloidal currents has been included by correcting the toroidal flux using the Grad-Shafranov equation as well as the pressure profile inferred from Thomson scattering data[5]. In this work we add the diamagnetic loop signal into MINERVA, and resolve the toroidal flux function within MINERVA, thus representing poloidal currents. We demonstrate that using these measurements alone enables inferences of the q-profile that are in reasonable agreement with those of EFIT++[2] where only solutions of the Grad-Shafranov equation are admitted.

The poloidal fields are modelled by poloidal field coils and a set of toroidal current beams distributed throughout the vacuum. Each beam is rectangular and carries a uniform current. The toroidal field is generated primarily by a vertical current passing down the central axis of the machine. This so-called vacuum toroidal field is supplemented within the plasma separatrix by a plasma-induced component. In this study the functional form $f(\psi_p)=RB_\varphi$, is used. This satisfies $\nabla\cdot J=0$ and $\nabla\cdot B=0$, but disallows radial currents that may arise due to the flow of energetic particles. The function $f(\psi_p)$ is represented by linear interpolation of a set of free parameters {f_i} distributed uniformally in poloidal flux between the magnetic axis and separatrix. Regularisation of the toroidal current beams and of {f_i} is carried out using a Conditional AutoRegressive (CAR) prior[3] which controls the variation of adjacent parameters via a hyper-parameter.

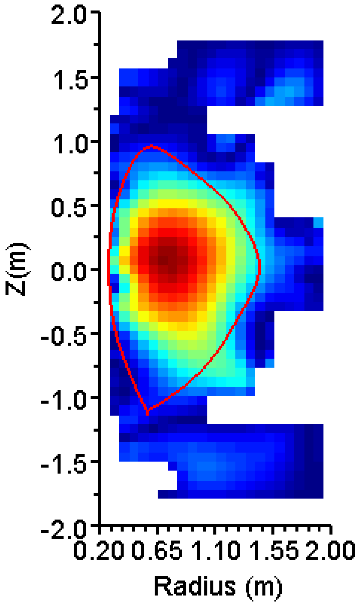

Figure 1. MAP toroidal current distribution; the location of the separatrix is drawn in red,

Figure 1 shows the MAP toroidal current distribution using the complete set of pickup coils and flux loops. Poloidal currents are not included in the inference. The results indicate the presence of currents flowing outside the separatrix region. Although permissible in the model this is not an expected result and it is instructive to establish if the result could be due to rogue signals. A measure of the information provided by each detector can be determined from the change in entropy of the system when a single detector is removed defined as[6]: $I=\int P_1 \log_2(P_1/P_2)dX$ in which P(X) is the posterior distribution, with suffices indicating evaluation with or without a particular detector and X denoting the free parameters. With this definition, the information of a rogue signal would be expected to be higher as it will not be corroborated by other signals. The information parameter I has been calculated for magnetic detectors and flux loops using a series of graphical models constructed without MSE and diamagnetic loop data. Ranked into order of decreasing magnitude reveals that (a) 10 detectors have significantly inflated values of I and (b) the quantity -log(I) varies linearly with detector index; variation for detector index<10 is six times greater than for higher detector indices. Use of this information measure can provide a regularisation termed Tychonoff Cross Validation[7] by which signals with the highest values of I are successively removed. Figure 3 plots flux surfaces and the toroidal current distribution where the 10 detectors with largest I are excluded. The "anomalous" currents have now disappeared, confirming the supposition that the previous result was generated by a few detectors. In this case detailed examination of the signals reveals that there are systematic errors in these detectors. In cases where systematic errors cannot be identified, the result would imply the need to increase the density of measurements locally.

Information about the poloidal currents arises from two sources: from MSE measurements and from the diamagnetic loop signal. The former provides a relational constraint between field components. For an MSE signal arising from point-wise interaction with the neutral beam, the relation on MAST is $\tan\gamma = a_1 B_z / \left(a_2 B_r + a_3 B_\varphi\right)$ where a_1, a_2, a_3, are geometric coefficients. The measurements are a non-linear function of the field components, resulting in a non-trivial relation between poloidal and toroidal currents. The diamagnetic loop signal is the total integrated plasma-induced toroidal field and can be considered to provide a scaling of any features in the poloidal current profile implied by the MSE signals.

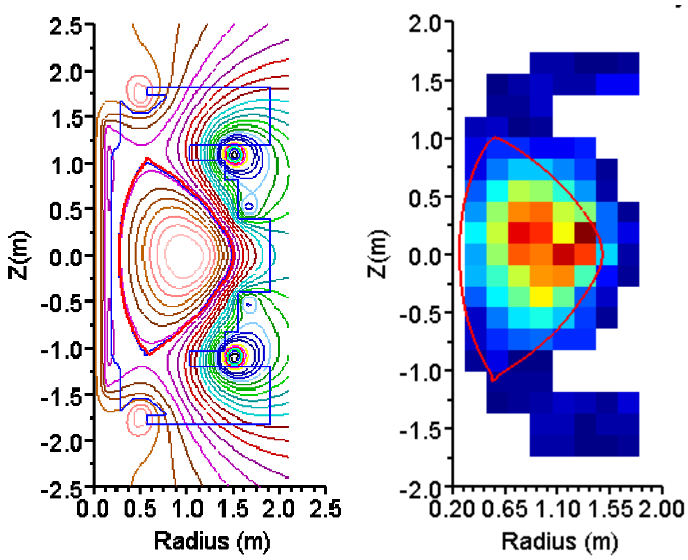

Figure 3: Map solution of flux surfaces (left) and toroidal current distribution (right). The separatrix from the EFIT++ solution is shown in red on the left plot.

The MAP solution is obtained using an optimisation based on an algorithm of Hooke and Jeeves. We use an uncorrelated Gaussian estimate of the posterior centred at the MAP: $[\sigma^2]_{ii} = [(\nabla\nabla L)^{-1}]_{ii}$ in which $\nabla\nabla L$ is the Hessian matrix of the log of the posterior distribution. Figure 4 shows the q-profile plotted as a function of poloidal flux for 100 solutions sampled around the MAP. The results are in reasonable agreement with the EFIT++ solution plotted on the same figure. Specific structural features may be connected with details of the MSE model. Figure 5 shows the separatrix location using the same posterior sample set. The EFIT++ result lies outside the scatter range particularly at the outboard mid-plane and towards the x-points. This is explained by two factors: the calculation of covariances may be conservative, and systematic errors in the data have been neglected. Both issues are currently being addressed with the use of MCMC sampling and with the adoption of non-Gaussian distribution functions.

In conclusion, we have built on existing work to demonstrate the use of Bayesian techniques to infer magnetic equilibrium on MAST. Experimental data has been utilised from magnetic probes, flux loops, MSE and a diamagnetic loop to model both poloidal and toroidal currents. We have described a statistical information measure which can provide automatic identification of rogue detectors. The approach could be extended to identify locations of critical diagnostic coverage.

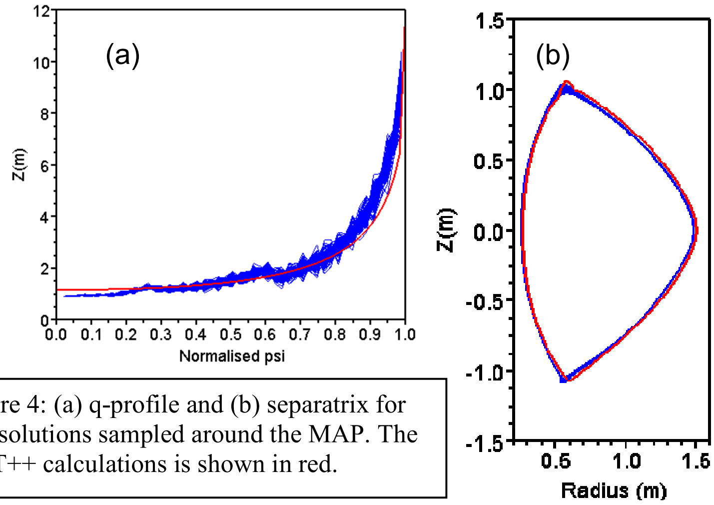

Figure 4: (a) q-profile and (b) separatrix for 100 solutions sampled around the MAP. The EFIT++ calculations is shown in red.

References

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