*36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-4.123 (2009)*

Stationary transport states with zonal flows in self-consistent 3-D drift wave turbulence simulations

A. Kammel, K. Hallatschek

Max-Planck-Institute for Plasma Physics, 85748 Garching, Germany

Introduction

For the first time, transport bifurcations have been found examining resistive drift wave turbulence in self-consistent simulations. The transport states are associated with asymmetric zonal flows (the negative ones being sharper and deeper), which gain energy from the turbulence and reduce the turbulence level in return.

Resistive (and also collisionless) drift wave turbulence is potentially relevant at internal transport barriers and to high gradient tokamak edge turbulence – and possibly to the drift wave analogon in planetary turbulence, geostrophic modes.

Numerical simulations

Using the Braginskii-based two-fluid code NLET[3], a turbulent sheared-slab cold-ion resistive drift-wave system consisting of the following Hasegawa-Wakatani equations has been examined:

$$d_t n = d_t \nabla_\perp^2 \phi \tag{1}$$

$$\hat{\rho}_s^{-3} d_t \nabla_\perp^2 \phi = -\partial_\parallel^2 (\phi - n) \tag{2}$$

where $d_t = \partial_t + \vec{z} \times \nabla_\perp \phi \cdot \nabla_\perp$, $\partial_\parallel = \partial_z - 2\pi s x \partial_y$ and $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ as well as $L_s = 1/2\pi s$ and $L_z (= 2\pi q R)$ as the parallel length scale.

Here, $\hat{\rho}_s = \rho_s / L_\perp$ - the single relevant parameter for the Hasegawa-Wakatani equations - is the dimensionless ratio of the 'ion sound Larmor radius' $\rho_s = m v_{th}/eB = m\sqrt{T_e}/eBm_i$ to the orthogonal length scale $L_\perp = R/L_n \, (\pi q/s)^{2/3} \left[c_s t_0 n e \eta_\parallel / 2B\right]^{1/3}$ (the scale of maximal drift wave growth where the relaxation frequency equals the diamagnetic drift frequency) where $\eta_\parallel$ marks the parallel resistivity and $L_n = -n(dx/dn)$.

Time is normalized to $t_0 = \rho_s / v_{dia,e^-}$ with $v_{dia,e^-} = \alpha(1+\eta_i)(1+\tau) t_0 L_\nabla / 2 L_0 L_n$.

Typical run parameters in the previously defined units are $n_x = n_y = 512$, $n_z = 32$, $L_x = L_y = 104.5\rho_s$, $L_z = 6.3qR$, grid step size $\approx 7.7 \cdot 10^{-3}$, time step $\approx 3.4 \cdot 10^{-4}$ and run time $\approx 8.8 \cdot 10^{1}$. Extensive consistency and convergence scans have been performed prior to the parameter scans for $\hat{\rho}_s^{-3}$.

Parameter studies

As is well-known, the linear properties of the flow states are best characterised by the eigenvalue of the unsheared system, since the sheared eigensystem cannot easily reproduce the development of the states. There is no feasible decomposition for this non-orthogonal, nearly collinear eigensystem, thus developing single eigenvectors on their own is rendered impossible. Strictly speaking, there are no growing eigenmodes for $s \neq 0$, thus the general growth rate of modes of the shearless, non-adiabatic case, derived from eqns. (1) & (2), is used

$$\gamma = \Im(\omega) \approx \left[k_\perp^2 + k_\parallel^2 \left(\frac{1}{k_\perp k_y} + \frac{k_\perp}{k_y} \right)^2 \right]^{-1} \quad (3)$$

which is approximated by $\gamma = \omega*^2/\omega_\parallel = k_\perp^2 / \left(k_\parallel^2 / (\hat{\rho}_s^{-3} k_\perp^2) \right) = \hat{\rho}_s^{-3} k_\perp^4 / k_\parallel^2$.

The mixing length anomalous heat diffusion coefficient $D = \gamma / \vec{k}_\perp^2$ depends on the orthogonal wavenumber, which is determined by one of two scales with a transition at approximately $\hat{\rho}_s \approx 0.12 - 0.20$ (coinciding with the onset of zonal flow formation). For the two regimes we find:

- relaxation scale $L_\perp$ dominant for $\hat{\rho}_s < 0.12$: $\hat{D} = \hat{\gamma}/\hat{k}_\perp^2|_{k_{phys}=L_\perp^{-1}} = \hat{\gamma}/\hat{k}_\perp^2|_{k_{units}=\hat{\rho}_s} \propto \hat{\rho}_s^{-1}$
- diam. drift scale ρ_s dominant for $\hat{\rho}_s > 0.2$: $D_\rho = \gamma_\rho / k_{\rho\perp}^2|_{k_{phys}=\rho_s^{-1}} = \gamma_\rho / k_{\rho\perp}^2|_{k_{units}=1} \propto \hat{\rho}_s^{-3}$

$$\Rightarrow \frac{D_\rho}{\hat{D}} = \hat{\rho}_s^{-2} \text{ (analytically)} \iff \frac{D_\rho}{\hat{D}} = \hat{\rho}_s^{-2\pm 0.1} \text{ (numerically)}$$

It has been verified thoroughly by a set of numerical parameter scans over $\hat{\rho}_s$ that D/D_ρ is asymptotically constant for small $\hat{\rho}_s$ and, vice versa, $D/\hat{D}$ for large $\hat{\rho}_s$.

Transport bifurcations

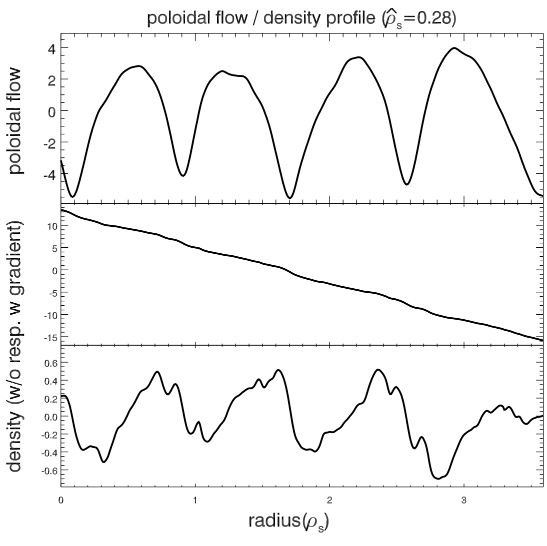

Figure 1: *Flow and density profiles*

To our knowledge, for the first time in self-consistent simulations, transport bifurcations containing two stable gradients have been found.

These density corrugations represent stationary transport states with regions of high diffusivity and low gradients at the location of the flows pointing in the electron diamagnetic drift direction (the positive flows) while low diffusivity and high gradients can be observed at the more sharply concentrated, radially tightened negative flows - the bifurcations are accompanied by an asymmetric flow pattern.

This flow structure emerges on time scales which are $\sim O(10^1)$ for a typical parameter $\hat{\rho}_s \approx 0.28$ (and gain approximately one order of magnitude for every doubling of $\hat{\rho}_s$) - this, in addition to a higher resolution, might indicate why they have not been observed in earlier studies[1].

Bifurcation mechanism

Using the drift wave action invariant N [2],

$$\partial_t N_{\vec{k}} = -\nabla_{\vec{x}} \left(N_{\vec{k}} \cdot \vec{v}_{gr,\vec{k}} \right) - \nabla_{\vec{k}} \left(N_{\vec{k}}(x) \cdot \dot{\vec{k}}(\vec{x},\vec{k}) \right) \tag{4}$$

negative flows are found to repulse the turbulence, while positive flows are attractive. (The flows can change the radial wavenumbers of the drift waves, thus acting like forcefields on the radially propagating drift waves.) Transport, in concurrence with turbulence levels, is thus reduced at the negative flows. Since the transport balance $\partial_x \Gamma(x) = 0$ is maintained in equilibrium, higher gradients at the location of the negative flows are required to counterbalance the transport which is reduced in concurrence with lower turbulence levels. Steeper gradients lead to an increased rate of drift wave generation, which are then repelled by the negative flows. These radially moving drift waves cause Reynolds stresses (via Poynting's theorem) which, in turn, fuel the flow up to its equilibrium level.

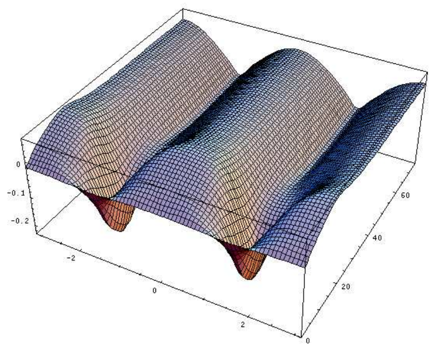

Figure 2: *Numerical solution of eqns (8) & (9) - flow vs. radius and time ($\gamma = 10^{-4}$, $\eta = 0.6$, $\alpha = 1.5$, $\beta = 2 \cdot 10^{-4}$)*

To study this mechanism analytically, an elementary nonlinear equation system (there cannot be a steady state with just the linear terms) has been constructed based on three balances:

$$\mu[N] = v_y + const. \quad \text{(negative flows correlate with higher drift wave intensity)} \tag{5}$$

$$\Gamma[N] = const. \quad \text{(transport balance in equilibrium)} \tag{6}$$

$$\dot{N} = \gamma[v_y, n'] \quad \text{(change in drift wave intensity acc. to the local growth rate)} \tag{7}$$

Empirical analysis shows that the occurence of density corrugations is independent from the development of the asymmetric flow structure, finally yielding the following coupled equation system for the drift wave intensity and the flow strength with various source and damping terms (where $\mu[N] = \eta N \left(\frac{\alpha-1}{\alpha} + \frac{N}{\alpha}\right)$ includes the stabilizing first nonlinear term for saturation):

$$\partial_t N = -\gamma(N - N_0) - \partial_x \left(-N \partial_x (\mu[N] - v_y)\right) - \beta \partial_x \left(N(\partial_x^3 N)\right) \tag{8}$$

$$\partial_t v_y = -\partial_x \left(-N \partial_x (\mu[N] - v_y)\right) - \beta \left(\partial_x^4 v_y\right) \tag{9}$$

The solution of these equations yields asymmetric flows. It increases with higher growth rate γ (as could be expected); other influencial parameters include a diffusion constant $D \neq 1$, and changing the relative amplitude of the nonlinear term. Thus, these quantitative considerations support the concept of the transport bifurcations found in the large-scale numerical simulations.

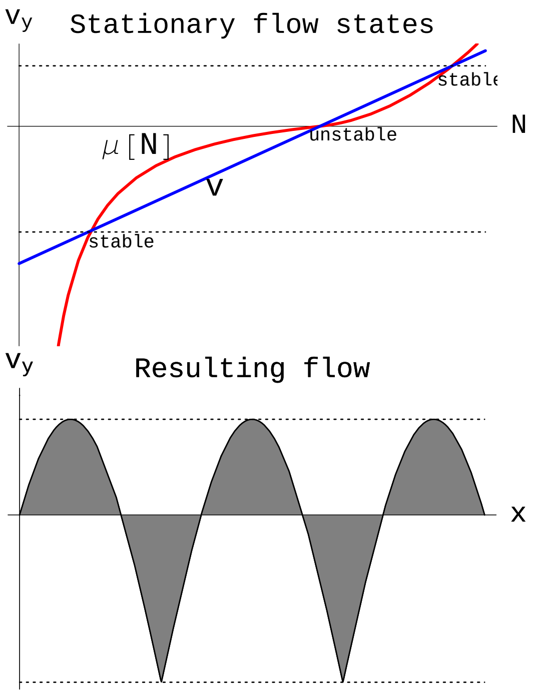

Figure 3: *Stationary flow states and corresponding asymmetric flow*

Plotting the balance between the momentum and drift wave flows given by the integrated Reynolds stress (Poynting's theorem, valid for $\gamma_{DW} = 0$), $v = N + const.$, against the nonlinear chemical potential $\mu[N]$ [Fig. 3] where $\mu \propto N + N^2$ for large N (determining the saturation of the positive flows) and $\mu \rightarrow -\infty$ for $N \rightarrow 0$ (in accordance with general behaviour of chemical potentials, preventing negative drift wave intensities) yields three intersections.

The central point marks the unstable $v = v_0$-state, with the two outer ones being stable – and corresponding to the developed flow states.

The intrinsic asymmetry in the chemical potential (strongly supported by NLET results) causes a corresponding asymmetry in the deviation of both the maximal positive and negative flows from the median – and thus, due to total flow conservation, an asymmetry of the radial flow length scale.

Summary

Our sheared slab drift wave turbulence runs yield the first example of transport bifurcations in self-consistent simulations.

These transport states and the associated flow asymmetry pose a robust phenomenon in a considerable parameter range and have been affirmed by means of an ansatz for an asymmetry mechanism as well as an analytical set of equations, derived from basic conservation principles.

References

[1] A. Zeiler, D. Biskamp, J.F. Drake and P.N. Guzdar, Physics of Plasmas **3**, 8, 2951-2960 (1996)

[2] K. Itoh, K. Hallatschek, S.-I. Itoh, P.H. Diamond and S. Toda, Physics of Plasmas **12**, 062303 (2005)

[3] K. Hallatschek and A. Zeiler, Physics of Plasmas **7**, 2554 (2000)