## Comparative Linear MHD Stability Analysis for ASDEX Upgrade and DIII-D Power Scan Studies

C. Konz<sup>1</sup>, P.B. Snyder<sup>2</sup>, L.D. Horton<sup>1</sup>, C.F. Maggi<sup>1</sup>, R. Groebner<sup>2</sup>, S. Günter<sup>1</sup>, P.J. Mc Carthy<sup>3</sup>, and the ASDEX Upgrade<sup>1</sup> and DIII-D Teams

<sup>1</sup>Max-Planck-Institute for Plasma Physics, 85748 Garching, Germany, EURATOM Association

<sup>2</sup>General Atomics, San Diego, CA 92186, USA

**Introduction:** The edge-localized modes (ELMs) which typically occur in the high confinement mode (H-mode) of tokamak plasmas are generally regarded as resulting from large-scale magneto-hydrodynamical (MHD) instabilities [1]. Within ideal MHD, instabilities associated with the edge transport barrier can either be driven by the edge pressure gradient (ballooning modes with high toroidal mode number n), a finite current density gradient in the edge (general peeling or edge kink modes with low n) or a finite current density at the plasma boundary (pure peeling modes with low n). These modes can couple to form intermediate-n peeling-ballooning modes [2] which may be responsible for type-I ELMs.

**Equilibrium Variations:** A thorough analysis of the linear MHD stability of the plasma edge of tokamak discharges typically involves the calculation of so-called j- $\alpha$ -diagrams, which show the location of the experimental reference equilibrium in a two-dimensional parameter scan of the edge pressure gradient (" $\alpha$ ") and the edge current density ("j") (e.g. [3, 4]). Until recently, the fixed boundary equilibrium code HELENA [5], used at AUG, only solved the Grad-Shafranov equation for axisymmetric equilibria for given input profiles  $p'(\psi)$  and  $FF'(\psi)$  or  $p'(\psi)$  and  $j_{tor}(\psi)$ , where  $\psi$  is the poloidal flux function.

A new version of the HELENA equilibrium code now allows for a self-consistent calculation of the equilibria for such j- $\alpha$ -diagrams by locally modifying the plasma edge while maintaining global plasma parameters like the plasma current  $I_p$  and the energy content  $W_{\rm MHD}$  of the plasma. To this end, it uses the pressure  $p(\rho_{\rm vol})$  and the toroidal current density  $j_{\rm tor}(\rho_{\rm vol})$  specified vs.  $\rho_{\rm vol} = \sqrt{V/2\pi^2R_{\rm geo}}$ . The radius  $\rho_{\rm vol}$  only mildly depends on the resulting equilibrium (unlike  $\psi$ ) and is therefore much better suited for equilibrium scans.

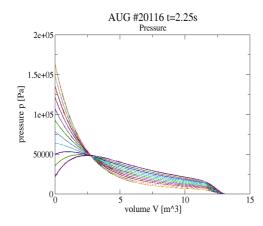
These profiles are modified through multiplication by the piecewise defined factors

$$f_{\rm p} = c_{p,1} \left( \Psi - \Psi_{\rm ped} \right)^2 H \left( \bar{\Psi}_{\rm ped} - \bar{\Psi} \right) + c_{p,2} \tag{1}$$

for the pressure profile  $p(\psi)$  and

$$f_{\rm j} = c_{j,1} \sqrt{|\psi - \psi_{\rm ped}|} H \left( \bar{\psi}_{\rm ped} - \bar{\psi} \right) + c_{j,2} \tag{2}$$

<sup>&</sup>lt;sup>3</sup>Department of Physics, University College Cork, Association EURATOM-DCU, Cork, Ireland



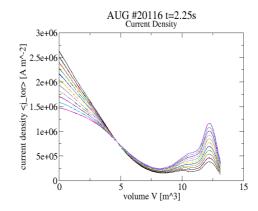


Figure 1: edge pressure variation

Figure 2: edge current variation

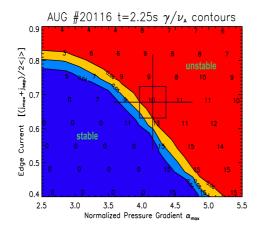
for the flux surface averaged toroidal current density profile  $j_{\text{tor}}(\psi)$ . Here  $H\left(\bar{\psi}_{\text{ped}} - \bar{\psi}\right)$  denotes the Heaviside function and  $\bar{\psi}_{\text{ped}}$  the normalized poloidal flux at the pedestal top. The factors  $c_{p,2}$  and  $c_{j,2}$  are direct scaling factors for the maximum edge pressure gradient and edge current density while  $c_{p,1}$  is determined by maintaining the energy content  $W_{\text{MHD}}$  of the plasma and  $c_{j,1}$  is determined by the total plasma current  $I_{\text{p}}$ .

With this extension, experimental equilibria can be consistently modified by varying the amplitude of the pedestal top pressure and peak edge current density within a margin of typically 50% to 150% of their reference values. Figs. 1 and 2 show the variation of the edge pressure gradient ( $\alpha$ -scan) and the edge current density (j-scan) vs. the plasma volume for an ASDEX Upgrade H-mode shot. Each of the visible curves consists of 11 variations of the respective other profile, i.e. of  $c_{j,2}$  for Fig. 1 and of  $c_{p,2}$  for Fig. 2, thereby demonstrating the possibility to vary either quantity independently of the other.

Stability Analysis: This method of varying experimental reference equilibria has been applied to a series of dedicated power scan experiments which were carried out at ASDEX Upgrade (AUG) and DIII-D to study the pedestal and core confinement dependence on the total plasma  $\beta_N$  in hybrid discharges [6].

For each of the experimental reference equilibria, a j- $\alpha$ -diagram is produced by varying the edge pressure gradient and edge current density in 10% steps from 50% to 150% of the reference values. The resulting 121 equilibria are then analyzed with the linear ideal MHD stability codes ILSA [7] (used in MISHKA mode [8]) and ELITE [2].

Figs. 3 and 4 show the stability diagrams for the low beta phase of an AUG improved H-mode power scan, calculated with ILSA (Fig. 3) and with ELITE (Fig. 4). The MHD stability is shown as a contour plot of the linear growth rate  $\gamma$  of the most unstable mode for each equi-



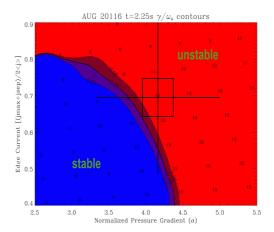


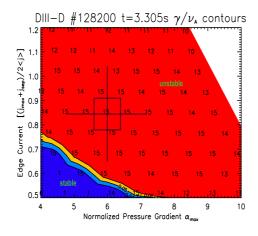
Figure 3: calculated with ILSA

Figure 4: calculated with ELITE

librium, normalized to the Alfvén frequency  $v_A = v_A/R$  with  $v_A = (B^2/\mu_0\rho)^{1/2}$  and B and R taken at the magnetic axis and the mass density  $\rho = \text{constant}$  in the simulations. The maximum pedestal pressure gradient  $\alpha_{\text{max}}$  is normalized as in [9]. The current densities are normalized in a somewhat arbitrary way as the sum of the maximum edge current density and the edge current density at the plasma boundary divided by twice the overall current density average. This normalization captures both the pure peeling term (finite surface current density) and the kink term (peaked current density in the edge) in the stability equations.

The toroidal mode number n of the most unstable mode for each equilibrium is noted on the contour plot, showing the transition from low-n peeling modes at high current density and low pressure gradient to intermediate-n peeling-ballooning modes at low current density and high pressure gradient. We define here the stability boundary of the j- $\alpha$ -diagram to be the region where the condition  $\gamma/\nu_A=0.04$ -0.06 is satisfied (marked by 3 black contours). This selection is intended to represent the level below which diamagnetic drift stabilization is dominant. A more detailed analysis including diamagnetic drift effects is planned for the near future. The experimental point is marked by a black box, with representative uncertainties of 20% both in current density and pressure gradient directions.

The experimental point of the low beta phase of the AUG improved H-mode shown in Fig. 3 lies very close to the calculated stability threshold. Thus good agreement is found between the peeling-ballooning model and experiment, insofar as the maximum pedestal pressure achieved prior to the type I ELM crash is predicted by the theory to be limited by ideal MHD instabilities. The stability diagrams shown here are the first consistent j- $\alpha$ -diagrams for AUG. Special care was taken at the construction of the reference equilibrium with the CLISTE free boundary code [10]. Comparison with the j- $\alpha$  diagram produced with the ELITE code (Fig. 4) shows very good



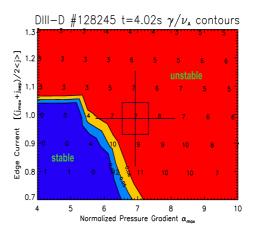


Figure 5: low  $\delta$ , "AUG shape",  $\beta_N = 2.5$ 

Figure 6: high  $\delta$ ,  $\beta_N = 2.6$ 

agreement in the neighbourhood of the reference point and only small differences in the high-n regime for low edge current densities (likely due to insufficient resolution in ILSA).

Figs. 5 and 6 show the stability diagrams for two DIII-D hybrid discharges at similar  $\beta_N$  but different plasma shape calculated with ILSA. Fig. 5 shows the stability diagram for a low triangularity (low  $\delta$ , "AUG shape") discharge, whereas Fig. 6 shows the equivalent diagram for a high triangularity discharge [6]. As expected, stronger shaping has a significant stabilizing influence on the plasma edge. The high  $\delta$  discharge exhibits a significantly larger stability region than the low  $\delta$  case. Also, a change of the most unstable mode to smaller toroidal mode numbers is found indicating a stronger relative importance of the edge current density in the high triangularity discharge. The reference point lies somewhat deep in the unstable region.

Overall, we find very good agreement of the predictions by the peeling-ballooning model with the experimental findings in terms of linear stability thresholds at AUG and DIII-D as well as very good agreement between the stability codes ILSA and ELITE.

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