

Validation of the beam tracing method for heating and diagnostics

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Abstract. The beam tracing approximate description of the propagation and absorption of wave beams is studied and compared to the corresponding exact solution of the wave equation for two simplified models relevant to electron cyclotron resonance heating and reflectometry diagnostics.

INTRODUCTION

The beam tracing method [1, 2] constitutes a well established tool for the description of the propagation of electron cyclotron (EC) wave beams in fusion plasmas. Its numerical implementation for EC wave, the code TORBEAM [3], has been fruitfully used in several different scenarios and benchmarked together with other major wave propagation codes under ITER relevant conditions [4].

Despite the rather advanced status of development of both theory and codes, in some scenarios, a further analysis of the validity of the beam tracing solution is required. More specifically, on one hand, the applicability of the beam tracing method for EC resonant heating and current drive in ITER has been questioned by Balakin et al [5]; on the other hand, the beam tracing method has attracted interest in the reflectometry community for a number of new applications [6] for which a precise study of the validity conditions of the method is lacking.

The criticisms of Balakin et al, in particular, deserve special consideration as they deal with the difficult problem of propagation near the EC resonance layer. Strictly speaking, in the beam tracing method one assumes that the wave is weakly damped and the medium is smooth, i.e., a scale length L can be identified such that (i) the gradients of plasma parameters are $\sim 1/L$ and (ii) $\kappa = \omega L/c \gg 1$ is a large parameter (here, ω is the fixed frequency of the beam and c the light speed in free space); such assumptions are typical for high frequency asymptotic analysis of wave equations, yet they are both violated near the EC resonance layer. Therefore, a mathematically precise approach to resonant wave propagation and absorption is a challenging task which, in our opinion, still waits for a convincing solution. On a heuristic level, the beam tracing method has been applied under such critical conditions on the basis of the fact that condition (i) and (ii) are violated only in a neighbourhood of the resonance layer where the wave is quickly absorbed and thus there is no need to resolve the details of the propagation. On the other hand, Balakin et al [5] have developed a novel approach to the problem based on a heuristic treatment of the wave equation and the numerical results based on

this method show significant differences with respect to the results of all other major EC codes [4]: specifically, a significant broadening of EC power deposition profiles is claimed, due to “aberration effects”.

Therefore, we have carried out an analytical study of the beam tracing solution aimed to validate the beam tracing method for the two above mentioned applications. As usual for such analyses, the strategy is the comparison of the beam tracing solution to the exact solution of some simplified models that, however, retain the relevant physical effects.

Here, we report shortly the main results of this study and we refer to our forthcoming article [7] for details. Neglecting polarization effects, we have considered the two-dimensional Helmholtz equation for the scalar field $u(x, y)$, i.e., $\Delta u + k_0^2 \varepsilon(x) u = 0$, with $k_0 = \omega/c$ and $\varepsilon(x)$ being the effective dielectric function. Only Gaussian beams are considered and the following two models for $\varepsilon(x)$:

(1) EC heating. We set $\varepsilon = 1 + i\gamma H(x)$ where $H(x)$ is the Heaviside step function; this represents an absorbing half-plane ($x \geq 0$) which models the resonance layer while refraction is neglected as $n^2 = 1$; such a profile violates both conditions (i) and (ii) above, and, since the wave is absorbed in a thin layer around the boundary $x = 0$, the finite width of the real resonance layer should make no significant difference; on the other hand, near the boundary, the discontinuous profile of γ amounts to energy losses due to reflection which are overestimated as the actual profile is expected to be smooth.

(2) Reflectometry. We study the classical linear layer model given by the real-valued dielectric function $\varepsilon(x) = n^2(x) = 1 - x/L$ where L is the typical scale length of the model (physically, the scale of the density profile); this model exhibits a cut-off at $x = L$ where the squared refractive index vanishes; near the cut-off the beam is reflected causing the beam trajectory (the reference ray) to curve abruptly: when the beam width is comparable to, or even larger than, the curvature radius of the trajectory the error of the beam tracing solution can be significant.

RESULTS FOR EC HEATING

Main results for the EC model (1) are reported in Fig. 1 for ITER relevant parameters (upper launcher setup [8]). The contour plot of the amplitude shows the propagation of the (focused) beam up to the boundary of the absorbing plane where most of the wave energy is dissipated in few wave lengths. The plot of the absolute error clearly shows the reflected wave which is two orders of magnitude less intense than the launched wave (thus it is not resolved by amplitude contours); that is the major source of error in the beam tracing solution.

Power deposition profiles are plotted (Fig. 1, right) for different values of the absorption coefficient γ . For low values of γ the deposition profiles have a long tail which is due to a deeper penetration of the beam into the half-plane; here, the beam tracing solution is superposed to the exact one within the resolution of the plot. Upon increasing the value of γ , the tail of the deposition profiles shortens and the profiles tend to be purely Gaussian: this happens when the beam has been completely absorbed in a thin layer around the illuminated portion of the boundary, and, thus, the deposition profiles follow the Gaussian beam profile projected onto the boundary. Upon increasing further on the absorption coefficient, the exact solution starts to deviate from the beam tracing

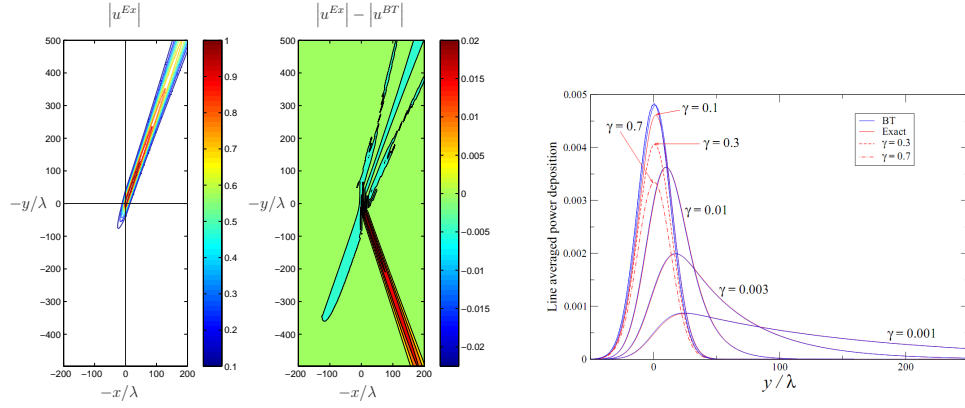


FIGURE 1. (Left) Amplitude contours of the exact solution $|u^{Ex}|$ and absolute error $|u^{Ex}| - |u^{BT}|$. (Right) Power deposition profiles versus γ . Beam parameters are chosen in the range relevant to the ITER upper launcher, i.e., frequency 170 GHz, beam width at the waist 1.48 cm, and injection angle 70° ; the value of the absorption coefficient that better represents the ITER case is $\gamma = 0.01$.

solution as a consequence of energy losses due to the reflected wave. Since the model is flawed by an unphysical discontinuity, the reflected power is overestimated, though it is very small for realistic values of the parameters ($\gamma = 0.01$ for the ITER case); analogous calculations with a linearly increasing absorption profiles, i.e., $\text{Im}(\epsilon) = \gamma x H(x)$, show that reflection can be safely neglected.

We can conclude that the beam tracing solution of the absorbing half-plane is accurate despite the violation of the two formal validity conditions addressed in the introduction. This also excludes the possibility of significant aberration effects due to spatial gradients and strong absorption and supports the implementation of TORBEAM [3]. The only remaining possible source of error for the beam tracing solution is the combination of strong absorption with spatial dispersion; on the other hand, such an effect is expected to be negligible in real conditions, yet further work is needed in this direction.

RESULTS FOR REFLECTOMETRY

The beam tracing solution of model (2) is found to resolve very well both the phase and the amplitude of the beam as far as the injection angle ϑ is not too close to the critical value $\vartheta = 90^\circ$; here, ϑ is defined as the angle of the group velocity at the launching position measured from the direction parallel to the cut-off (2D geometry): thus, $\vartheta = 90^\circ$ means that the beam is launched head-on into the cut-off layer. The main result of the study of model (2) for reflectometry applications is displayed in Fig. 2. One is basically interested in an estimate of the position in x and size of the caustic region where the wave field attains its maximum and, thus, where the beam is likely to interact significantly with the background plasma density fluctuations. In Fig. 2 the normalized position x/L and the normalized width w of the caustic are displayed as a function of the injection angle ϑ .

The red curves refer to the exact values while blue curves correspond to the beam

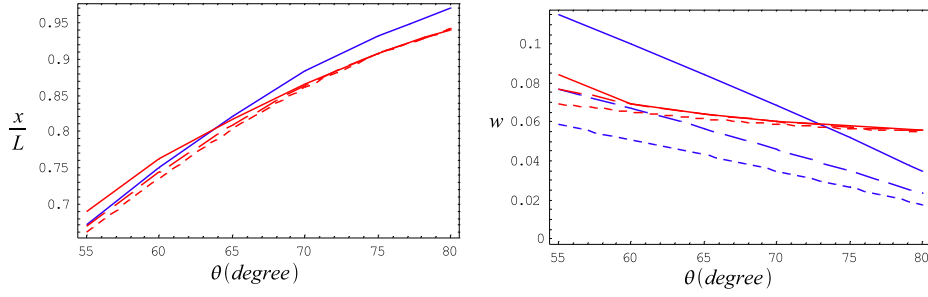


FIGURE 2. Caustic position and normalized width as a function of the injection angle ϑ .

tracing solution. Three different values of the initial beam width have been chosen, corresponding to a very narrow beam (solid line), an intermediate beam (coarse-dashed line), and a rather wide beam (fine-dashed line); the precise values of the width and other initial conditions are not reported here for brevity and can be found in our forthcoming paper [7]. Let us note that the beam tracing solution for the caustic position corresponds just to the location of the classical turning point and thus does not depend on the initial width (only one curve is shown); on the other hand the exact value depends weakly on the initial beam width and tends to a common value for $\vartheta \rightarrow 90^\circ$; the error of the beam tracing solution for the caustic position stays below few percent. As for the width, one can see that for both the narrow and the wide beam the error can be significant; for intermediate beam widths, however, the error remains below acceptable levels up to angles around $\vartheta \approx 65^\circ$: the validity conditions of the beam tracing method are strongly violated for steep injection angles resulting in a significant loss of accuracy. In this case, the beam width is larger than the radius of curvature of the reference ray (the beam trajectory) thus yielding the breakdown of the local coordinate system used in the construction of the beam tracing solution.

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