Three Dimensional Effects in Tokamaks

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Abstract

The paper deals with three dimensional effects in tokamaks that can be approximated by slowly varying 3d equilibria so that their theoretical description and corresponding code developments can benefit from stellarator research. We investigate the effect of magnetic field ripple and magnetic islands on fast particle orbits, and find a strong synergistic effect. – The coupling of the plasma perturbation to realistic tokamak wall structures introduces 3d elements into the treatment of Resistive Wall Modes (RWMs). We have developed and successfully benchmarked a RWM code allowing for 3d plasma equilibria and wall geometries. We show that for realistic wall structures the 3d effects remove the degeneracy of +/-n modes and can give rise to significant coupling of modes with different toroidal mode number n. – Kinetic damping of RWMs by plasma rotation is investigated in the low rotation regime relevant for ITER and DEMO. It is found that the inclusion of drift wave physics results in an increased damping for plasma rotation velocities in the range of the diamagnetic frequency due to the resonant excitation of electrostatic plasma waves.

1. Introduction

Although tokamak plasmas are usually considered to be axisymmetric, three dimensional effects are often important. The most obvious is due to the finite number of toroidal field coils and the resulting magnetic field ripple which gives rise to a diffusion of deeply trapped particles. In this paper we discuss several other 3d-effects in tokamaks, emphasizing, in particular situations where synergies have been possible with stellarator research in the development and use of computational tools. We restrict ourselves to situations, where the 3-d perturbations grow slowly ($\gamma << R/v_A$), so that the plasma passes through a sequence of ideal-MHD equilibrium states, with the time-dependence determined by the resistive decay of plasma or wall currents (excluding thereby Alfvén-type modes).

3d perturbations lead to losses of fast particles, generally not associated with resonances in velocity space, but similar to those in non-optimized stellarator equilibria. In this context we have investigated the influence of field perturbations like magnetic islands and magnetic field ripple onto the fast particle orbits. Even ideal-MHD unstable low-n modes can be reduced in growth rate by the presence of sufficiently close resistive walls so as to correspond to slowly developing equilibrium states. The consideration of realistic wall structures (with port-holes) in these cases leads, however to a coupling of different toroidal modes, effectively suggesting the use of stellarator codes for their analysis. We have therefore developed a fully 3d linear MHD stability code that deals with realistic structures of the resistive walls (including holes) and have applied it to the planned ITER and a modified ASDEX Upgrade wall geometry. We find that the 3d wall structures in general break also the degeneracy of +/-n modes in rigorous toroidal symmetry, and give rise to two eigenmodes with close, but distinct growth rates. In addition - in particular for the planned ASDEX Upgrade wall with its large holes - the coupling between modes with various n numbers becomes significant. These 3d effects, but also the possibility that more than one mode can become unstable in a given situation necessitate a feedback system capable of dealing with multiple modes. We have included such a feedback system into our code and have successfully demonstrated feedback stabilization of multiple modes.

Plasma rotation has a strong effect on the predicted mode growth rate, but the magnitude of the rotation and the critical field amplitude for mode locking depend themselves strongly on the damping of rotation by 3d perturbations. In addition, even at a given rotation speed the dispersion relation of the mode depends on kinetic effects. We have therefore utilized the truly (gyro-) kinetic stability code LIGKA - developed originally for fast particle driven instabilities - to the analysis of resistive wall mode damping.

2. Influence of 3d magnetic field perturbations on the orbits of fast ions

Magnetic islands not only influence the confinement properties of the thermal plasma, but also that of supra-thermal particles[1,2,3]. The particle orbits develop drift islands around surfaces where the unperturbed drift orbits are closed (rational q values for particle motion). For sufficiently large particle velocities, the drift islands deviate from the corresponding islands in the magnetic field structure. For sufficiently large island sizes, and in particular for trapped particles, these drift orbits can become stochastic [4].

With a new fast ion loss detector recently installed in ASDEX Upgrade, losses of fast ions caused by NTM-produced magnetic islands have been observed, both for well passing and trapped ions. It has been shown [5,6] that well passing ions can get lost on a surprisingly short time scale (order of a few micro seconds), not compatible with diffusive losses. Numerical simulations were able to explain these findings[7], showing that islands in the particle orbits caused by helical field perturbations can come close to or even hit the wall structures. In addition, particle losses on a slower time scale have been observed that could be explained by diffusion due to drift orbit stochastization, caused by both field ripple and magnetic islands.

Ripple losses of energetic particles are also a concern for ITER, but will be reduced by ferritic inserts that partially compensate the ripple. In addition to the magnetic field ripple, however, field perturbations caused by magnetic islands and perturbation fields applied for ELM mitigation might cause losses of fast ions. In order to investigate these effects we have followed fast particle orbits (NBI generated as well as α particles) in 3d ITER equilibria, investigating the (combined) effect of (2,1) magnetic islands and magnetic field ripple. Details of the generation of the 3d equilibrium magnetic field structure are given in [7]. The large distance of the ITER wall from the separatrix prevents the drift islands of purely parallel 1 MeV D-ions from reaching the wall structures. For deeply trapped particles, with turning points in the region where the field ripple is significant, we have found however, a strong synergetic effect of magnetic islands and field ripple (for the discussion in this paper we have not considered the reduction of the ripple by ferritic inserts). Fig. 1 shows a Poincaré plot of the particle orbits in the perturbed equilibrium. Although the separate effects of field ripple and magnetic islands are small, the combined effect can lead more easily to ergodic particle orbits and, consequently, an increased perpendicular transport of fast ions.

3. Three dimensional effects on Resistive Wall Modes (RWMs)

Advanced tokamak scenarios aim at steady state operation with a bootstrap current reaching up to 80 - 90% of the total plasma current. This requires a large normalized plasma pressure

($\beta = p/(B^2/2\mu_0)$), requiring a careful tailoring of the q-profile to avoid core localized instabilities. The ultimate limit to the normalized plasma pressure is then given by the onset of external kink modes. The growth time of these ideal plasma modes (a few hundred microseconds) can be slowed down significantly (to a few milliseconds) by the presence of conducting walls close to the plasma. The interaction between the plasma and the conducting wall structures, however, introduces additional 3d effects, as the walls will inevitably have holes for heating and diagnostic access. To deal with these effects, we have developed - based on the linear stellarator stability code CAS3D[8] - the STARWALL code[9], which calculates the growth rates of RWMs for walls with arbitrary shape and conductivity in the thin wall approximation. Due to the small growth rate of RWMs, inertia does not play any role for these instabilities and is therefore neglected. The STARWALL code does not need to assume the plasma to be axisymmetric, and it can - in contrast to other attempts to include realistic 3d wall structures - allow for the coupling of modes with different toroidal mode numbers n introduced by the wall structures and the feedback coil system.
The effect of active feedback has been studied using the OPTIM code[10,11], which takes into account all the unstable, and a large set of relevant stable modes found by STARWALL. Like this it treats not only the effect of the feedback system on the unstable modes, but allows also for the changes in the mode structure induced by it and even for the possibility that new modes are driven unstable.

A variety of linear stability codes has been developed to investigate growth rates of RWMs. To test these codes, a benchmark case was defined, based on the ITER steady state scenario (scenario 4)[12]. As most of the codes are restricted to exact axisymmetry, toroidally symmetric walls were considered for the benchmark exercise. The results of the benchmark calculations are given in Fig. 2 for β_N values below the ideal wall limit. We have added to this benchmark suite the results of the CASTOR-FLOW[13] and the STARWALL codes. Both codes agree very well with the results given by MARS[14], VALEN[15] and KINX[16]. To reduce the effort, we have restricted most of the STARWALL calculations to the consideration of only the inner ITER wall. This is well justified, except for very small growth rates, as the perturbed flux does not easily penetrate the first wall.

For the ITER scenario 4 equilibrium, the main concern is the n=1 RWM, as modes with toroidal mode numbers larger than 2 are stable up to much higher β_N values ($\beta_N<3.4$) which is due to the special shape of the pressure profile. This situation might, however, not be realistic for true steady state scenarios, as for the particular pressure profile the bootstrap current and the total current density are not well aligned. To improve this alignment, the region of large pressure gradients has to be shifted outwards what might lead to a destabilization of n=2, and even n=3 modes. Such a situation would be much more demanding for a feedback system as it would have to deal with several unstable modes simultaneously. In order to investigate if the latter situation could be handled under realistic conditions, we have generated an equilibrium[17] for which all modes with n≤3 have to be taken into account. Without conducting walls, this equilibrium is linearly unstable against n=1 (β_N >2.2), n=2 (β_N >2.4), and n=3 (β_N >2.4) modes. An ideal wall at a distance of the inner ITER wall would increase these β_N values to 3.65, 2.65, 2.8 for n=1, n=2 and n=3, respectively. Note that the quite large distance between the plasma boundary and the ITER walls does not allow for a significant stabilizing effect onto the n>2 modes and thus limits the achievement of high β_N values for this particular model equilibrium. For walls much closer to the plasma, like envisaged in corresponding DEMO design studies, the achievable β_N values might be significantly larger.

A corresponding equilibrium has been generated as well for ASDEX Upgrade[17] with the total pressure scaled to achieve similar values of β_N in both machines. For ASDEX Upgrade we use only the additional wall structures that are planned to be inserted for RWM investigations[18], as the distance between the plasma and the present wall is too far to have a significant effect on stability.Table 1 gives the growth rates of the most unstable modes both for an (idealized) toroidally symmetric wall as well as for the realistic 3d structures. The violation of axisymmetry removes the original degeneracy of +/- n modes, resulting in two modes with different phasing to the wall structures and slightly different growth rates. As discussed above, the coupling to 3d wall structures results also in a coupling of modes with different toroidal mode numbers already in a linear calculation. This effect is quite prominent for the n=1 and n=2 modes in the case of the ASDEX Upgrade wall (Fig. 3, Fig. 4). The eigenfunction in Fig. 4 exhibits pronounced spikes at the rational surfaces of the respective Fourier harmonics. These spikes are a result of the neglect of plasma inertia in the STARWALL code. Although this approximation is well satisfied considering the very slow growth of the modes, it allows for a strong acceleration of the plasma - otherwise limited by inertia - within the resonant surfaces associated with a finite radial displacement. This effect necessitates a very high spatial resolution at the rational surfaces, but does not significantly influence the growth rates, as demonstrated in the careful benchmarking efforts discussed above.

The simultaneous occurrence of several modes is highly demanding for an effective feedback system. To demonstrate that such a situation can, however, in principle be dealt with by an optimized feedback system, we have applied the OPTIM code to the most demanding case of four unstable modes in ASDEX Upgrade (Table 1). Using a single toroidal array of 8 equidistantly placed sensors and the feedback coil system discussed in[18] we were able to demonstrate that the feedback coil system as designed is able to stabilize all modes simultaneously. The details of the applied feedback algorithm are described in [10].

4. Rotation damping of RWMs

As RWMs are unstable only if perturbed magnetic flux is able to penetrate the wall, they cannot grow while rotating with respect to the wall with a frequency significantly larger than the inverse resistive wall time.. On the other hand - as the plasma usually rotates much faster - perturbations nearly frozen into the wall frame will be subject to a number of damping mechanisms in the plasma. As the magnetic perturbation and the plasma rotation happen on time scales slow compared to the Alfvén time, the driving MHD part can, in principle, again be considered as a 3d perturbation of the equilibrium. Direct damping of the perturbation in the plasma can then occur by collisional dissipative effects, or by resonances with particle motions. This kind of damping can be well analyzed by a perturbative approach, neglecting its effect onto the structure of the MHD perturbation[19,20]. The motion of the MHD perturbation enforced by its near-freezing to the wall frame can, however, also excite plasma waves through continuum[21,22,23] or discrete resonances, and extract energy in this way.

An instrument suited to study all these effects is the gyrokinetic linear code LIGKA[24]. Although originally developed to study fast particle driven instabilities, it has recently been extended towards low frequency modes[25] so that it can now treat rigorously the coupling of the shear Alfvén waves to drift and sound waves. As it does, in its present form, not include a vacuum region or plasma rotation we use it in its antenna version[25], imposing the rotating MHD perturbation via boundary conditions (or a virtual antenna), mimicking solutions obtained with the free-boundary CASTOR-FLOW code. Since LIGKA is based on a comprehensive physics model, it can give information on all kind of kinetic damping mechanisms, both acting directly via particle – MHD perturbation resonances, and those corresponding to resonant excitation of secondary waves. The inclusion of diamagnetic drift effects makes it well suited to investigate the low rotation regime of recent experiments with balanced NBI injection[26] which is most important as well for ITER and DEMO.

Resonant excitation of secondary modes with n=1 at low frequencies is possible at rational surfaces only where the phase velocity of the modes can equal the plasma rotation frequency. For the equilibrium discussed above, the continuum spectrum in this region and its modification by diamagnetic drift effects are shown in Fig. 5. The damping of a wave imposed by the antenna is given by the power absorbed in the plasma, which, for the low frequency range, is shown in Fig. 6. In addition to a number of known resonances at higher frequencies (not shown), we observe, in particular, a broad resonance region around the diamagnetic drift frequency. The mode structure at the frequency of maximum absorbed power is shown in Fig. 7. The inclusion of diamagnetic drifts obviously allows for the excitation of electrostatic plasma waves with frequencies below ω^*, localized at the rational surface. The excitation of these waves significantly increases the absorbed power over quite a broad frequency range. As the spectral width of this range is large compared to the typical inverse resistive wall time, this resonance and the concomitant damping of the MHD perturbation could have a significant influence on the damping of RWMs at low rotation frequencies.

5. Summary and Discussion

Our investigation of the combined effects of magnetic ripple and additional magnetic perturbations on fast ions shows a significant effect already for relatively small islands (in our case of a width of less than 5% of the plasma minor radius). Although the results presented do not take account of the reduction in ripple due to ferritic inserts, the synergetic loss effects, in particular in combination with the resonant perturbation fields envisaged for ELM mitigation should be assessed for the finally proposed configuration.

The prediction of growth rates of resistive wall modes and of the maximum β values achievable with active feedback requires the consideration of realistic wall structures. The STARWALL code, based on a stellarator code development, solves this problem in the most comprehensive way. Other codes, dealing with realistic 3d walls, either do not allow for a modification of the eigenfunction by wall structures and feedback system at all[15] or at least not for the coupling of modes with different toroidal mode numbers[27].

Damping of RWMs by plasma rotation has been investigated with the LIGKA code that is particularly well suited to investigate the kinetic damping effects either by direct wave-particle resonances or by the excitation of secondary plasma waves. It has been shown that the inclusion of diamagnetic effects into the analysis might significantly increase the damping rates in the range of low plasma rotation as relevant for ITER and DEMO, due to the possibility of excitation of electrostatic drift waves in the region close to resonant surfaces. This analysis so far has been limited to the investigation of the damping of an antenna produced perturbation. The most important restriction of this approach is the neglect of sheared plasma rotation, corresponding to different rotation speeds at the respective rational surfaces. Therefore we envisage in the near future the inclusion of a vacuum region and of plasma rotation into the LIGKA code.

Figure Captions

Fig. 1: Poincaré plot of characteristic guiding centre orbits of α particles (energy: 3.5 MeV, pitch angle: $\mathit{arccos}(v_{\|}/v_{\perp)}=70^\circ$) in a 2d equilibrium, but perturbed by a (2,1) magnetic island (left), in an equilibrium with field ripple only (middle) and with the combined perturbation by ripple and magnetic island (right).

Fig. 2: Results of a benchmark effort of various RWM stability codes[12]. The growth rates of an n=1 RWM are given vs. β_N in the region between the ideal and the no wall limit. In the left figure both ITER walls are considered. The results of our codes CASTOR_FLOW and STARWALL agree very well with previous results. The influence of the outer wall is very small, except for extremely small growth rates (right).

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Fig. 3: Contour plot of the current potential Φ ($\vec{j} = \vec{n} \times \nabla\Phi$) induced by an unstable RWM (normalized growth rate 5.88, see Table 1) on the wall in ASDEX Upgrade geometry. The vertical and horizontal axes represent the poloidal and toroidal angle, respectively. Note that the mode structure has a pronounced n=1 structure on the outer midplane ($\theta/2\pi=0.5$), but an obvious n=2 contribution on the upper and lower regions of the wall structures.

Fig. 4: Fourier harmonics of the radial displacement vs. normalised radius for the most unstable RWM in ASDEX Upgrade geometry.

Fig. 5: (a) Spectrum of continuum modes in the region of the rational surfaces for low frequencies. (b) Zoom in the frequency region of the diamagnetic drift frequency around the q=3 surface. Note that the inclusion of diamagnetic drift effects significantly alters the mode spectrum.

Fig. 6: Absorbed power vs. frequency with and without inclusion of diamagnetic drifts.

Fig.7: (a) Mode structure (electrostatic potential) at the frequency of maximum absorbed power in Fig. 6. (b) Zoom into the region of the excited the electrostatic mode.

Table 1: Normalised growth rates $\gamma\mu_0\sigma d\,(1/m)$ for toroidally symmetric walls and wall geometry of **Error! Reference source not found.** respectively. Note that the values given in

n	ITER 2D	ITER 3D	AUG 2D	AUG 3D
1	0.72	1.46	1.54	7.70
1	0.72	1.46	1.54	7.50
2	1.55	4.03	1.67	6.24
2	1.55	4.03	1.67	5.88

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