## Algorithms for the Control of NTM by Localized ECRH. Principles and Requirements of the Real Time Diagnostic and Control System

G. D'Antona<sup>1</sup>, S. Cirant<sup>2</sup>, D. Farina<sup>2</sup>, F. Gandini<sup>2</sup>, E. Lazzaro<sup>2</sup>, W. Treuterer<sup>3</sup> and A. Manini<sup>3</sup>

<sup>1</sup>Politecnico di Milano - Dip. di Elettrotecnica - P.za L. da Vinci,32 − 20133 Milano − Italy <sup>2</sup>Istituto di Fisica del Plasma − Ass. CNR-ENEA-EURATOM − Via Cozzi 53 − 20125 Milano − Italy 3Max Plack Institut für Plasmaphysik − Boltzmannstraβe 2 - D-85748 Garching, Germany

**Abstract.** The diagnostics requirements for the control of NTM instabilities is outlined stressing the importance of correctly managing the estimate uncertainty by the control system. A methodology for the Bayesian assimilation of model predictions and observations is outlined together with an example of application.

**Keywords:** Bayesian estimation, *NTM* control.

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## INTRODUCTION

In this paper the diagnostic requirements for the control strategy and the suppression of *NTM* instabilities in a tokamak will be outlined. The topic will be assessed as a decision making problem under risk and uncertainty. In its most essential formulation the decision problem can be stated as the way to maneuver the *ECRH* antenna angles, subject to some static and dynamic actuator constrains, given the uncertain knowledge concerning the amplitude and position of the *NTM* instabilities and about the locations of the *ECRH* power deposition in the plasma.

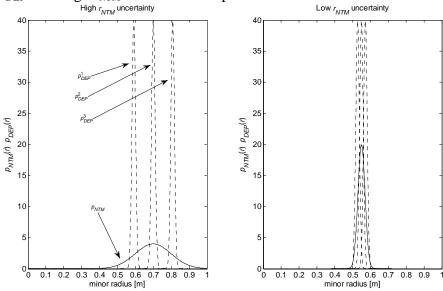
The actuator control variables in this case are the antenna steering angle  $\alpha$  for each *ECRH* line and the power modulation  $\mathbf{P}_{ECH}$  for each gyrotron. As aforementioned the information available for the control/decision task are the amplitude  $W_{NTM}$  and radial position  $r_{NTM}$  of the NTM instability and the deposition radiuses  $\mathbf{r}_{DEP}$  together with  $\mathbf{P}_{ECH}$  and  $\alpha$ . In the following we will suppose for simplicity that there is only one instability subject to control, but the approach outlined can be extended to the case of more instabilities.

If the information concerning  $W_{NTM}$ ,  $r_{NTM}$ ,  $r_{DEP}$ ,  $P_{ECH}$  and  $\alpha$  were perfectly known, i.e. with negligible uncertainty, together with a dynamical model  $M(\cdot)$  of the instability growth:

$$\begin{bmatrix} \dot{W}_{NTM}(t) \\ \dot{r}_{NTM}(t) \end{bmatrix} = M(W_{NTM}, r_{NTM}, \mathbf{r}_{DEP}, \mathbf{P}_{ECH}, \boldsymbol{\alpha}, t), \begin{bmatrix} W_{NTM}(t_0) \\ r_{NTM}(t_0) \end{bmatrix} = \begin{bmatrix} W_0 \\ r_0 \end{bmatrix}$$
(1)

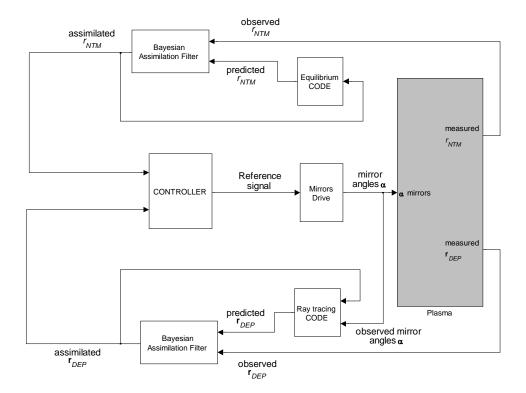
classical control theory would allow the design of an open loop control law with the objective of minimizing  $W_{NTM}$  given the actuators constraints. If the uncertainty of the model (1) was too large a feedback loop control strategy would be necessary. In this case the success of the control objective would be attained only if little uncertainty was affecting the knowledge of the quantities  $W_{NTM}$ ,  $r_{NTM}$ ,  $r_{DEP}$ ,  $\alpha$  and  $P_{ECH}$  (actually for NTM stabilization, since part of the control problem is tracking  $r_{NTM}$  by  $\mathbf{r}_{DEP}$ , it would be sufficient to have a low uncertainty on the tracking error  $\mathbf{e} = r_{NTM} - \mathbf{r}_{DEP}$  only, even with large uncertainties on  $r_{NTM}$  and  $\mathbf{r}_{DEP}$ ).

In practice the uncertainty affecting the knowledge of  $W_{NTM}$ ,  $r_{NTM}$ ,  $\mathbf{r}_{DEP}$  and  $\alpha$  is far to be negligible, and thus the likely of a wrong decision taken by the control system is large. For this reason it is mandatory to explicitly process together with the estimates of the quantities involved in the decision procedure also their uncertainties. For example figure 1 shows the instability tracking with two different levels of  $r_{NTM}$  uncertainty. When the uncertainty is high the ECRH deposition radiuses are far from each other and the ECRH power is switch off. When the  $r_{NTM}$  uncertainty drops the various  $\mathbf{r}_{DEP}$  converge  $r_{NTM}$  and the ECRH power is switched on.



**FIGURE 1**. Steering of  $\mathbf{r}_{DEP}$  for the case of high and low  $r_{NTM}$  uncertainty

Formally the information about all the quantities whose knowledge is affected by uncertainty will be described by random vector variables characterized by a proper joint probability density function (pdf). From this point of view there is no reason to discriminate among uncertain information coming from either direct measurements or physical models: it is always a proper pdf. When both these two sources of information are available their respective information must be assimilated together. For NTM stabilization we can count on the plasma equilibrium code for predicting  $r_{NTM}$  and on a ray tracing code for  $\mathbf{r}_{DEP}$ ; figure 1 shows the block diagram of the diagnostic and control system including two assimilation algorithms.



**FIGURE 2.** Block diagram of the *NTM* control system with two model-data assimilation filters for  $\mathbf{r}_{DEP}$  and  $\mathbf{r}_{NTM}$ 

## BAYESIAN MODEL-DATA ASSIMILATION PARADIGM

The assimilation algorithm is named Bayesian filter in figure 2 since it is based on the Bayes' formula:

$$p(r \mid \mathbf{d}) = \frac{L(\mathbf{d} \mid r) \cdot p(r)}{p(\mathbf{d})} \propto L(d \mid r) \cdot p(r)$$
 (2)

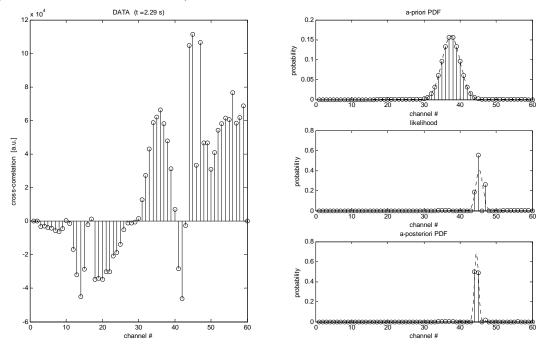
In (2) the likelihood function  $L(\mathbf{d}|r)$  is the stochastic model of the diagnostic system generating the data vector  $\mathbf{d}$ , the a-priori  $pdf \, p(r)$  is the stochastic model predicting the position r either of the NTM instability  $(r=r_{NTM})$  or of the ECRH deposition radius  $(r=r_{DEP})$ , while the a-posteriori  $pdf \, p(r!\mathbf{d})$  is the estimate of r given the observation data vector  $\mathbf{d}$ . The number  $p(\mathbf{d})$  at denominator of (2) is the evidence pdf representing the agreement among the observation vector  $\mathbf{d}$  and the a-priori model of the quantity r which have generated the data  $\mathbf{d}$ . The evidence can be evaluated as:

$$p(\mathbf{d}) = \int_{R} L(\mathbf{d} \mid r) p(r) dr$$
 (3)

The evidence (3) is also used by the control system since action is taken, i.e. *ECRH* power is switch on, only if there is a high agreement among physical models and observations.

Figure 3 shows an example of application of equations (2) and (3) for the estimation of  $r_{DEP}$  for one *ECRH* line using a rough time invariant a-priori pdf p(r) and the correlation among the *ECRH* power modulation and the *ECE* channels response as

the data vector **d** for shot No. 17107 in ASDEX Upgrade. Figure 4 shows the estimate (mean and standard deviation) and the its evidence versus time.



**FIGURE 3.**  $r_{DEP}$  estimate for shot No. 17107 in ASDEX Upgrade by PECH/ECE cross-correlation at time 2.89 s

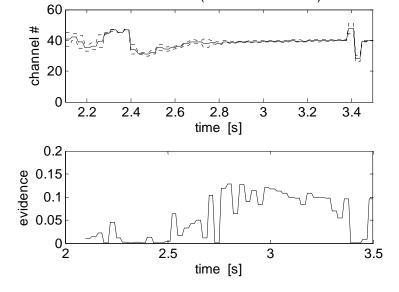


FIGURE 4.  $r_{DEP}$  estimate (mean and standard deviation) and its evidence for shot No. 17107 in ASDEX Upgrade by PECH/ECE cross-correlation

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