Simulations of the bootstrap current in small rotating magnetic islands

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Abstract

The bootstrap current in small magnetic islands of neoclassical tearing modes is studied in numerical simulations whith the guiding center particle code HAGIS. The contributions of both, electrons and ions, are included, as well as the island rotation and its electric field. The case of islands that are smaller than the ion banana orbit width is studied. We find that the size of the bootstrap current in small islands depends strongly on the rotation frequency of the island.

1 Introduction

Neoclassical tearing modes play an important role in the hot plasma of present large tokamak experiments and the planned International Thermonuclear Experimental Reactor (ITER), since they often limit the achievable pressure to a lower level than other instabilities [1]. The neoclassical tearing mode is a perturbation creating a helical magnetic island structure, which would be stabilized by the equilibrium current profile, but is driven instable by the loss of the bootstrap current inside the island, which corresponds to a current perturbation in the opposite direction driving the growth of the island [2]. The bootstrap current is lost by the flattening of the density and temperature profiles due to the strong transport along field lines. Here, we study the bootstrap current in small islands including the island rotation and the electric field. Both ions and electrons are treated kinetically including electron-ion collisions for determining the current.

2 The simulation model

The model consists of the unperturbed magnetic field $\mathbf{B} = \nabla\phi \times \nabla\psi + q(\psi)\nabla\psi \times \nabla\theta$ and a perturbation with single helicity corresponding to a rotating island, $\delta\psi = \hat{\psi}\cos(m\theta - n\phi - \omega t)$, with mode numbers m and n and mode frequency ω. Here ψ ist the poloidal flux, $q = 1 + 2\psi/\psi_{\rm edge}$, is the safety factor, and θ and ϕ are poloidal and toroidal angles. The perturbation creates an island of half-wdith $w_\psi = \sqrt{4\hat{\psi}q_s/q'_s}$ (half of the poloidal flux difference across the island), where $q_s = m/n$ is the value of q at the resonant surface and the prime denotes the derivative with respect to the poloidal flux. We introduce a helical angle $\xi = m\theta - n\phi - \omega t$, and a normalized helical flux $\Omega = 2(\psi - \psi_s)^2/w_\psi^2 - \cos\xi$, where the index s denotes the resonant surface. On the perturbed flux surfaces Ω is constant, $\mathbf{B} \cdot \nabla\Omega = 0$, and $\Omega = 1$ defines the island separatrix, while a minimum value of $\Omega = -1$ is obtained at the O point. We adopt the usual approximation for the time-dependent electric potential of the rotating island (obtained from $E_\parallel = 0$) [3],

$$\Phi = \frac{q\omega}{m}\left\{(\psi - \psi_s) \pm \frac{w_\psi}{\sqrt{2}}(\sqrt{\Omega} - 1)\Theta(\Omega - 1)\right\} \tag{1}$$

(positive sign for $\psi < \psi_s$), with $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$ and ω/m is the poloidal rotation frequency of the island. The electric field $-\nabla\Phi$ vanishes far away from the island, where the plasma is assumed to be at rest (relative to the island). The first part of Φ is constant on the unperturbed flux surfaces and the corresponding electric field $E_r = -(q\omega/m)\nabla\psi \approx -(q\omega/m)RB_p$ causes the plasma inside the island to co-rotate with the island. In our simulations the finite width of the particle orbits smoothes out the effect of the jump of the electric field at the separatrix.

Since the effect to be studied depends on the banana orbit width, we perform simulations with the Hamiltonian guiding center particle code HAGIS [4], which was augmented by a Monte Carlo procedure for pitch-angle collisions [5, 6]. Momentum conservation in the ion-ion collisions and the electron-electron collisions is achieved by adjusting the particle weights. The necessary velocity space integrals are calculated separately in volume elements obtained by dividing the plasma volumes between helical flux surfaces into ten parts of equal extent in helical angle ξ. The code solves the equations of motion of either ion or electron marker particles which trace the trajectories in phase space. Integrals of the distribution functions of ions (f_i) or electrons (f_e) are calculated by summing up the contributions of all marker particles in a finite phase space volume element. The electric current is computed in two steps [6]:

First the equations for the ions are solved to provide the distribution function f_i. In the second step the equations for the electrons are solved, where the collision operator depends also on f_i. This is possible, since in the ion equation the ion-electron collisions can be omitted, i.e. $C(f_i) = C_{ii}(f_i)$, because the momentum loss caused by them is negligible, whereas for the electrons the electron-ion collisions are crucial: $C(f_e) = C_{ee}(f_e) + C_{ei}(f_e, f_i)$. With this procedure there is no need to follow the electrons for several ion collision times, but a few electron collision times are sufficient. HAGIS is used as a δf code, i.e. f is split into two parts, $f = f_0 + \delta f$, where f_0 is a Maxwellian, and only δf is represented by marker particles. For the ions f_{i0} is a Maxwellian centered around $v = 0$, hence the collision operator is reduced to $C_{ii}(\delta f_i, f_{i0})$. For the electrons f_{e0} is a shifted Maxwellian centered at the ion flow velocity Within $C_{ei}(f_e, f_i)$ we approximate f_i by a Maxwellian (the exact form of f_i is not important here) such that $C_{ei}(f_{e0}, f_i) = 0$ holds. Then the collision operator is $C_{ee}(\delta f_e, f_{e0}) + C_{ei}(\delta f_e, f_i)$. The contribution of f_{e0} to the electric current just cancels out the ion current $en_i u_{i\parallel}$, such that the electric current is given by an the integral over δf_e.

3 The small island effect

The island of a neoclassical tearing mode is normally rotating with respect to the surrounding plasma and a radial electric field is present inside the island that acts to force the plasma to co-rotate with the island. However, since the trapped particles cannot follow, on average, the poloidal $\boldsymbol{E} \times \boldsymbol{B}$ drift and by collisions the poloidal rotation of the passing particles is also damped, a contribution to the parallel flow with velocity $u_\parallel = \langle E_r/B_p\rangle$ arises [7], where B_p is the poloidal magnetic field and the brackets denote the flux surface average. These contributions to the parallel flows of ions and electrons are equal if the island width is large compared to the

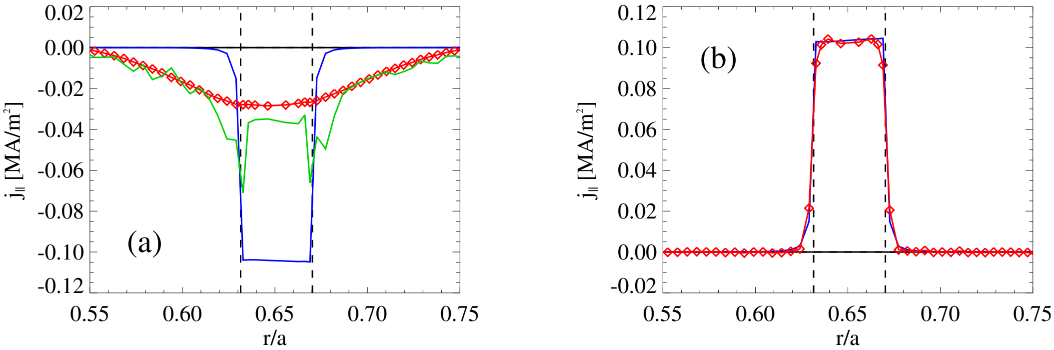

Figure 1: Parallel currents (symbols) of ions (a) and of electrons w/o e-i collisions (b) versus the radius through the O point of a small island ($w/w_b = 0.6$) rotating in the electron diamagnetic drift direction. Also shown by solid lines are $\langle enE_r/B_p\rangle$ (a) and $\langle -enE_r/B_p\rangle$ (b) and the neoclassical current obtained from the perturbed gradients (l.h.s., grey line).

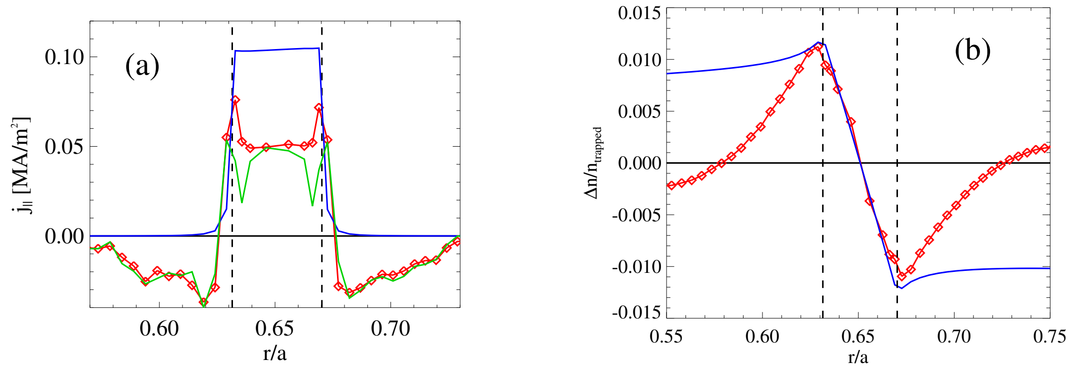

Figure 2: Left: Surface averaged electric current (symbols) in the island of Fig. 1. Also shown are $\langle -enE_r/B_p\rangle$ (dark line) and the bootstrap current calculated with the perturbed density gradients (grey line). Right: Density perturbation normalized to the trapped ion density (symbols) in this island compared to $-e\Phi/kT_i$ (solid line).

ion banana orbit width. In small islands, however, this is not true as shown below.

We illustrate this by particle simulations of a small island rotating in the direction of the electron diamagnetic drift in a plasma with constant density and temperature. The ratio of island width to the orbit width is $w/w_b = 0.6$. The parallel ion current in steady state after several collision times is shown in Fig. 1(a). The ion flow is strongly reduced in the island compared to the neoclassical velocity $\langle E_r/B_p\rangle$, because the ions do not feel the strong electric field all along the orbit, but only inside the island. The electron current obtained *without e-i collisions* is shown in Fig. 1(b). The electron velocity is close to $\langle E_r/B_p\rangle$ due to the small electron orbit width. Since the ion flow velocity is much smaller, an electric current in the island of the order of $\langle -enE_r/B_p\rangle$ arises. The friction between electrons and ions reduces this current to the value shown in Fig. 2(a). where the current obtained *with e-i collisions* is shown.

The ion current goes along with a density perturbation $\Delta n/n_{\mathrm{trapped}} = -e\Phi/kT_i$ (Fig. 2(b)). This agrees with the result from Ref. [8], that the ions with a large orbit width move radially to

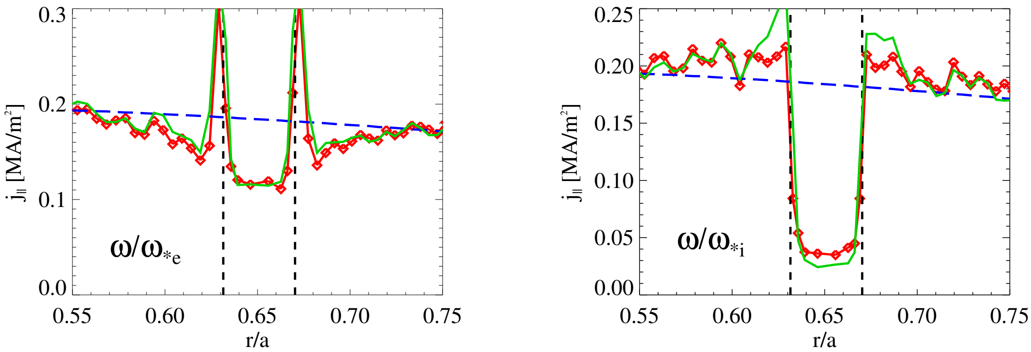

Figure 3: Bootstrap current (symbols) in a small island rotating at $\omega = \omega_{*e}$ (left) or $\omega = \omega_{*i}$ (right). Dashed line: bootstrap current without island, solid grey line: bootstrap current calculated with the perturbed gradients. Vertical dashed lines indicate the position of the island.

acquire a Boltzmann distribution inside the island, since most trapped ions belong to this group of ions. (The density perturbation and the electric current have the opposite sign if the island is rotating in the direction of the ion diamagnetic drift, since the potential is proportional to ω).

Surprisingly, the ion current and the electric current agree with the corresponding neoclassical results obtained from the *perturbed* density gradient (except close to the separatrix), although the island is smaller than the ion orbit width. On the other hand, since the density variation is only a percent or two over the island width, the gradient length remains rather large. However, one could expect some nonlocal behaviour, since the gradients inside and outside the island are very different.

4 The bootstrap current

In our simulations ω is a free parameter, but in the experiment it is the result of a complex process. Analytic estimates for the rotation frequency of neoclassical tearing modes [9] find that ω should be near either of the diamagnetic frequencies,

$$\omega_{*i/e} = \pm mkT_{i/e}n_0'/qen_0 \qquad (n_0' = dn_0/d\psi). \tag{2}$$

Then the current treated above is of similar size as the bootstrap current, which therefore can differ considerably from the result for big islands: the electric current is enhanced if the island rotation frequency is near the electron diamagnetic frequency, but it is decreased in case of rotation at the ion diamagnetic frequency.

We performed simulations for islands of the same size as before, but with finite density and temperature gradients in the unperturbed plasma ($\eta = T'/n' = 1$). In Fig. 3 the results for the surface averaged parallel electric current are shown, the corresponding density profiles in Fig. 4. In an island rotating with $\omega = \omega_{*e}$ a large fraction of the unperturbed bootstrap current is preserved. Here the small island effect tends to enhance the bootstrap current. The trapped particle density perturbation is the important one and is similar to that in Fig. 2(b) as shown in Fig. 5. In the case of rotation at $\omega = \omega_{*i}$ only a small residual parallel current remains, although previously a restoration of the ion current in small non-rotating islands was found [10]. This fits to the result that the current treated in the last paragraph is opposite to the bootstrap current.

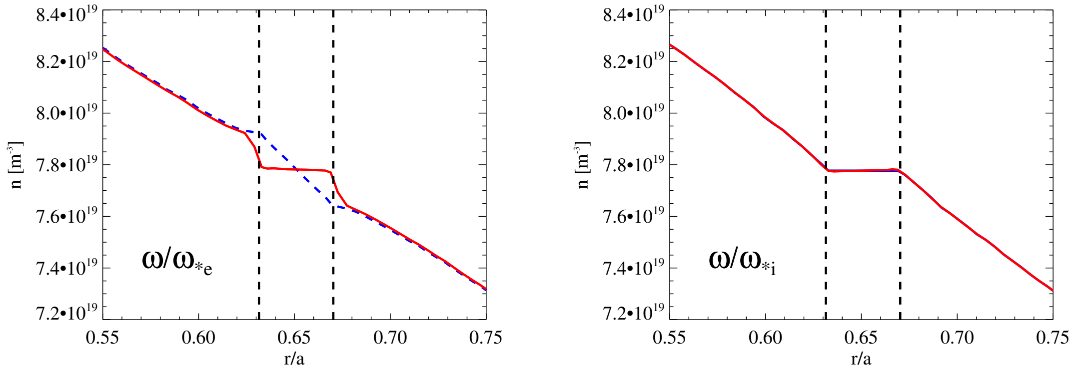

Figure 4: Density profile of ions (dashed) and electrons (solid) for the two cases in Fig. 3, $\omega = \omega_{*e}$ (left) and $\omega = \omega_{*i}$ (right). Vertical dashed lines indicate the position of the island.

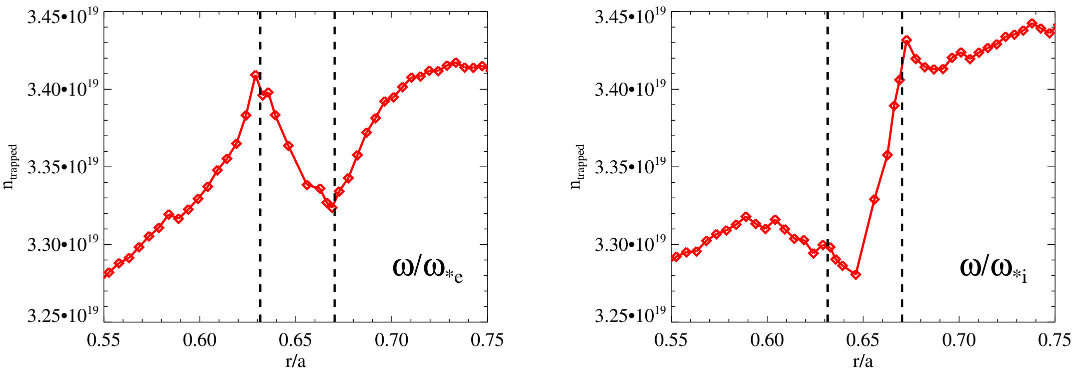

Figure 5: Density profiles of trapped ions for $\omega = \omega_{*e}$ (left) and $\omega = \omega_{*i}$ (right). Vertical dashed lines indicate the position of the island.

The density profile is completely flattened like in big islands, while the density perturbation found in the previous paragraph has the same sign, but is smaller in magnitude. The trapped particle density perturbation is reversed compared to Fig. 2(b), since ω has the opposite sign. We find again that the parallel electric current agrees with the neoclassical result calculated with the perturbed gradients (Fig. 3).

The violation of quasi neutrality in the case $\omega = \omega_{*e}$ indicates that in such islands the electric field must be different from that derived for big islands as given by Eq. (1). In order to get an idea of how the electric field has to change, we did simulations with different electric fields obtained by multiplying the potential in Eq. (1) by a factor (this results in a finite parallel electric field). For the case $\omega = \omega_{*e}$ and $w/w_b = 0.6$ the mismatch of ion and electron densities can be strongly reduced by setting the potential to zero. Then the density profiles are close to the unperturbed profiles and the bootstrap current (Fig. 6) is even larger than with the electric potential. Apparently the steepening of the ion density in Fig. 4 can not fully compensate the flattening of the electron density. The current is lower than the unperturbed one, since in all three cases the electron temperature profile is flattened (Fig. 6). Only a small residual gradient remains, while the ion temperature profile is unperturbed due to the large orbit width. If the unperturbed temperature profile was flat ($\eta = 0$) the full bootstrap current would be preserved.

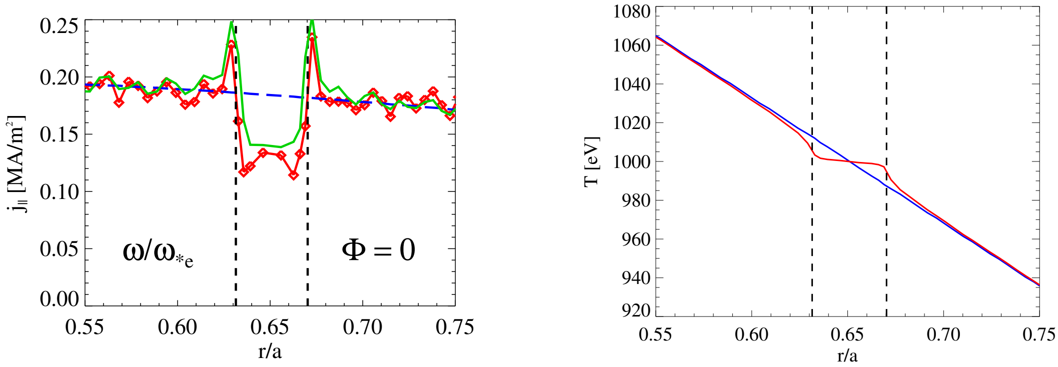

Figure 6: Left: Parallel current (symbols) in island rotating at the electron diamagnetic frequency with vanishing electric potential (see text). Dashed line: unperturbed bootstrap current. solid grey line: bootstrap current calculated with the perturbed gradients. Right: Temperature profile of ions (dashed) and electrons (solid). Vertical dashed lines indicate the position of the island.

Of course, simply multiplying the potential in Eq. (1) by a factor can only give an approximation to the true electric potential, which is likely to be finite, but the result obtained indicates that the electric field in a small island could be much smaller than in big islands.

5 Summary

The bootstrap current in islands that are smaller than the ion banana orbit width was calculated. It is small if the island is rotating at the ion diamagnetic frequency, but in the case of rotation at the electron diamagnetic frequency a large fraction of the bootstrap current is preserved. In both cases the current agrees with the neoclassical result calculated with the perturbed gradients.

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