Kinetic Effects on Slowly Rotating Magnetic Islands in Tokamaks

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INTRODUCTION

The full comprehension of the dynamics of magnetic islands is a critical point in order to predict and to improve the performance of a tokamak reactor, since their appearance can lead to a substantial deterioration of the radial confinement of both particles and energy. The presence of an island in the plasma generates a parallel current perturbation, through several physical mechanisms. This current affects in turn the stability of the island itself [1]. In this paper, we focus on the current connected with the rotation of the island with respect to the surrounding plasma. In particular, the rotation frequency range is extended beyond the standard assumption that, for ions, the island frequency is larger than the parallel streaming along the island itself for passing particles and than the magnetic precession frequency for trapped particles. In this case, the standard polarization current contribution becomes smaller, and other electric and magnetic effects play a role [2]. An analytical approach is employed, which consists in a two-parameter series expansion of the drift-kinetic equation [3, 4]. When the island propagation frequency drops below the parallel streaming of the ions, the main current contribution is shown to be linked to the interaction of the toroidal electric field generated by the island and the magnetic toroidal precession of trapped particles. A resonance mechanism between preceeding trapped particles and the island is also identified and discussed. The contribution of passing particles is on the other hand shown to be secondary. Numerical calculations performed with the drift-kinetic Hamiltonian code HAGIS [5] support the analytical results.

APPROACHING THE PROBLEM

We suppose a large-aspect-ratio tokamak, with circular cross section and circular concentric equilibrium magnetic surfaces. Equilibrium geometric coordinates are the poloidal flux χ, the poloidal angle θ and the toroidal angle ζ. We suppose that $\theta = 0$ corresponds to the outer midplane. To include the magnetic island, it is convenient to define the helical angle $\xi = m\theta - n\zeta - \omega t$, where m, n are the poloidal and toroidal mode number, respectively, and ω is the island propagation frequency. The total magnetic field has the expression:

$$\mathbf{B} = I(\chi)\nabla\zeta + \nabla\zeta \times \nabla(\chi + \tilde{\psi}\cos\xi)$$

where $I(\chi) = RB_\zeta$ and $\tilde{\psi}$ is constant according to the well-known constant-ψ approximation [1]. We define a perturbed flux surface label:

$$\Omega = \frac{2(\chi - \chi_s)^2}{W_\chi^2} - \cos\xi,$$

where W_χ is the island half-width expressed in χ units, and the subscript s labels quantities evaluated on the resonant surface. The electrostatic potential is not calculated self-consistently, but an analytical expression is given supposing the electrons immediately shorting out every parallel electric field [4]:

$$\Phi = \frac{\omega q}{mc}\left[\chi - \chi_s - h(\Omega)\right], \tag{1}$$

where $h(\Omega)$ is an integration constant determined from boundary conditions.

In this paper, we solve the drift-kinetic equation [6] by means of a δf method, which consists in writing the distribution function f as $f = F_0 + g$ where F_0 is known, and supposed to be a homogeneous isotropic Maxwellian F_M (in order to neglect perturbed bootstrap current effects [3, 4]), while g is the perturbation on the distribution, supposed to be small.

The problem is solved numerically using the drift-kinetic Hamiltonian code HAGIS [5], and analytically by a double-parameter series expansion of g [3, 4]. In particular we write

$$g = \sum_{n,m}^{\infty} g^{(n,m)}\delta^n\Delta^m, \qquad \delta = \frac{\rho_b}{w} \qquad \Delta = \frac{w}{a}, \tag{2}$$

where ρ_b is the ion banana width, w the island half-width and a the tokamak minor radius. Within these assumptions, the drift-kinetic equation takes the form

$$\begin{aligned} -\omega\frac{\partial g}{\partial\xi} + \frac{v_\parallel}{Rq}\frac{\partial g}{\partial\theta} + k_\parallel v_\parallel \left.\frac{\partial g}{\partial\xi}\right|_\Omega + m\frac{c}{B}\frac{I}{Rq}\frac{\partial\Phi}{\partial\chi}\frac{\partial g}{\partial\xi} + \\ -m\frac{c}{B}\frac{I}{Rq}\frac{\partial\Phi}{\partial\xi}\frac{\partial g}{\partial\chi} + \frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial g}{\partial\chi} - m\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial g}{\partial\xi} \\ -\frac{q_i}{m_i}\left[\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial\Phi}{\partial\chi} - m\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial\Phi}{\partial\xi}\right]\frac{\partial g}{\partial v} = \\ = -\frac{q_i F_M}{T}\left[\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial\Phi}{\partial\chi} - m\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\chi}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial\Phi}{\partial\xi}\right]. \end{aligned} \tag{3}$$

In the course of the paper, we will assume different scalings for the island propagation frequency ω, discarding the standard assumption $\omega > k_\parallel v_{\rm th}$, defining $k_\parallel = -m(\chi - \chi_s)/Rq\, q_s'/q_s$, where the apex $'$ indicates a derivative with respect to χ. This exploration of the parameter space is meaningful because a theory which gives a theoretical value of ω is not available yet. Collisions are not considered in our analytical treatment. Their role will be clarified when we compare our results to numerical simulations (which include pitch-angle scattering). Our final purpose is to discuss the breaking down of the quadratic dependence of the polarization current on the frequency [4] for low values of ω, which was already pointed out in Ref.[2].

THE $\omega \sim \omega_*$ REGIME

A standard assumption in NTM literature consists in supposing $\omega \sim \omega_*$, where ω_* is the equilibrium electron diamagnetic frequency [4]. This corresponds to the following ordering for the terms in Eq.(3):

$$\Delta : 1 : \Delta : \Delta : \Delta : \delta : \Delta\delta : \Delta\delta : \Delta^2\delta : \delta : \Delta\delta.$$

It is important to note that the assumption $\omega \sim \omega_*$ implies $\omega \sim k_\| v_\|$, supposing $v_\| \sim v_{\rm th}$. In the drift-kinetic calculation performed by Wilson *et al.*[4], the further assumption $\omega > k_\| v_\|$ was made in order to isolate the contribution of the polarization current. The aim of this section is to focus on the same calculation *retaining the terms in* $k_\| v_\|$, to analyse their effect on the perturbed current.

In Ref.[4], solving the drift-kinetic equation, the lowest-order perturbed distribution turned out to be (neglecting equilibrium gradients):

$$g^{(1,0)} = -I\frac{v_\|}{\omega_c}\frac{\partial\Phi}{\partial\chi}\frac{q_i F_M}{T} + \overline{h}_P^{(1,0)} + \overline{h}_T^{(1,0)}, \tag{4}$$

where the bar over a function indicates that it is independent on θ. The subscripts P and T refer to to the passing and trapped region of phase space, respectively. Here,

$$\overline{h}_P^{(1,0)} = -\frac{4I}{W_\chi^2}\frac{\omega q}{mc}\frac{\mathrm{d}h}{\mathrm{d}\Omega}\frac{q_i F_M}{T}\left\langle\frac{1}{\omega_c}\right\rangle_\theta\left[\left\langle\frac{1}{v_\|}\right\rangle_\theta\right]^{-1}\left[\chi - \langle\chi\rangle_\Omega\right]. \tag{5}$$

in $\omega > k_\| v_\|$ limit [4], while we obtain

$$\overline{h}_P^{(1,0)} = -\frac{4I}{W_\chi^2}\frac{\omega q}{mc}\frac{\mathrm{d}h}{\mathrm{d}\Omega}\frac{q_i F_M}{T}\left\langle\left(\frac{\mathrm{d}h}{\mathrm{d}\Omega}\frac{\omega}{m\tilde{\psi}}\frac{Rq}{v_\|}+1\right)\frac{v_\|}{\omega_c}\right\rangle_\theta \left[\left\langle\frac{\mathrm{d}h}{\mathrm{d}\Omega}\frac{\omega}{m\tilde{\psi}}\frac{Rq}{v_\|}+1\right\rangle_\theta\right]^{-1}\left[\chi - \langle\chi\rangle_\Omega\right]. \tag{6}$$

retaining the terms in $k_\| v_\|$. The contribution of $\overline{h}_T^{(1,0)}$ will be discussed in the following section. The main difference between these two different situations is that, retaining terms in $k_\| v_\|$, the perturbed distribution gets a resonant denominator, which corresponds of course to the case $\omega \approx k_\| v_\|$.

The perturbed distribution is used to calculate the parallel current perturbation (which is what influences island stability [1]) by means of quasi-neutrality equation ($\nabla \cdot \mathbf{J} = 0$)

[4]. *What can be found is that, after the integration in the velocity space, the effect of the resonant denominator is small compared to the standard polarization current contribution.* As a matter of fact, one can see that expanding the resulting perpendicular current in the quasi neutrality equation, the resonant denominator starts to contribute from a higher-order term in ε, where ε is the inverse aspect ratio, while the lowest-order term does not show any difference with respect to the $\omega > k_{||}v_{||}$ case. This physically depends on the fact that the contribution of the resonant particles changes its sign with the parallel velocity, so the current contribution of co-passing particles cancels the one of counter-passing particles.

THE $\omega \sim \omega_D$ REGIME

We now assume ω to be of the same order as the toroidal precession frequency of trapped particles due to the equilibrium magnetic drift, ω_D:

$$\omega_D = \frac{q}{Rr\omega_c}\frac{1}{2\theta_b}\int_{-\theta_b}^{\theta_b}\left[\mu B + v_{||}^2\right]\cos\theta\,\mathrm{d}\theta \tag{7}$$

where θ_b is the bounce angle. For thermal particles, $\omega_D < k_{||}v_{\mathrm{th}}$. Another equilibrium toroidal precession effect for trapped particles, due to magnetic shear [7], is also present in tokamaks. We define the corresponding frequency as ω_s, and we introduce $\omega_{tp} = \omega_D + \omega_s$.

The resulting ordering of the terms in Eq.(3) with this frequency scaling is

$$\Delta\delta : 1 : \Delta : \Delta\delta : \Delta\delta : \delta : \Delta\delta : \Delta\delta^2 : \Delta^2\delta^2 : \delta^2 : \Delta\delta^2.$$

The resulting lowest-order solution can be approximated by:

$$J_{\perp}(v) = q_i\left\langle g^{(1,0)}\frac{\mathrm{d}\Omega}{\mathrm{d}t}\frac{\mathrm{d}r}{\mathrm{d}\Omega}\right\rangle_{\theta} = \frac{m^2}{q^2}\frac{q_i^2\Phi}{T}\omega_D\frac{\omega_{tp}}{\omega + \frac{m}{q}\omega_{tp} + \frac{m}{q}\omega_E}\frac{dr}{d\Omega}F_M\sin\xi. \tag{8}$$

We name this contribution *precessional current*. Here, ω_E is the toroidal precession frequency of trapped particles due to the presence of the island radial electric field [8]. This current is not divergence free, so it causes a closure parallel current which can affect the island stability. No analytical evaluation of this contribution is available yet.

For positive values of the frequency no resonance is possible, since in this case the island propagates in the direction opposite to the toroidal precession of trapped ions, so the dependence of this current on the velocity is quite smooth. On the contrary, a resonance occurs for negative frequencies, as shown in Fig.1a. The physical interpretation of this effect is connected to the interaction of the island toroidal electric field with the toroidal magnetic precession [9]. When $\omega > 0$, all particles are “faster” than the island. Where the toroidal electric field is positive, they are accelerated and so they tend to disperse. On the other hand, a negative electric field brakes them, so there they tend to accumulate. The situation is more complicated for $\omega > 0$, because there are particles which are

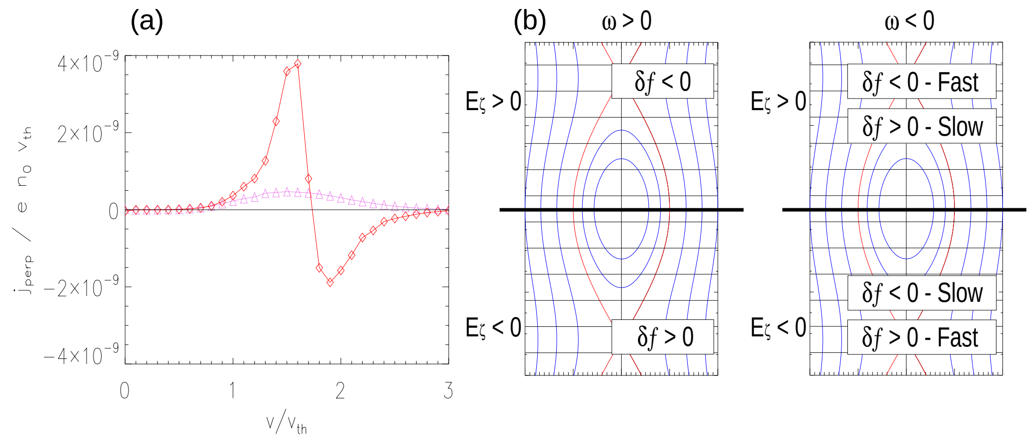

FIGURE 1. HAGIS Results: Perpendicular current versus velocity for $\omega = 300\ rad/s$ (triangles) and for $\omega = -300\ rad/s$ (diamonds) for a very low value of collisionality (a). Schematic draw of trapped particles' interaction with the island toroidal electric field (b).

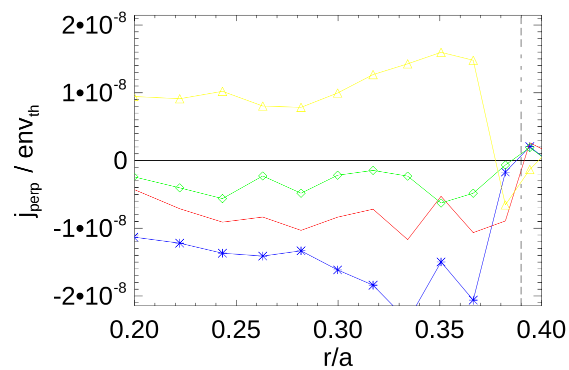

FIGURE 2. HAGIS Results: Current on the X-point helical cell (triangles), on the O-point cell (stars), on the intermediate cell (diamonds) and their sum (solid) for $\omega = -300\ rad/s$.

"slower" or "faster" than the island, since ω_{tp} depends on the particle's velocity. Where slow particles are accelerated, *they actually decrease their relative speed with respect to the island*, and on the contrary they increase their relative speed as the electric field brakes them. So their contribution has the opposite sign with respect to the fast particles' one. This situation is summarized in Fig.1b.

The contribution of ω_E is such to locally change the number of "faster" and "slower" particles while moving from island's X-point (where the radial electric field is minimum) to O-point (where the radial electric field is maximum), so that the overall current along the island comes out from a very complicated balance of these contributions, as shown in Fig.2.

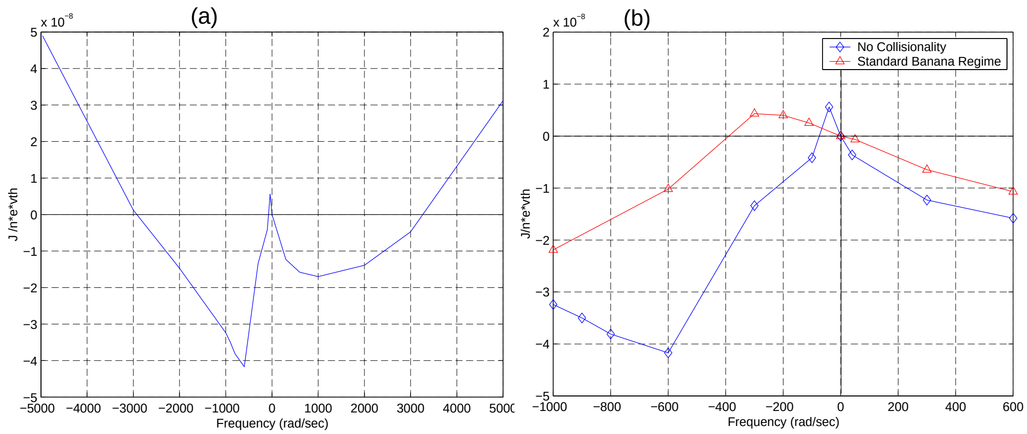

FIGURE 3. HAGIS Results: Perpendicular current versus island propagation frequency in the non-collisional regime (a). Comparison of the perpendicular current versus frequency in a standard banana regime and in the non collisional regime (b).

SUMMARY

Fig.3a summarizes all our results. For high absolute values of ω, the polarization current, which scales like ω^2 in absence of gradients [4], prevails. For lower values of ω the precessional current overcomes the standard polarization current. It changes sign in $\omega = 0$, because the electric field reverses there its sign (see Eq.(1)), and for a small negative value of ω, when the contribution of "slower" particles starts to prevail on the "faster"'s one.

Collisions can be shown to be very important, allowing the integral in the velocity space of this precessional current to exist. A change in the collisional frequency can affect the sign of the current integrated over velocity space, with important consequences on the stabilizing power of the subsequent parallel current, see Fig.3b.

REFERENCES

1. P. H. Rutherford, Phys. Fluids **16** 1903 (1973)
2. E. Poli *et al.*, Nucl. Fusion **45** 384 (2005)
3. R. Carrera, R. D. Hazeltine, M. Kotschenreuther ,Phys. Fluids **29** 899 (1986)
4. H. R. Wilson *et al.*, Phys. Plasmas **3** 248 (1996)
5. S. D. Pinches *et al.*, Comput. Phys. Commun. **111** 133 (1998)
6. R. D. Hazeltine and J. D. Meiss, *Plasma Confinement* (Addison-Wesley, New York, 1992)
7. A. G. Peeters, PhD Thesis, *Technische Universiteit Eindhoven*, unpublished (1994)
8. F. L. Hinton, J. A. Robertson, Phys.Fluids **27**(5) 2958 (1984)
9. M. Siccinio, E. Poli, *submitted to* Plasma Phys. Control. Fusion (2008)