35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-1.062 (2008)

Sawtooth Crash as a Result of Quasiperiodic Transition to Chaos in ASDEX Upgrade

V. Igochine[1], O.Dumbrajs[2,3], H. Zohm[1], ASDEX Upgrade team[1]

[1]*MPI für Plasmaphysik, Euratom-Association, D-85748 Garching, Germany*
[2]*Helsinki University of Technology, Association Euratom-Tekes, P.O.Box 2200, FIN-02015 HUT, Finland*
[3]*Institute of Solid State Physics, Association Euratom-University of Latvia, Kengaraga Street 8, LV-1063, Riga, Latvia*

Introduction

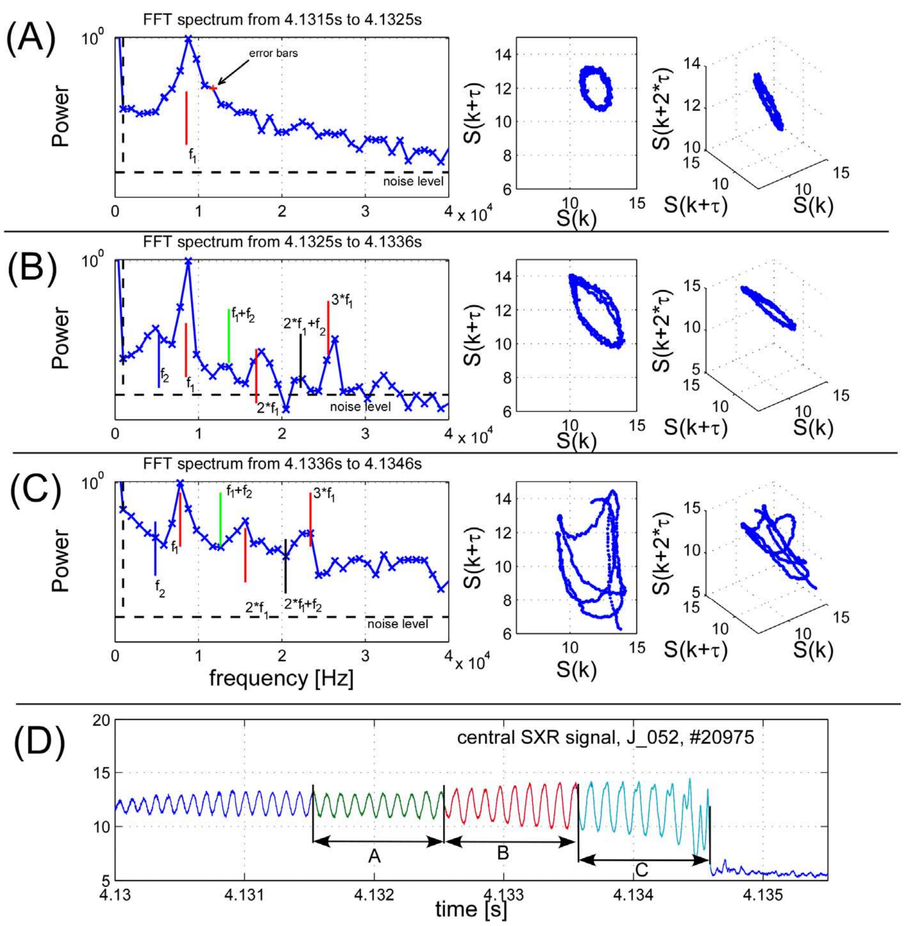
In magnetically confined fusion plasmas, a variety of magnetohydrodynamic (MHD) instabilities can occur, driven by gradients of kinetic pressure or current density. The sawtooth oscillation is one of the fundamental instabilities in tokamaks which is often observed but still has no definitive explanation for crash process. This phenomenon is characterized by a repetitive and rapid crash of the central electron temperature. The Kadomtsev model, in which the (m,n)=(1,1) island turns to a new magnetic axis after the reconnection process, has provided a starting point for understanding the sawtooth, but not for explanation of the phenomenon. Several experiments show that this model is in clear contradiction with experimental observations. For instance, it can explain neither the measured safety factors [1,2,3], nor the existence of (1,1) mode after the crash [4,5]. As a result, a number of different theories were proposed to explain the crash dynamics and the fast crash time. Recent 2D ECE measurements [6,7] of the crash allow one to prove some predictions of these models. It was demonstrated that a sawtooth crash is localized in the toroidal direction which immediately withdraws all symmetrical hypotheses like Kadomtsev model or quasi-interchange model. Other proposed models are based on the idea of stochastization of magnetic field lines during the sawtooth crash [8,9]. This variant of the crash requires no preferable poloidal position for the crash and can be also non-symmetric in the toroidal direction. The important parameters for such scenario are: (i) amplitude of the perturbations; (ii) safety factor profile; (iii) number of perturbations with different helicities; (iv) coupling of the perturbations. It was shown recently that amplitudes of the primary (1,1) mode together with its harmonics are sufficient to stochastize the region if the central q is less than 0.85-0.9, which is in good agreement with measurements of the safety factor profile and allows one to explain the existence of the mode after the sawtooth collapse [5]. In that work influence of first three parameters were investigated with the field line tracing technique. The last condition, coupling of perturbations, cannot be described by means of this technique because in this type of analysis all resonances are coupled by definition. In reality, different resonances have different rotation frequencies which screen perturbations from each other and strongly reduce the amplitude of the magnetic fluctuations in tokamaks. Such a screening effect vanishes only if the perturbations are coupled to each other. It is obvious that the primary (1,1) mode is coupled to its harmonics (2,2) and (3,3), but the coupling to other low order rational surfaces is not trivial and requires special investigation. In contrast to our earlier work [5], here we focus our attention on the dynamics of the instability just before the sawtooth crash and investigate transition to the stochastic stage. We present a clear indication of the transition into stochastic (chaotic) stage which supports the stochastisation hypothesis of the sawtooth crash.

Identification of the transition to stochastic phase

As was mentioned above, we think that evolution of the (1,1) instability leads to stochastic stage during the sawtooth crash. The stochastic stage is very short (20-100 μs in ASDEX Upgrade) and it is accompanied by the strong reduction of the temperature due to the heat flow in the stochastic magnetic field. Thus, all temperature measurements show only decrease of the plasma temperature which is a standard signature of the sawtooth crash from Soft X-ray (SXR) and electron cyclotron emission (ECE). Due to these problems, the stochastic phase itself can not be resolved and investigated. At the same time, transition to stochastic stage can be examined if one uses time traces from the SXR and ECE diagnostics. These two diagnostics give two independent measurements of the temperature perturbations as line integral measurements (SXR) and as local temperature measurements (ECE). We use these signals to identify the type of transition to the stochastic stage.

Intensive investigations of completely different mathematical and experimental chaotic systems demonstrate that there are three possible roads to chaos [10]: (i) period doubling, when the period is doubling many times during the transition (ii) intermittency, which is characterized by sudden changes from non-chaotic to chaotic behavior and back; (iii) quasiperiodicity which is characterized by appearing of two incommensurable frequencies [11]. Each of these roads to chaos has a set of unique signatures which appear in the system independently from its nature. It could be a physical, biological or any other system, but the roads remain the same [10,12,13,14]. Thus, a set of transition signatures is a universal invariant which could be used to verify the nature of the system and clarify the type of the transition to chaos. Analysis of the experimental SXR and ECE signals before a sawtooth crash shows neither period doubling (power spectrum at frequencies $f_1/2^n$, $n=1,2,3,...$ with decreasing amplitudes), nor sudden jumps between chaotic and non-chaotic behavior. This eliminates the roads to chaos via period doubling or intermittency. The characteristics of the quasiperiodic transition are different. “In the quasiperiodic regime, an experimental spectrum is a series of peaks at all integer combinations of two incommensurate frequencies f_1 and f_2.”[11]. To understand this type of transition we assume for a moment an arbitrary system which has two basic frequencies f_1 and f_2. Such system would have a trajectory in 3D-phase space which is lying on the torus. In case of the rational ratio between the two frequencies ($f_1 n = f_2 m$, with integer *n* and *m*), the system has periodic behavior (it repeats itself after a fixed period). Poloidal cross-section of the torus is Poincare plot of the system which consists of a set of repetitive points on a curve. The system trajectory is closed in this case and does not cover the torus surface completely. If the two frequencies are incommensurate, the torus surface would be cover completely and the Poincare points will never (in principle) repeat. Eventually, the Poincare points fill in the curve in the Poincare plane. In such a situation, an arbitrary small change in the system (third frequency, frequency locking, etc.) destroys the surface of the torus and converts the motion into 3D motions in the phase space which is chaotic motion. It was demonstrated both numerically and experimentally that the more irrational ratio between the frequencies, the more easy such a system goes over to the chaotic stage and computation experiments often use the conjugate golden ratio $\left(G=\left(\sqrt{5}-1\right)/2\right)$ which is the most irrational number, since it has the slowest convergence.

We investigate several typical H-mode discharges which were performed during 2006-2007 campaigns (I_p = 0.8MA, P_{NBI}=5 MW, with and without ECRH power). We analyze the central SXR signal (2MHz sampling frequency, 500kHz upper filter frequency) before a sawtooth crash in ASDEX Upgrade by means of spectral analysis and reconstruction of trajectory by means of delay coordinates which are the standard techniques for stochastic systems and have been used for identification of the transition to chaos in different physical systems[15,16].



<u>Figure 1.</u> Analysis of the sawtooth crash in ASDEX Upgrade (A-D). Normalized spectral amplitude and reconstruction of trajectory by means of delay coordinates (number of time points for the time delay: τ=600) are shown for single mode regime (A), slightly quasiperiodic regime which is close to the crash time (B), and strongly quasiperiodic regime just before the sawtooth crash (C). Smallest resolvable frequency and noise level are indicated by dashed lines. The SXR signal is shown in figure (D).

One can clearly see a transition of the system from the single frequency state (Fig. 1A) to the slightly quasiperiodic state (Fig.1B) and then to the strongly quasiperiodic state (Fig. 1C) where the whole lower part of the spectrum is strongly enhanced and only the strongest frequencies can be seen. Such increase of broad band low frequency noise is typical, when the chaos is about to appear [10]. The frequency locking (mode coupling) occurs in the stochastic stage which is accompanied by the strong reduction of the temperature. The frequency of the primary (1,1) kink mode is marked here as f_1. (The signal itself and all stages are shown in Figure. 1D.) Another important observation is the ratio between the two primary frequencies before the sawtooth crash in Figures 1B. It is equal to the conjugate golden ratio $f_2/f_1 = 0.59 \pm 0.03$ (within error bars of the measurements). To show the “ideal situation” we adjust in Figure 1 position of the primary peak f_1 to the (1,1) mode frequency and mark automatically all other frequencies by using the golden mean ratio and linear combinations of the frequencies. One can see that the frequency marks fit all the experimentally measured peaks within error bars of the measurements. The lowest resolvable frequency 977Hz (fundamental frequency of the Fourier transform) is shown together with the noise level by dashed lines. Thus, the low frequency spectrum is completely described by two primary frequencies and their linear combinations. In typical experiments with transition to chaos, the scan of the frequencies is made to obtain the irrational frequency ratio [14]. In our case, the most irrational frequency ratio develops naturally in the system. This fact indicates that the chaos in the system is approached in the most "intense" way (the most robust way). Other confirmation of the transition comes from reconstructions of trajectories by means of delay coordinates in 2D and 3D phase space. These plots were constructed from the same signal using the fixed time delay τ and are analogous to Poincare plots (2D) and torus phase structures (3D) in phase space which were discussed before. In these phase plots our transition to chaos should have the following steps: (single frequency) → (2D torus) → (3D chaos). Indeed, in the case of the single frequency f_1 we observe a planar periodic cycle which is typical signature of pure periodic behavior (Fig.1A). In the slightly quasiperiodic case open orbits are observed (Fig. 1B). In the last pre-crash phase strongly quasiperiodic behavior is seen which is characterized by a non planar 3D structure (Fig.1C) but the torus structure is not completely destroyed. (Completely chaotic stage is characterized by a cloud of the trajectory points in 2D plot and an attractor structure or cloud of points in 3D plot depending on system's nature.) The analysis of the ECE temperature signals (32 kHz sampling rate, 16kHz Nyquist frequency) in the slightly quasiperiodic stage shows the same frequency spectra as in the SXR signal and it also consists of the same frequencies $f_1 = 8.7kHz$ and $f_2 = 5.26kHz$ ($f_2/f_1 = 0.605 \pm 0.025$).These measurements resolve also other $(f_1 - f_2)$ and $(2f_1 - f_2)$ resonances as shown in Figure 2.

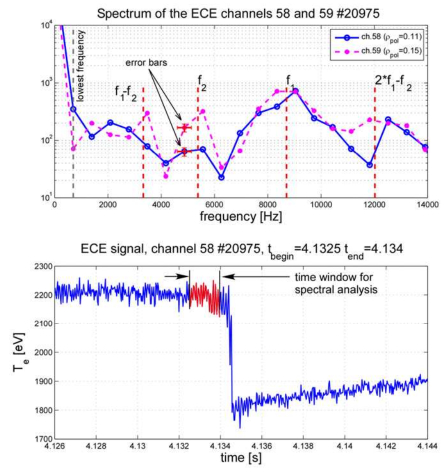


Figure 2. ECE power spectrum for two neighbouring channels and the one of the signals for the same sawtooth crash as shown in figure 1. The two frequencies are shown and the linear combination of these frequencies can be seen ($f_2/f_1 = 0.605 \pm 0.025$). The lowest resolvable frequency is marked.

Depending on the duration of the slightly quasiperiodic phase, different numbers of resonances can be resolved (longer time interval in Fourier analysis provides better frequency resolution). We have found that longer (1,1) precursor phase before the crash typically corresponds to a longer phase of the quasiperiodic motion. All these observations strongly support the hypothesis of the quasiperiodic transition to chaos during the sawtooth crash. More detailed variant of this work has been published as a Letter in Nuclear Fusion [17].

References

[1] F.M. Levinton, et. al., *Phys.Rev.Lett.,***72,**2895 (1994).
[2] M.Yamada, et.al, *Physics of Plasmas,* **1,** 3269 (1994).
[3] H.Soltwisch, *Rev. Sci. Inst.,* **59,** 1599 (1988).
[4] A. Letsch, et. al., *Nucl. Fusion* **42,** 1055 (2002).
[5] V. Igochine, et. al., *Nucl. Fusion* **47**, 23-32A (2007).
[6] H.K. Park, et.al., *Phys. Rev. Let.* **96,** 195003 (2006).
[7] H.K. Park, et.al., *Phys. Rev. Let.* **96,** 195004 (2006).
[8] A. J. Lichtenberg*, Nucl. Fusion* **24,** 1277 (1984).
[9] A. J. Lichtenberg, et.al. *, Nucl. Fusion* **32,** 495 (1992).
[10] H. G. Schuster and W. Just, "Deterministic Chaos"
[11] D. Rand et. al. *Phys. Rev. Let.* **49,** 132 (1982).
[12] R. Hilborn, "Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers", 2000
[13] J. Argyris, et.al., "An exploration of Chaos", 1994
[14] A. P. Fein et. al.*, Physica Scripta ,* **T9,** 79 (1985).
[15] S. Martin et. al. *Phys. Rev. Let.* **53,** 303 (1984).
[16] A. Brandstater et.al. *Phys. Rev. Let.* **51,** 1442 (1983).
[17] V. Igochine et.al.*, Nucl. Fusion* **48,** 062001 (2008).