Algebraic and iterative tomography of fast-ion velocity-space distributions from synthetic fast-ion diagnostics

<u>M. Salewski</u>[1], B. Geiger[2], S.K. Nielsen[1], H. Bindslev[3], M. García-Muñoz[2],

W.W. Heidbrink[4], S.B. Korsholm[1], F. Leipold[1], F. Meo[1], P.K. Michelsen[1],

D. Moseev[2,5], M. Stejner[1], G. Tardini[2], and the ASDEX Upgrade team[2]

[1] *Association Euratom - DTU, Technical University of Denmark,*

Department of Physics - DTU Risø Campus, DK-4000 Roskilde, Denmark

[2] *Association Euratom - Max-Planck-Institut für Plasmaphysik,*

D-85748 Garching, Germany

[3] *Aarhus University, DK-8000 Aarhus C, Denmark*

[4] *University of California, Irvine, California 92697, USA*

[5] *FOM Institute DIFFER, Dutch Institute for Fundamental Energy Research, Association*

Euratom-FOM, Trilateral Euregio Cluster, Nieuwegein, The Netherlands

Introduction

The tokamak ASDEX Upgrade is equipped with several fast ion diagnostics including collective Thomson scattering (CTS) [1-5] and fast-ion D_α (FIDA) [6], neutral particle analyzers (NPA) [7] and fast-ion loss detectors (FILD) [8]. This offers unique possibilities to study fast-ion distribution functions in plasmas. In CTS and FIDA measurements, one pre-selects a projection direction through geometric arrangement of the experiment and measures a 1D function g of the fast-ion 2D velocity-space distribution function f in a volume of roughly 10 cm size. Two CTS receivers and two FIDA optical heads are available at ASDEX Upgrade after installations in 2012 (before there was one each). Other tokamaks have also been equipped with multiple FIDA views, for example DIII-D [9, 10], NSTX [11] or MAST. Here we study methods to calculate tomographies of 2D fast-ion velocity distribution functions from CTS and FIDA measurements. Two new prescriptions to compute tomographies have been developed recently [12, 13], and a third prescription had been developed in a previous work [14]. We discuss these three prescriptions, their differences and similarities and their promise and limits. With our prescriptions we can compute tomographies for any set of fast-ion measurements, in particular those obtained with CTS or FIDA or other fast-ion charge exchange spectroscopy (FICXS) that detects other light than D_α. Tomographies based on a mix of diagnostics would also be possible as will be relevant to the CTS/FIDA system at ASDEX Upgrade and the CTS/FICXS system at LHD [15, 16]. One could also include NPAs or other fast-ion diagnostics in such mixes.

Velocity-space tomographic reconstruction prescriptions

The final goal of velocity-space tomography is to determine a 2D velocity-space distribution function f from 1D CTS or FIDA measurements g. The forward problem to determine g from f is straightforward since forward models for CTS and FIDA are available. The inverse problem to determine tomographies f from measurements g is more complicated and has no unique solution. Nevertheless, one can find optimum solutions by minimizing cost functions. Here we discuss the three proposed prescriptions to compute a tomography.

The first reconstruction prescription of f was restricted to 1D CTS measurements g [14]. The inverse problem was made tractable by expansion of the g's as well as f into orthonormal sets of base functions: $g = \sum_i a_i^g G_i$ and $f = \sum_i a_i^f F_i$. They used Bessel functions but other choices would be possible. The expansion coefficients a_i^g and a_i^f are then related through a transfer matrix A in the form $a_i^g = A_{ij} a_j^f$. A also depends on the choice of the base functions. The tomography could be found from the Moore-Penrose pseudoinverse of A. It was possible to reconstruct coarse features of a smooth α-particle distribution function in JET in an idealized situation – the numerical grids for the original and the tomography were identical. This prescription failed to reconstruct the function close to the $v_\parallel$-axis for normalized $v_\perp$ coordinates up to 0.2-0.3.

The idea of weight functions [17, 18], which relate the CTS or FIDA measurements to 2D fast-ion velocity distribution functions, was proposed after this first attempt and lead to the second prescription for tomography [12]. In reference [12] weight functions are used as base functions to construct f even though weight functions are not orthonormal. The velocity distribution is then related to the expansion coefficients and the weight functions by $f_{kl} = \sum_i \sum_j a_{ij} w_{ijkl}$. Expansion coefficients a_{ij} are found by iteration in the following way: After an initial guess, the projections g are computed from the iterated f. The next iteration is obtained by adding or subtracting small multiples of the weight functions for each u and ϕ to f, depending on whether the corresponding iterated $g(u, \phi)$ is larger or smaller than the target $g(u, \phi)$, respectively. The iterated solution depends on the initial guess and is not unique. Nevertheless, the coarsest features of f are reproduced with this tomographic prescription. Computation of the tomography by iteration takes considerably longer than by algebraic techniques. Figure 1b shows an iterated tomography of a beam ion velocity distribution function (Figure 1a). The grid of the tomography was different from that of the original (realistic experimental conditions), and we used a realistic two-view CTS system as defined in reference [13]. It is also possible to combine fast-ion diagnostics with this prescription as long as we can formulate a weight function.

Weight functions also considerably simplify algebraic reconstruction prescriptions because a transfer matrix taking f into g can now immediately be written down [13]. If f and g are

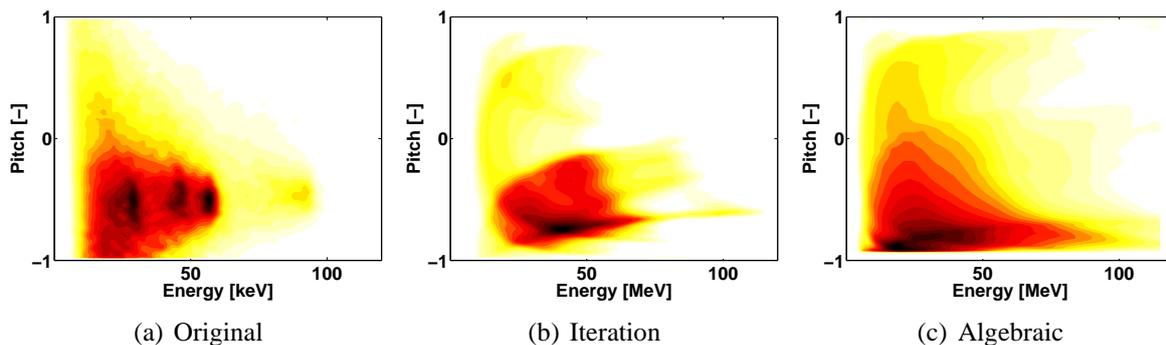(a) Original   (b) Iteration   (c) Algebraic

Figure 1: Original function (a) and two-view tomographies found by the iteration (b) and algebraically (c) for realistic experimental conditions. The original function is a typical beam ion velocity distribution function for beam sources S3 and S8 computed with TRANSP.

written as column matrices, the weight functions go into the rows of the transfer matrix W, and we get $Wf = g$. Since the iterative prescription exploited only qualitative knowledge of velocity-space interrogation regions, weight functions describing these could be qualitative. Contrarily, weight functions for the algebraic prescription must be quantitatively correct. If such quantitative weight functions are available, unique tomographies can be computed from the Moore-Penrose pseudoinverse W^+ as $f^+ = W^+g$ for a given number of singular values. Analytic expressions for CTS weight functions have been found [12] whereas FIDA weight functions are found numerically, and presently they agree with the FIDASIM code to within 10%. The quantitative weight functions make the problem inherently tractable, and no expansion in basis functions is necessary. Figure 1c shows an example. In reference [13] it is demonstrated that completely accurate tomographies can be computed for one-view, two-view, three-view, and four-view systems under idealized conditions. For simulated experimental conditions two-view tomographies do not reproduce the details of f but nevertheless identify that the majority of ions has negative pitch. The pitch of the beam injection source is off by about 0.25 compared with the underlying original function. A four-view tomography of a realistic beam ion velocity distribution function at ASDEX Upgrade resembles the original function well in general shape and location of the beam injection sources at full and half energies [13]. The combination of CTS and FIDA measurements, which could result in four views at ASDEX Upgrade, will be discussed in future work.

Conclusions

We have discussed three prescriptions for tomographic reconstruction of f from 1D synthetic CTS or FIDA measurements. The algebraic prescriptions have advantages over iterative prescriptions: First, the algebraic prescriptions give unique tomographies in the sparse data problem whereas the iterative prescriptions depend on the start conditions. Second, algebraically

computed tomographies usually resemble the original function more than those computed with any iterative prescription we have tested though we have not attempted to quantify this. Third, tomographies using the algebraic prescription require significantly less computer time than iterative prescriptions. The algebraic prescription based on weight functions is more effective than the earlier algebraic prescriptions since it works well for all $v_\perp$ for arbitrary functions and no expansions of f and g into base functions are necessary. A difficulty of all prescriptions is systematic handling of noise. Using tomographies one could study selective reorganization or depletion of fast ions in velocity space that is typical for instabilities (e.g. a sawtooth crash [19, 20]). The final goal of our work is the purely experimental determination of a tomography of the 2D fast-ion distribution function and of such selectivity in velocity space [13].

Acknowledgements

References

[1] M. Salewski *et al.*, Nuclear Fusion **50** 053012 (2010)

[2] F. Meo *et al.*, Review of Scientific Instruments **79** 10E501 (2008)

[3] F. Meo *et al.*, Journal of Physics: Conference Series **227** 012010 (2010)

[4] V. Furtula *et al.*, Journal of Instrumentation **7** C02039 (2012)

[5] V. Furtula *et al.*, Review of Scientific Instruments **83** 012507 (2012)

[6] B. Geiger *et al.*, Plasma Physics and Controlled Fusion, **53** 065010 (2011)

[7] S. Äkäslompolo *et al.*, Europhysics Conference Abstracts **34A** P5.113 (2010)

[8] M. Garcia-Munoz *et al.*, Nuclear Fusion, **51** 103013 (2011)

[9] W.W. Heidbrink *et al.*, Review of Scientific Instruments **79** 10E520 (2008)

[10] C.M. Muscatello *et al.*, Review of Scientific Instruments **81** 10D316(2010)

[11] A. Bortolon *et al.*, Review of Scientific Instruments **81** 10D728 (2010)

[12] M. Salewski *et al.*, Nuclear Fusion **51** 083014 (2011)

[13] M. Salewski *et al.*, "Tomography of fast-ion velocity-space distributions from synthetic CTS and FIDA measurements" submitted to Nuclear Fusion (2012)

[14] J. Egedal and H. Bindslev, Physics of Plasmas **11**(5) 2191-2198 (2004)

[15] S. Kubo *et al.*, Review of Scientific Instruments **81** 10D535 (2010)

[16] T. Ito *et al.*, Plasma and Fusion Research, **5** S2099 (2012)

[17] W.W. Heidbrink *et al.*, Plasma Physics and Controlled Fusion **49** 1457-1475 (2007)

[18] W.W. Heidbrink, Review of Scientific Instruments **81** 10D727 (2010)

[19] S.K. Nielsen *et al.*, Plasma Physics and Controlled Fusion **52** 092001 (2010)

[20] S.K. Nielsen *et al.*, Nuclear Fusion **51** 063014 (2011)