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Model Order Reduction of an Electro-Thermal Package Model ⋆

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Abstract: Model order reduction of an electro-thermal package model is discussed. This model is an electro-thermal coupled problem with geometrical variation. Due to the coupling and parameter variation, the package model leads to a system of nonlinear parametric differential-algebraic equations (DAEs). This model can be very large-scale which makes simulation computationally expensive. This calls for the application of model order reduction techniques, particularly for parametric model order reduction (pMOR) methods. However, many pMOR methods are restricted to linear parametric models. This motivates us to propose an approach which involves first decoupling the nonlinear parametric DAEs into differential and algebraic parts. Then, the differential and algebraic parts can be reduced separately.

Keywords: differential-algebraic equations, nonlinear parametric model, model order reduction

1. INTRODUCTION

An electro-thermal package can be modeled as an electro-thermal coupled problem. For general complex geometries, an accurate, physical model is desired. Spatial discretization using the finite-element method (FEM), finite volume method (FVM), or finite integration technique (FIT) leads to a large-scale parametric DAE model.

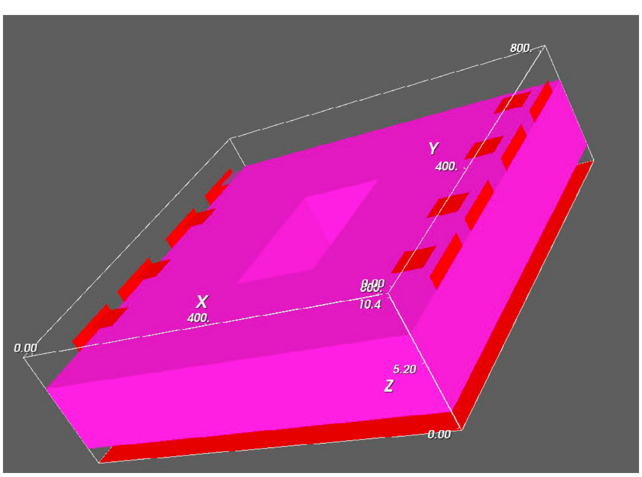

Fig. 1. Electro-thermal package

In Fig. 1, a FIT model of an electro-thermal package is shown. If we consider meshes that are topologically equivalent for different package thicknesses p, the parametric dependence will take the form

$$\mathbf{M}(p) = \mathbf{M}_0 + p\mathbf{M}_1 + \frac{1}{p}\mathbf{M}_2, \tag{1}$$

where $\mathbf{M}_i$ are constant coefficient matrices or tensors. If the parameter p symbolically appears in the package model, then its mathematical model can be written as a parametric nonlinear DAE of the form

$$\mathbf{E}(p)\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{A}(p)\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathcal{F}(p)\mathbf{x} + \mathbf{B}(p)\mathbf{u}, \tag{2a}$$

$$\mathbf{y} = \mathbf{C}(p)\mathbf{x} + \mathbf{D}(p)\mathbf{u}, \tag{2b}$$

where the matrix $\mathbf{E}(p) \in \mathbb{R}^{n\times n}$ is singular for every parameter p, $\mathbf{A}(p) \in \mathbb{R}^{n\times n}, \mathbf{B}(p) \in \mathbb{R}^{n\times m}, \mathbf{C}(p) \in \mathbb{R}^{\ell\times n}, \mathbf{D}(p) \in \mathbb{R}^{\ell\times m}$, and the tensor $\mathcal{F}(p) \in \mathbb{R}^{n\times n\times n}$ can be written in the form (1). This tensor is a 3-D array of n matrices. The state vector $\mathbf{x} = \mathbf{x}(t,p) \in \mathbb{R}^n$ includes the nodal voltages $\mathbf{x}_v \in \mathbb{R}^{n_v}$ and the nodal temperatures $\mathbf{x}_T \in \mathbb{R}^{n_T}$, i.e., $\mathbf{x} = (\mathbf{x}_v^{\mathrm{T}}, \mathbf{x}_T^{\mathrm{T}})^{\mathrm{T}}$. $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y} = \mathbf{y}(t,p) \in \mathbb{R}^\ell$ are the inputs and the desired outputs, respectively. In order to obtain accurate models, fine meshes must be used which lead to very large n compared to the number of inputs m and desired outputs ℓ. Despite the ever increasing computational power, simulation of these systems in acceptable time is very difficult because of the storage requirements and expensive computations, in particular if multi-query tasks are required. This calls for the application of parametric model order reduction (pMOR) methods. PMOR replaces (2) by a reduced-order model

$$\mathbf{E}_r(p)\frac{\mathrm{d}\mathbf{x}_r}{\mathrm{d}t} = \mathbf{A}_r(p)\mathbf{x}_r + {\mathbf{x}_r}^{\mathrm{T}}\mathcal{F}_r(p)\mathbf{x}_r + \mathbf{B}_r(p)\mathbf{u}, \tag{3a}$$

$$\mathbf{y}_r = \mathbf{C}_r(p)\mathbf{x}_r + \mathbf{D}_r(p)\mathbf{u}, \tag{3b}$$

where $\mathbf{E}_r(p), \mathbf{A}_r(p) \in \mathbb{R}^{r\times r}, \mathbf{B}_r(p) \in \mathbb{R}^{r\times m}, \mathbf{C}_r(p) \in \mathbb{R}^{\ell\times r}, \mathbf{D}_r(p) \in \mathbb{R}^{\ell\times m}$ and the tensor $\mathcal{F}_r(p) \in \mathbb{R}^{r\times r\times r}$ can

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also be written in the form (1), so that the parameter p is symbolically preserved in the reduced-order model. The dimension $r \ll n$ of the reduced-order model (3) is now much smaller than that of the original model (2). A good reduced-order model is one with small approximation error $\|\mathbf{y} - \mathbf{y}_r\|$ in a suitable norm $\|.\|$ for every arbitrary input $\mathbf{u}(t)$ and parameter p. There are many existing pMOR methods such as the robust pMOR algorithm in Benner and Feng (2014), based on implicit moment matching. However, these methods are limited to linear parametric models, i.e., when (2) has no tensor $\mathcal{F}(p)$. With the tensor included it is difficult to use the existing methods in their current form. This motivates us to propose a new approach which involves first decoupling the nonlinear parametric DAE model (2) into differential and algebraic parts. Then, the two parts can be reduced separately. We note that this approach has the same underlying ideas as index-aware MOR methods in Banagaaya (2014), but here different decoupling approaches must be used.

2. MODEL ORDER REDUCTION OF AN ELECTRO-THERMAL PACKAGE MODEL

In this section, we introduce a new reduction technique which can be used to reduce electro-thermal package models of the form (2). This is done by first decoupling (2) into differential and algebraic parts. Taking advantage of the natural structure of the matrices $\mathbf{E}(p) = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_T(p) \end{pmatrix}$, $\mathbf{A}(p) = \begin{pmatrix} \mathbf{A}_v(p) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_T(p) \end{pmatrix}$ and the tensor $\mathcal{F}(p)$ in (2), we obtain the following decoupled algebraic (electrical) and differential (thermal) parts,

$$\mathbf{0} = \mathbf{A}_v(p)\mathbf{x}_v + \mathbf{B}_v(p)\mathbf{u}, \tag{4a}$$

$$\mathbf{E}_T(p)\frac{\mathrm{d}\mathbf{x}_T}{\mathrm{d}t} = \mathbf{A}_T(p)\mathbf{x}_T + \mathbf{x}_v^{\mathrm{T}}\mathcal{F}_T(p)\mathbf{x}_v + \mathbf{B}_T(p)\mathbf{u}, \tag{4b}$$

$$\mathbf{y} = \mathbf{C}_v(p)\mathbf{x}_v + \mathbf{C}_T(p)\mathbf{x}_T + \mathbf{D}(p)\mathbf{u}, \tag{4c}$$

where $\mathbf{A}_v(p) \in \mathbb{R}^{n_v \times n_v}$, $\mathbf{B}_v(p) \in \mathbb{R}^{n_v \times m}$, $\mathbf{E}_T(p) \in \mathbb{R}^{n_T \times n_T}$ is a nonsingular matrix for every parameter p, $\mathbf{A}_T(p) \in \mathbb{R}^{n_T \times n_T}$, $\mathbf{B}_T(p) \in \mathbb{R}^{n_T \times m}$, $\mathbf{C}_v(p) \in \mathbb{R}^{\ell \times n_v}$, $\mathbf{C}_T(p) \in \mathbb{R}^{\ell \times n_T}$, and $\mathcal{F}_T(p) \in \mathbb{R}^{n_v \times n_v \times n_T}$ is a nonzero part in the tensor $\mathcal{F}(p)$. We note that the total dimension of the decoupled system (4) is equal to the dimension of (2), that is, $n = n_v + n_T$. From, (4), we observe that the initial condition of (2) must be a consistent initial condition, that is, $\mathbf{x}(0,p) = (\mathbf{x}_v(0,p)^{\mathrm{T}}, \mathbf{x}_T(0,p)^{\mathrm{T}})^{\mathrm{T}}$, where $\mathbf{x}_v(0,p) = -\mathbf{A}_v(p)^{-1}\mathbf{B}_v(p)\mathbf{u}(0)$ and $\mathbf{x}_T(0,p)$ can be chosen arbitrarily. The next step is to apply pMOR to (4a) and (4b), separately. Considering the series expansion of x_v w.r.t the parameter p in (4a), the implicit moment-matching pMOR method proposed in Benner and Feng (2014) can be applied to compute a projection matrix $\mathbf{V}_v$ and to get a reduced parametric algebraic model of (4a). If we first reduce (4a), this reduction induces a reduction in the differential part (4b). We note that pMOR of (4a), introduces an approximation to the nonlinear term in (4b). However, the dimension of (4b) is unreduced but the dimension of the matrix blocks of the tensor $\mathcal{F}_T(p)$ is reduced. In order to reduce the dimension of the differential part (4b), the same pMOR approach in Benner and Feng (2014) can be applied by ignoring the nonlinear term (coupled term) to construct another projection matrix $\mathbf{V}_T \in \mathbb{R}^{n_T \times r_2}$. The two separate reduced-order models are further coupled into a single system which leads to a reduced-order model of the electro-thermal package model given by

$$\mathbf{0} = \mathbf{A}_{v_r}(p)\mathbf{x}_{v_r} + \mathbf{B}_{v_r}(p)\mathbf{u}, \tag{5a}$$

$$\mathbf{E}_{T_r}(p)\frac{\mathrm{d}\mathbf{x}_{T_r}}{\mathrm{d}t} = \mathbf{A}_{T_r}(p)\mathbf{x}_{T_r} + \mathbf{V}_T^{\mathrm{T}}\mathbf{x}_{v_r}^{\mathrm{T}}\mathcal{F}_{T_r}(p)\mathbf{x}_{v_r} + \mathbf{B}_{T_r}(p)\mathbf{u}, \tag{5b}$$

$$\mathbf{y}_r = \mathbf{C}_{v_r}(p)\mathbf{x}_v + \mathbf{C}_{T_r}(p)\mathbf{x}_{T_r} + \mathbf{D}(p)\mathbf{u}, \tag{5c}$$

where $\mathbf{A}_{v_r}(p) \in \mathbb{R}^{r_1 \times r_1}$, $\mathbf{B}_{v_r}(p) \in \mathbb{R}^{r_1 \times m}$, $\mathbf{E}_{T_r}(p)$, $\mathbf{A}_{T_r}(p) \in \mathbb{R}^{r_2 \times r_2}$, $\mathbf{B}_T(p) \in \mathbb{R}^{r_2 \times m}$, $\mathbf{C}_{v_r}(p) \in \mathbb{R}^{\ell \times r_1}$, $\mathbf{C}_{T_r}(p) \in \mathbb{R}^{\ell \times r_2}$, and a tensor $\mathcal{F}_{T_r}(p) \in \mathbb{R}^{r_1 \times r_1 \times n_T}$. $r_1 \ll n_v$ and $r_2 \ll n_T$ is the reduced dimension of (4a) and (4b), respectively.

3. NUMERICAL RESULTS

We consider an electro-thermal package model from MAGWEL NV. It is a system of the form (2) with dimension $n = 9193$, $m = 34$ inputs and $\ell = 68$ outputs. The system is excited by the input $\mathbf{u}(t) = (\mathbf{u}_1(t), \ldots, \mathbf{u}_{34}(t))^{\mathrm{T}}$, where $\mathbf{u}_1(t) = 1, \mathbf{u}_2(t) = \cdots = \mathbf{u}_{17}(t) = 0$,

$$\mathbf{u}_{18}(t) = \begin{cases} 75 \times 10^8 t + 75 & \text{If } t \le 10^{-8} \\ 150 & \text{If } t > 10^{-8} \end{cases},$$

$\mathbf{u}_{19}(t) = \cdots = \mathbf{u}_{34}(t) = 75$. We use $\mathbf{x}_T(0,p) = 75$ as the initial condition for the thermal part. We were able to decouple this system into the form (4) leading to an algebraic part and a differential part of dimension $n_v = 1122$ and $n_T = 8071$, respectively. Using the approach discussed in Sec. 2, we have reduced the dimension of the algebraic part and that of the differential part to $r_1 = 32$ and $r_2 = 577$, respectively. The reduced-order model can be written into the form (5) of dimension $r = r_1 + r_2 = 609$ and it leads to an accurate solution with output error $\|\mathbf{y}-\mathbf{y}_r\|_2/\|\mathbf{y}\|_2 \le 3.3\times 10^{-6}$. As an illustration we compare in Fig. 2, the output solution of $y_{48}(t)$ computed by full and reduced simulation, respectively.

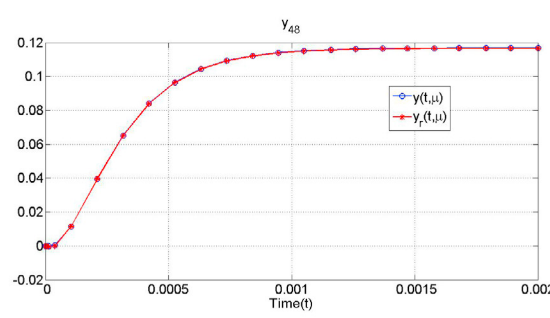

Fig. 2. Output comparison of $y_{48}(t)$.

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