Bootstrap current derived from different model collision operators

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Introduction

The bootstrap current is an important quantity for the construction of the advanced Tokamak reactor. For this reason it is often studied experimentally and theoretically. The comparison between experiment and theory is always done using the famous formula of Hirshman and Sigmar (1) which is based on an asymptotic solution of the kinetic equation in the limit of very small collision frequency. This calculation was extended by Kikuchi (2) to the range of higher collisionality. In this theory are all collisions described by the Lorentz operator containing only pitch angle scattering.

In order to proof the accuracy of the coefficients entering the bootstrap current the kinetic equation was solved for arbitrary collisionality without the assumption of large aspect ratio and without neglecting energy scattering. Is is statet in the literature (1) that energy scattering has no effect on the parallel viscosity. On the other hand the results from a precise numerical calculation presented here show a small change of the viscous force due to energy scattering such, that the contribution of bootstrap current related to the temperature gradient is in the range of low collisionality reduced by about 30 % compared with the results from the pitch angle scattering model. For the coefficient of the pressure gradient in the bootstrap current one finds always the same value. This quantity is obviously insensitive against a change in the collision model.

Drift kinetic equation and collision operator

The linearized drift kinetic equation for a pure plasma consisting of electrons and ions with charge Z = 1 takes the form given elsewhere in the literature (3). It calculates the solution for the distribution function $g_{\prime} = g_{\prime}(\eta, w)$ defined as the deviation from a Maxwellian ($f = f_M^{(0)} + g_{\prime}$) according to an expansion into the gyroradius parameter $\delta = \rho_\Theta / L$. The velocity variables used here are the random velocity $\vec{w}$ and $\eta = w_{\|}/w_{\prime}$, which is the pitch angle. The dependence of $g_{\prime}$ from the random velocity involves the condition

$$\int w_{\prime\prime} g_{\prime} \delta^3 w = 0, \tag{1}$$

where $w_{\prime\prime}$ is the random velocity parallel to the magnetic field (the condition perpendicular to the magnetic field $\int \vec{w}_\perp g_{\prime} \delta^3 w = 0$ is fulfilled automatically, because $g_{\prime}$ is independet from the gyrophase). In this work, circular magnetic surfaces are assumed with the magnetic field having the standard form

$$(B_\rho, B_\Theta, B_t) = \left(0, B_{\Theta_{\prime o}} R_o/R, B_{t,o}\frac{R_o}{R}\right),$$

where $R = R_o(1 - \epsilon cos\Theta)$ is the major radius and $\epsilon = \rho/R_o$ the inverse aspect ratio. The collision operator for like particle collisions used here is a model operator close to that as described in the paper of Bolton-Ware (4). This operator has all the properties of the full collision operator: conservation of particles, momentum and energy. It contains also the decrease of the Coulomb cross section with velocity of the colliding particle as w^{-3}. It reads for the electrons

$$I_{e,e} = \frac{3\sqrt{2\pi}}{4}\nu_{e,i}\Bigg(\frac{A_o(x)}{x^3}\frac{\partial}{\partial\eta}\Big((1-\eta^2)\frac{\partial g_1}{\partial\eta}\Big) - \tilde{\lambda}B_o(x)g_1$$
$$+ \frac{4}{3\sqrt{2\pi}}xP_1f_M^{(o)}\int_o^\infty\Big(A_o + 0.5\tilde{\lambda}x^3B_o\Big)a_1e^{-x^2/2}\delta x - f_M^{(o)}\frac{P_o}{\sqrt{2\pi}}(x^2-5)\int_o^\infty\tilde{\lambda}B_oa_ox^2e^{-x^2/2}\delta x$$
$$+ f_M^{(o)}\frac{P_o}{3\sqrt{2\pi}}(x^2-3)\int_o^\infty\tilde{\lambda}B_oa_ox^4e^{-x^2/2}\delta x\Bigg) \tag{2}$$

where $x = w/(T_e/m_e)^{1/2}$. The functions A_o, B_o are given in paper (4). The same expression holds for the ions, replacing $\nu_{e,i}$ with $\sqrt{2}\nu_{i,i}$ the Braginskii ion-ion collision time (in the case of ions one has $x = w/(T_i/m_i)^{1/2}$). The first term in equ. (2) is the well known pitch angle cross section which enters also the electron-ion collision term $I_{e,i}$. The second term looks like a modified Krook term which contains energy scattering, but contributes also to momentum exchange. The correction terms proportional to the Legendre ploynomials P_o and P_1 are needed to guarantee the conservation properties. The parameter $\tilde{\lambda}$ is used to modify the energy scattering effect.

Method of Solution:

With this model collision operator, the distribution function g_1 could easily be expanded in terms of orthogonal functions: That is, Legendre polynomials in η

$$g_1 = f_M^{(o)}\sum_{n=o}^{N}a_n(\Theta,x)P_n(\eta) \tag{3}$$

and a Fourier series in the poloidal angle Θ. Each Legendre polynomial coefficinet a_n consists of $2M+1$ Fourier modes where M is the Fourier mode number. In the equations for the Legendre ploynomials P_o and P_1 there enter the unknown integral relations from equ. (2). These where calculated from a system of $6M+1$ linear equations which exists between these $6M+1$ variables: In order to find this system the solution for the Legendre polynomials $a_2\ldots,a_N$ was inserted into the equ. for P_1 and the remaining equations for a_o and a_1, where resolved such, that the quantities a_o and a_1 could be expressed by the driving terms and the unknown integral variables. Multiplying each Fourier quantity of a_o and a_1 with the functions within the integrals of equ. (2) and integration over the volocity space established the system of equations. The side condition (1) which reads

$$\int_o^\infty e^{-x^2/2}x^3a_1(\Theta,x)\delta x = 0 \tag{4}$$

is a condition for only <u>one</u> constant C_o which is posed to be zero: From the zero moment equation for particle conservation it is found that the solution of the kinetic equation fulfills the relationsship $< \int_o^\infty e^{-x^2/2}x^3 a_l \delta x >= C_o$, where $< \ldots >$ is the flux average.

With $C_o = 0$ the momentum is conserved and the solution takes the form (given here for the electrons):

$$\frac{eU_L q}{2\pi T}C_1 + 2\alpha \tilde{d}_{//}^{(o)} C_2 + \frac{2\alpha q \pi P'}{B_{t,o} e n_e (T_e/m_e)^{1/2}} C_3 + \frac{\rho_g q T'}{12T} C_4 - \frac{q}{\epsilon}\tilde{V}_{\Theta,o} C_5 = 0 \qquad (5)$$

(that is: $e-$ the charge (here is $e > 0$), n_e- density, $P'-$ pressure gradient, U_L the Loop Voltage, $\alpha = \frac{3\sqrt{2\pi}}{4} R_o q \nu_{e,i}/(T_e/m_e)^{1/2}, q-$ safety factor, $\tilde{d}_{//}^{(o)} = (V_{e,//}^{(o)} - V_{i,//}^{(o)})/(T_e/m_e)^{1/2}$, $\tilde{V}_{\Theta,o} = V_{\Theta,o}^{(e)}/(T_e/m_e)^{1/2}, \rho_g-$ electron gyroradius)

In this formula the visours force and the $e-i$ friction force are not seperated. The ion momentum law looks much simpler neglecting terms of order $(m_e/m_i)^{1/2}$, that is

$$\frac{\rho_g^{(i)} q T_i'}{12T_i} C_4 + \frac{q}{\epsilon}\tilde{V}_{\Theta,o}^{(i)} C_5 = o \qquad (6)$$

The most difficult parameter ranges in the calculation are where collision frequency is very small. Here about 50 Legendre polynomials and 11 to 13 Fourier components (corresponding to M = 5 or 6) are needed to obtain convergence.

Results

The bootstrap current is calculated with the formula

$$j_{bs} = -\frac{q}{\sqrt{\epsilon}B_t}\left(\frac{L_{31}}{\sqrt{\epsilon}}P' + \frac{L_{32}^{(e)}}{\sqrt{\epsilon}} n_e T_e' - \delta_3 \frac{L_{31}}{\sqrt{\epsilon}} n_e T_i'\right) \qquad (7)$$

(here $P = P_e + P_i$ is the total pressure).

Values for the coefficients are given in the tabels 1 to 3. The energy scattering parameter $\tilde{\lambda}$ varies from 10^{-2} to 5.4. The results for $\tilde{\lambda} = 10^{-2}$ (the programm does not allow for $\tilde{\lambda} = 0$) agree perfect with that given by Kikuchi (2). The parameter $\tilde{\lambda} = 5.4$ was choosen by Bolton-Ware (4) because this value fits perfect to the Spitzer-Härm parallel conductivity.

ν^*	$\tilde{\lambda} = 10^{-2}$	$\tilde{\lambda} = 1.0$	$\tilde{\lambda} = 5.4$
0.025	1.41	1.42	1.46
0.05	1.34	1.35	1.38
0.1	1.24	1.25	1.27
0.2	1.11	1.11	1.11
0.3	1.02	1.01	0.99

Table 1: The bootstrap current coefficient $L_{31}/\sqrt{\epsilon}$ for $\epsilon = 0.22$.

ν^*	$\tilde{\lambda} = 10^{-2}$	$\tilde{\lambda} = 1.0$	$\tilde{\lambda} = 5.4$
0.025	−0.29	−0.434	−0.715
0.05	−0.161	−0.30	−0.541
0.1	−0.020	−0.14	−0.34
0.2	0.13	0.026	−0.12
0.3	0.21	0.12	−0.001

Table 2: The bootstrap current coefficient $L_{32}^{(e)}/\sqrt{\epsilon}$ for $\epsilon = 0.22$.

ν^*	$\tilde{\lambda} = 10^{-2}$	$\tilde{\lambda} = 1.0$	$\tilde{\lambda} = 5.4$
0.025	0.62	0.68	0.73
0.05	0.56	0.615	0.65
0.1	0.48	0.53	0.55
0.2	0.39	0.43	0.41
0.3	0.33	0.36	0.32

Table 3: The coefficient δ_3 of the ion poloidal rotation $V_\Theta^{(i)} = \delta_3 T_i'/eB$ for $\epsilon = 0.22$

Conclusion

The effect of energy scattering reduces the amount of bootstrap current related to the temperature gradient: For equal electron and ion temperature one can write $j_{bs} = -q/\sqrt{\epsilon} B_T (L_n \cdot T \cdot n_e' + L_T n_e T')$.

For the example $\nu^* = 0.025$ and $\epsilon = 0.22$ one finds from the results in the tables:

$L_n = 2.82$ $\quad$ $L_T = 1.66$ (for $\tilde{\lambda} = 10^{-2}$)

and $L_n = 2.92$ $\quad$ $L_T = 1.14$ (for $\tilde{\lambda} = 5.4$)

Central peaking of density is needed in order to achieve a reasonable large bootstrap-current.

References

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