Three Dimensional Computation of Collisional Drift Wave Turbulence and Transport in Tokamak Geometry

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Computations of fluid drift turbulence in a nonlocal model tokamak geometry are presented. Profiles and fluctuations interact with each other self-consistently. A self-consistent limiter scrape-off layer (SOL) is incorporated. Electron drift wave dynamics dominate the fluctuation energetics. Parallel correlation lengths in the SOL are in agreement with experimental observations, even though the dynamics is still three-dimensional. The results reproduce well the qualitative and quantitative properties of observed fluctuations, but predict overly strong transport in the SOL.

It is of general interest to directly simulate the process of anomalous transport of thermal energy and particles in a magnetically confined plasma, with a fluid model system of equations. Ideally, the domain of computation would extend from center to edge, and the run time would be of order a global confinement time. As this is not yet feasible, the traditional approach is to carry a computation for a small domain whose extent perpendicular to the magnetic field lines is much less than the profile scale length, $L_\perp$. The fluctuation dynamics would be local, with profiles given as system parameters. An alternative approach is to carry an intermediate domain, whose extent is one or two $L_\perp$, so that the system's thermodynamics can be studied while still faithfully representing the scale, $\Delta \ll L_\perp$, at which the turbulence actually occurs [1]. This is already feasible for small tokamaks, from which we have the best data.

The computations are done in the geometry of an axisymmetric tokamak, with shifted-circular flux surfaces and a toroidal limiter in an outer portion. The reference flux surface is the last closed flux surface (LCFS), with minor and major radii a and R_0, and volume $V_a = 2\pi^2 a^2 R_0$. The coordinate system begins with the Hamada (V, θ, ζ), with θ and ζ periodic on $[0, 1]$, and then transformed via $\xi = \zeta - q\theta$, with $q(V)$ the field line pitch parameter. Only one contravariant component of the magnetic field, $B^\theta = \chi'$, with $\chi(V)$ the poloidal flux, is nonzero. The field line connection length is $L_\parallel = B_0/\chi' \approx 2\pi q R_0$, with B_0 the average field strength. The periodicity constraint, $f(V, \theta + 1, \xi - q) = f(V, \theta, \xi)$, serves as the boundary condition in θ in the closed field line region; in the SOL the standard Debye sheath model is used. The dependent variable V-derivatives are assumed to vanish on inner and outer boundaries, at which sources are prescribed. Domain sizes are quoted in terms of V_a and a toroidal periodicity, k_0, in terms of which ξ is periodic on $[0, k_0^{-1}]$, equivalent to keeping only multiples of k_0 in a Fourier expansion in ξ (as well as mode zero). This amounts to a flux-tube truncation which retains the periodicity constraint.

The equations are those of Braginskii [2], with perpendicular dissipation neglected and with a model for electron and ion Landau damping in the form of limits placed on the parallel dissipation. The ions are cold. The electrostatic drift approximation is used, such that a parameter $\delta_0 = \rho_0/a \ll 1$, with a the area-averaged minor radius of the LCFS and ρ_0 the ion gyroradius at reference parameters [n_0, T_0, and B_0 at $r/a = (V/V_a)^{1/2} = 1$]. This gives the electron and ion perpendicular velocities and heat fluxes in terms of their drifts: $\mathbf{v}_\perp{}^\mu = \mathbf{v}_E^\mu + \mathbf{v}_*^\mu$ with $ne\mathbf{v}_*^\mu = \hat{F}^{\mu\nu}\nabla_\nu p_e$ and $\mathbf{q}_{e\wedge}^{\ \mu} = (5/2e)nT_e\hat{F}^{\mu\nu}\nabla_\nu T_e$ for the electrons, and $\mathbf{u}_\perp{}^\mu = \mathbf{v}_E^\mu$ for the ions, where $\mathbf{v}_E^\mu = -\hat{F}^{\mu\nu}\nabla_\nu\phi$ is the ExB velocity of both, and $\hat{F}^{\mu\nu} = (c/B^2)\epsilon^{\mu\nu}{}_\eta B^\eta$ is the drift operator. The ion polarisation drift, due to the inertia, is retained: $\mathbf{v}_p^\mu = -\hat{F}^\mu{}_\nu\left[(\partial/\partial t) + \left(\mathbf{v}_E^\eta + u_\parallel b^\eta\right)\nabla_\eta\right]\mathbf{u}_\perp{}^\nu$, where $u_\parallel = b^\mu u_\mu$ and $b^\mu = B^\mu/B$. The equations appear as

$$
\begin{aligned}
-\nabla_\mu ne\mathbf{v}_p^\mu &= \nabla_\mu\left(b^\mu J_\parallel - ne\mathbf{v}_*^\mu\right)\\
\frac{\partial n}{\partial t} &= -\nabla_\mu n\left(b^\mu v_\parallel + \mathbf{v}_\perp{}^\mu\right)\\
1.5\frac{\partial T_e}{\partial t} &= -1.5\left(b^\mu v_\parallel + \mathbf{v}_\perp{}^\mu\right)\nabla_\mu T_e - T_e\nabla_\mu\left(b^\mu v_\parallel + \mathbf{v}_\perp{}^\mu\right) - n^{-1}\nabla_\mu\left(b^\mu q_{e\parallel} + \mathbf{q}_{e\wedge}^{\ \mu}\right)\\
\frac{\partial u_\parallel}{\partial t} &= -\left(b^\mu u_\parallel + \mathbf{v}_E^\mu\right)\nabla_\mu u_\parallel - (nM_i)^{-1}\left(\nabla_\parallel p_e + \nabla_\mu b^\mu nM_i D_{i\parallel} b^\nu\nabla_\nu u_\parallel\right)\\
J_\parallel &= ne(u_\parallel - v_\parallel) = neD_{e\parallel}\left(1.71T_e^{-1}\nabla_\parallel T_e + n^{-1}\nabla_\parallel n - eT_e^{-1}\nabla_\parallel\phi\right)\\
q_{e\parallel} &= -1.6nD_{e\parallel}\nabla_\parallel T_e + 0.71nT_e(v_\parallel - u_\parallel)
\end{aligned}
$$

where the first is an equation for ϕ, $\nabla_\parallel = b^\mu\nabla_\mu$, and the dissipation parameters are given by $D_{e\parallel}^{-1} = 0.51\nu_e V_e^{-2} + (L_\parallel V_e)^{-1}$ and $D_{i\parallel} = L_\parallel c_s$ with $V_e^2 = T_e/m_e$ and $c_s^2 = T_e/M_i$.

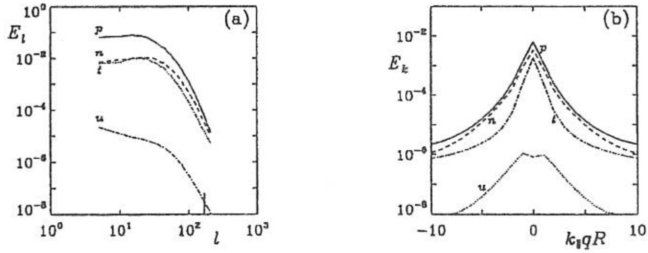

Figure 1. Fluctuation spectra, perpendicular (a) and parallel (b) wavenumbers, of the mean squared amplitudes of $\widetilde{\phi}$ (p), $\widetilde{n}$ (n), $\widetilde{T}$ (t), and $\widetilde{u}_\parallel$ (u). The long vertical hash denotes $k_\xi\rho_s \sim 1$. The source and sink curves [1] show ∇T as the strongest free energy source, low-k_ξ as a sink region, and low-$k_\parallel$ as the only source region.

Most of the results obtained from this system so far have been limited by the use of drift-ordering, which drops terms formally small by $\Delta \ll L_\perp$ (*e. g.*, $u_\parallel\nabla_\parallel$ in the fluctuations or magnetic divergences like $\nabla_\mu ne\mathbf{v}_*^\mu$ in the profiles) while still still allowing the profiles and fluctuations to self-consistently interact [1]. An example of such a result is shown in Fig. 1, the parallel and perpendicular spectra of fluctuations in a domain $V/V_a \in [0.9, 1.1]$ and $k_0 = 5$ for TEXT edge parameters ($a = 27\,\mathrm{cm}$, $R_0 = 4.0$, $B_0 = 2\,\mathrm{T}$, $T_0 = 30\,\mathrm{eV}$ and $n_0 = 3\times10^{12}\,\mathrm{cm}^{-3}$). The radial extent was two $L_\perp$. The collisionality parameter was $C_0 = (0.51\nu_e L_n)(m_e/M_i)(qR/L_n)^2 = 5.8$ and δ_0 was 0.002. The profiles were maintained by feedback control, where n/n_0 and T_e/T_0 were damped towards 2.5 and 0.4 at the inner and outer boundaries, respectively (numerical details appear in [1]). Fig. 2 shows the parallel transfer between neighboring regions of parallel wavenumber space and the scaling with C_0 of such "local" runs. The runs with the tokamak metric were compared to counterparts with a sheared slab metric, both with and without the magnetic divergences ("ballooning"). This shows that the importance of ballooning depends strongly on the geometry; it is relatively unimportant compared to the tokamak's poloidal variation of the drift scale $\rho_s = c(T_e M_i)^{1/2}/eB$ in exciting asymmetries in the more collisional regime, but as C_0 drops the range of $k_\parallel$-interaction narrows and the ballooning is relatively more important. The parallel transfer rates were found to be more than twice as strong in tokamak as in slab-metric geometry. The slab-metric runs reproduce the gist of other current work [3].

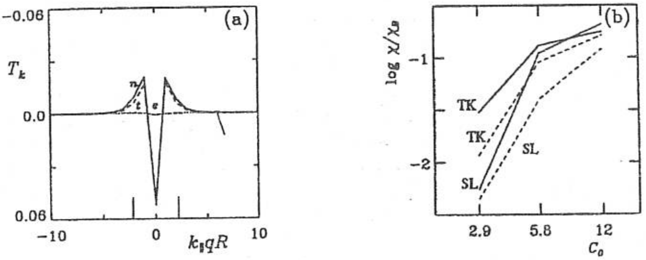

Figure 2. (a) Normalised parallel transfer rates, in ExB (e), density (n), and temperature (t) free energy components. Transfer is out of the $k_\parallel = 0$ region, with the ExB contribution negligible. (b) Dependence of energy transport, $\chi/\chi_B = (q_H/n\nabla T)(eB/cT)$, on the drift wave collisional parameter, C_0, for tokamak (TK) and slab (SL) geometry, with (solid) and without (dashed) the magnetic divergences (see text).

Another run was carried out, with $V/V_a \in [0.9, 1.1]$ and $k_0 = 10$, for ASDEX-Upgrade Ohmic edge parameters ($a = 60\,\mathrm{cm}$, $R_0 = 2.75$, $B_0 = 2\,\mathrm{T}$, $T_0 = 60\,\mathrm{eV}$, and $n_0 = 10^{13}\,\mathrm{cm}^{-3}$), with a SOL for $V/V_a > 1.0$. The drift wave parameters were $C_0 = 7.5$ and $\delta_0 = 0.0012$, not very different from the TEXT case. The SOL was determined by an inboard toroidal limiter at $\theta = \pm 0.5$. An inner-edge heat flux of $Q_0 = 0.6\,\mathrm{MW}(4\pi^2 aR)^{-1}$ was set, and the SOL was left as the sink according to the Debye-sheath boundary conditions. The profiles are shown in Fig. 3; these were not equilibrated over the $\sim 0.8\,\mathrm{msec}$ run time, but there has been reaction to the turbulence and to Q_0. In the SOL itself the confinement time is much shorter, as the gradients were flattened within 0.2 msec. The fluctuations were somewhat different in the closed and open field line regions; the turbulence and transport were stronger in the SOL. The parallel correlation length, $\lambda_\parallel$, for $\widetilde{T}$ was twice as long in the SOL. For $\widetilde{n}$ and $\widetilde{\phi}$ the difference was closer to 50%. The main point of this, however, is that the three-dimensional parallel dynamics, *i. e.*, the exchange of free energy between $k_\parallel = 0$ and $k_\parallel = \pm qR$ components, continues to play a strong role even in the SOL and even with the long $\lambda_\parallel$. Although the turbulence notices the existence of open field lines, there is no specialised role for the flute-like target-plate instability [4,5], since the collisional drift wave nonlinear instability is more robust.

It is important at this stage to point out two discrepancies between these simulations and the situation observed in tokamak edges. The profiles were found to be too flat in the

SOL, indicating that the turbulence is too strong there in the numerical model. The amplitude itself was in good agreement but the coupling among $\widetilde{\phi}$, $\widetilde{n}$, and $\widetilde{T}$ appears to be too immediate in the collisional drift wave model. It may be that the addition of electromagnetic effects, usually considered to be small for these parameters ($\hat{\beta} = 4\pi nTL_{\parallel}^2/B^2L_{\perp}^2 \ll 1$), may subtly interrupt the dissipative coupling one has in the electrostatic model; in pure two-dimensional simulations this appears to be the case [6]. The other discrepancy is the lack of a particle pinch. When there are no sources and no particle flux, ∇n is close to zero. Although the need for a particle pinch in tokamak edges depends on inferences from a (perhaps irrelevant) diffusion model [7], it is well known that in the interior there is a $\nabla n \neq 0$ well within the source regions. The mechanism for that will have to be something other than collisional drift wave physics. Finally, it is to be noted that this type of turbulence is not effective at all above about 120 eV in the ASDEX-Upgrade case, and the model indeed loses validity at about this point. This makes the collisional drift wave model by itself invalid for the study of the L-to-H transition.

It is important as well to note the successes: the fluctuation amplitudes agree well with observations ($e\widetilde{\phi}/T \sim 0.2 \gtrsim \widetilde{n}/n \gtrsim \widetilde{T}/T$), the computed thermal energy flux on closed field lines is about 0.3 to 0.5 of observed values. Concerning the parallel mode structure, observed fluctuations in both tokamak and stellarators preserve a correlation of 0.8 over 10 m [5], and this has been used to advance two-dimensional flute-like turbulence models with the inference that $k_{\parallel}$ should be nearly zero [4,5]. However, the correlation function measured here drops to 0.8 in about $0.2L_{\parallel}$, or about 8 m, in good agreement. This does however correspond to enough of a parallel gradient that collisional drift wave physics is robustly excited, and the special case of collisional drift waves on open field lines such that $k_{\parallel} \to 0$ with the Debye sheath providing dissipative coupling among $\widetilde{\phi}$, $\widetilde{n}$, and $\widetilde{T}$, elsewhere called the target-plate instability model [5], never enters.

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