Neoclassical Transport in a Strong Magnetic Field

by

Christian T. Dum and Dieter Pfirsch

Max-Planck-Institut für Plasmaphysik
Garching bei München, Westgermany

Abstract: The effect of a strong magnetic field $\Omega_e > \omega_e$
on collisions is investigated. These results are applied to
neoclassical transport. Except for extremely strong magnetic
fields, neoclassical transport is not modified significantly.

So far in neoclassical transport theory a collision term had
been used which is appropriate for a uniform plasma in the
absence of a magnetic field, either a Lorentz model collision
term which describes pitch angle scattering [1] or the Landau
collision integral [2]. In some current and planned experiments
as well as future fusion reactors the electron cyclotron
frequency Ω_e exceeds the electron plasma frequency ω_e. Since
the duration of a collision is of the order of a plasma period
one must expect modifications of the collision process
for $\Omega_e \gtrsim \omega_c$. We have generalized the collision term to take
account of the magnetic field, anisotropy and inhomogeneity of
the plasma. From a finite Larmor radius expansion it can be
shown that the flux across a magnetic surface consists of a
classical flux which is the same as for a uniform magnetic
field, and a neoclassical flux which directly depends on the
nonuniformity of the magnetic field. For the following it is
very important to note that for the neoclassical flux we need
only terms of the collision integral which depend on velocity
gradients, i.e. correspond to a homogeneous plasma.
We consider transport processes in an axisymmetric system.
For sufficiently small inverse aspect ratio $\epsilon = r/R$ (r minor
radius, R major radius) the neoclassical flux dominates the
classical flux. In the banana regime of weak collisions

$$D_{neocl}/D_{cl} \approx \left(\frac{2\pi}{L}\right)^2 \epsilon^{-3/2} \tag{1}$$

where $\frac{2\pi}{L} = q \cdot O(1)$ is the so called safety factor. Thus in the
following we shall investigate the neoclassical flux only.
Guiding center flux in a strong magnetic field has been
investigated by several authors [3,4,5,6].
The dominant contribution to the neoclassical flux comes from
the boundary layer between trapped and untrapped particles [1].
Trapped particles have a very small velocity parallel to the
magnetic field, $v_\parallel \lesssim \sqrt{2\epsilon}\, v$. It is to be expected that just
these particles will be most strongly affected by the magnetic
field. We describe collisions by the analog of the Lorentz
collision term for a plasma in a magnetic field.

$$\frac{\delta f}{\delta t} = \frac{1}{2\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \; \nu_{\theta\theta} \frac{\partial f}{\partial\theta} \tag{2}$$

where θ is the angle between the velocity and the magnetic
field. The collision frequency $\nu_{\theta\theta}$ is given by

$$\nu_{\theta\theta} = \frac{3\omega_e}{\Lambda v^3} H(\alpha v, \vartheta_\parallel) \tag{3}$$

where

$$H(\alpha v, \vartheta_\parallel) = \frac{4\vartheta_\perp}{\vartheta_\perp^2} \int_0^{\alpha v \Lambda} dx\, x \sum_{n=1}^{\infty} \frac{n^2 J_n^2(\vartheta_\perp x)}{[\vartheta_\parallel^2(\alpha^2 v^2 + x^2) + n^2]^2} \tag{4}$$

and $\Lambda = 12\pi n \lambda_D^3$, λ_D Debye length, the velocity v is normalized to
the thermal velocity and $\vartheta_\parallel = v_\parallel/v$, $\vartheta_\perp = v_\perp/v$, $\alpha = \omega_e/\Omega_e$. For
$\alpha v \to \infty$, $H \to \log\Lambda$ the usual Coulomb logarithm.

We assume a magnetic field of the form $B = B_0/h$, $h = 1 + \epsilon\cos\delta$

where δ is the small azimuth. For trapped particles (and
boundary layer) the flux is proportional to

$$I_{TP} = \int_{1-\epsilon}^{1+\epsilon} d\lambda \; \lambda \langle h \hat{\nu}_{\theta\theta} v/v_\parallel \rangle \tag{5}$$

where $\lambda = h^2 \sin^2\theta$, $v_\parallel^2 = v^2(1 - \lambda/h)$
and

$$\hat{\nu}_{\theta\theta} = \int_0^{\infty} dv \; 4\pi v^2 \left(\frac{1}{2\pi}\right)^{3/2} e^{-v^2/2} \nu_{\theta\theta} \tag{6}$$

is the "energy scattering frequency". (The collision term
conserves energy). The average in δ is taken over orbit of a
trapped particle with given λ and r. Thus it is seen that the
particle flux is proportional to the "energy dissipation"

over a trapped particle orbit. A similar expression may be
obtained for the untrapped particles, but their contribution
is much smaller. To find the effect of a strong magnetic field
on neoclassical transport we thus have to investigate the
dependence of $\nu_{\theta\theta}$ on the pitch angle for $0 < \vartheta_\parallel < \sqrt{2\epsilon}$.
We have succeeded in an approximate evaluation of (3) by various
methods. We find that the collision frequency goes to zero as
$\vartheta_\parallel$ for $\vartheta_\parallel \lesssim 1/\alpha v \Lambda$. Since Λ is a very large number, typically
10^7, this velocity is so small that we have to take account
of the ion thermal motion and higher order correlations.
For parallel velocities in the range $1/\alpha v \Lambda < \vartheta_\parallel < 1/2\alpha v$
we have

$$H \approx \frac{1}{\vartheta_\perp^3} \log(\alpha v \vartheta_\parallel \Lambda) \tag{7}$$

Most of the trapped particle motion falls into this range.
Performing the velocity integration (6) will approximately
replace αv by α. Since the dependence of (7) on the magnetic
field is logarithmic we see that only for extremely strong
magnetic fields $\alpha \lesssim 1/(e^{1/L}\Lambda)$ do we have to expect a significant
modification of transport. The logarithmic dependence on the
magnetic field may be understood from the fact most of the
electrons are not magnetized, $kr_L > 1$. Magnetized electrons
change only their guiding center positions and do not contri-
bute to (3). Guiding center diffusion does not take place if
the interaction time is short compared to a gyro period. For
velocities approximately parallel to the magnetic field, e.g.
this requires impact parameters less than v/Ω_e, in any case
less than the shielding length λ_D.
The minimum impact parameter is λ_D/Λ. From these considera-
tions we can understand the magnetic field dependence

$$H \approx \frac{1}{2} \log\left(\frac{\alpha^2 v_\perp^2 \Lambda^2}{1 + \alpha^2 v^2}\right) \tag{8}$$

for $\vartheta_\perp \lesssim 1/\alpha v \Lambda$.

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