Development and Saturation of the Cross-Field Current Driven Instability

by

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Anomalous resistivity observed in collisionless shock waves and Θ-pinches is generally believed to be caused by an electrostatic instability driven by an electric current perpendicular to the magnetic field. In contrast to earlier concepts it was shown only recently that the magnetic field plays an important role in the heating process of the electrons [1]. In a previous paper we investigated quantitatively the dependence of the growth rate of the instability on parameters such as m_e/m_i, Ω_e/ω_{pe}, v_d/v_{the}, v_d = drift velocity, using a simple numerical model which served as a substitute of a dispersion relation. It was found that over a wide range $\gamma \approx \omega_{pi} \left(\frac{\Omega_e}{\omega_{pe}} \frac{v_d}{v_{the}}\right)^{1/2}$, i.e. the growth rate is only weakly depending on the magnetic field strength. In the present work we report results of numerical simulations concerning the strongly nonlinear phase of the instability, in particular the effective collision frequency and the switch-off drift velocity.

The numerical set-up is either a one-dimensional system of 512 λ_D or a two-dimensional system of 128 x 128 λ_D^2. By imposing appropriate time varying electric fields the initial magnitude and direction of the electric current is manitained constant, $v_d = v_{the\,o}$, i.e. no current is allowed to build up in the perpendicular direction, which corresponds to the case of a plane perpendicular shock wave or magnetic sheath.

We first discuss the one-dimensional case. The main results are: The phase, where the field energy grows exponentially ("linear"phase) is terminated by the onset of ion trapping which strongly reduces the growth rate. In the"nonlinear"phase which follows, the ratio of field-over thermal energy $\langle\tilde{E}^2\rangle/8\pi nT_e$ remains approximately constant. The electron temperature increases proportional to t^2, i.e. $v_{the} = \sqrt{T_e/m_e} \approx \alpha t$. Changing the strength of the magnetic field the coefficient α is found to be proportional to Ω_e. These results are in agreement with the theory of electron heating given in Ref. 1. An electron trapped in a potential well of a wave suffers an increase of perpendicular velocity $\dot{v} \approx \Omega_e v_{ph} \approx \Omega_e v_d$ (since v_{ph} = phase velocity ≈ v_d in the frame of the electrons). Since the number of trapped electrons is determined by the ratio $\langle\tilde{E}^2\rangle/8\pi nT_e$ which is nearly constant, one has $n_o T_e \approx n_{trapped}\, m_e (\Omega_e v_d)^2 t$. As a consequence, the effective collision frequency ν_{eff} increases linearily with time, $n_o T_e \approx \eta j^2$ (since $T_e \gg T_i$), $\eta = 4\pi \nu_{eff}/\omega_{pe}^2$. This is in contrast to the predection of a simple "quasilinear" picture, $\nu_{eff} \approx 2\,\omega_{pe} \langle E^2\rangle/8\pi nT_e$. The maximum value of ν_{eff} is determined by the ratio v_d/v_{the} where the instability is effectively switched off. There has been some discussion wh ether this velocity is given by the velocity of sound $v_{dcrit} \approx c_s \approx \sqrt{T_e/m_i}$ or by the critical velocity of the linear electron-cyclotron drift instability $v_{dcrit} \approx v_{the}\, \Omega_e/\omega_{pe}$. In Ref. 2 we found that the switch-off velocity is not given by the latter value $v_{the}\Omega_e/\omega_{pe}$. In the present work we have followed a number of runs with different Ω_e/ω_{pe} and m_e/m_i up to saturation and find $v_{dcrit} \simeq 2c_s$. So the maximum value of ν_{eff} is $\nu_{max} \propto \Omega_e \sqrt{m_i/m_e}$, which is of course completely different from the conventional estimate $\nu_{eff} \sim \gamma \sim \omega_{pi}$ In particular, in some of the numerical runs ν_{max} was much larger than ω_{pi}.

The two-dimensional computations have been restricted so far to small initial ion damping, $T_{eo} \gg T_{io}$. We find that ν_{eff} is smaller than in one dimension. Furthermore there is a significant difference whether the plane containing j and B or the plane perpendicular to B is considered, ν being by a factor of 3 smaller in the latter case. The reason is that the coherent electron heating process is perturbed by a broad wave spectrum perpendicular to B. The scaling of ν_{eff} with Ω_e and m_i is consistent with the one-dimensional results.

1) D.W.Forslund, R.L.Morse and C.W.Nielson, Phys.Rev.Lett.27, 1424 (1971),
2) D.Biskamp and R.Chodura, to be published in Nuclear Fusion