Neoclassical Transport Predictions for Stellarators in the Long-Mean-Free-Path Regime

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The off-diagonal term in the neoclassical transport matrix, which is related to the particle flux, becomes essential in the stellarator long-mean-free-path (LMFP) regime. Strong temperature gradients can drive the density profile hollow, i.e., a positive density gradient related to the diagonal term in the transport matrix has to compensate this off-diagonal drive in order to fulfil the particle balance. As a consequence, central heating with peaked temperature profiles can make an active density profile control by central particle refuelling mandatory. This effect will become essential for the larger stellarator devices of the next generation since recycling as well as gas puffing can affect only the plasma edge region of typically a few centimeters. This neoclassically predicted outward particle flux driven by the temperature gradients is experimentally confirmed in W7-AS discharges in the LMFP regime [1].

Necessity of an Active Density Profile Control

With a particle source, S_p, within the bulk plasma, e.g., by NBI and/or by pellets, a particle flux density, $\Gamma_{\rm ex} = \frac{1}{r}\int_0^r r' S_p\, dr'$, is externally driven. Then, the ambipolarity condition, $\Gamma_e = \Gamma_i = \Gamma_{\rm ex}$, with the neoclassical particle fluxes is given by

$$\Gamma_{\rm ex} = -n\Big\{D_{11}^{e,i}\big(\frac{n'}{n} \pm \frac{E_r}{T}\big) + D_{12}^{e,i}\frac{T'}{T}\Big\}, \tag{1}$$

with + (-) for electrons (ions). Here, $T_e = T_i = T$ and $n_e = n_i = n$ was assumed for simplicity. The neoclassical transport coefficients, D_{jk}^{α} (with $j,k = 1,2$ and $\alpha = e,i$) are obtained by energy convolution of the mono-energetic transport coefficients. For various magnetic field configurations, the databases of these mono-energetic transport coefficients calculated by DKES code [2] are fitted based on traditional analytic theory [3] with axisymmetric contributions in the plateau collisionality regime taken into account. Inverting eq. (1) leads to

$$\left\{\begin{matrix} n'/n \\ E_r/T \end{matrix}\right\} = -\frac{1}{2}\Big\{\frac{D_{12}^e}{D_{11}^e} \pm \frac{D_{12}^i}{D_{11}^i}\Big\}\frac{T'}{T} - \frac{\Gamma_{\rm ex}}{2n}\Big\{\frac{1}{D_{11}^e} \pm \frac{1}{D_{11}^i}\Big\}, \tag{2}$$

where the ratio of the transport coefficients is much less sensitive to the radial electric field, E_r, than the D_{jk}^{α} itself. Several roots of eq. (2) with respect to E_r may exist: the "electron root" at large $E_r > 0$ with both the electron and ion transport coefficients being significantly reduced, the "ion root" at moderate $E_r < 0$ (for $T_e \simeq T_i$) where mainly the ion D_{jk}^i are decreased, and an unstable root inbetween. The 1st term of the r.h.s. of eq. (2) (with the - sign for E_r) is typically positive, and the "electron root" is forced for very small $\Gamma_{\rm ex}$. For large $\Gamma_{\rm ex}$, the "ion root" is obtained due to $D_{11}^e < D_{11}^i$.

Assuming "pure" collisionality regimes, the normalized off-diagonal terms, $\delta_{\rm off}^{\alpha} = D_{12}^{\alpha}/D_{11}^{\alpha}$, are easily obtained: $\delta_{\rm off}^{\alpha} = 7/2$ for the $1/\nu$ regime, $\delta_{\rm off}^{\alpha} = 3/2$ in the plateau regime, $\delta_{\rm off}^{\alpha} = 1/2$ for the $\sqrt{\nu}$ regime, and, finally, $\delta_{\rm off}^{\alpha} = -1/2$ for the tokamak-like ν regime. As these regimes overlap in the energy convolution, the values of $\delta_{\rm off}^e$ and $\delta_{\rm off}^i$ as given in Fig. 1 for an ambipolar particle flux $\Gamma_{\rm ex}$ estimated from the condition $n' = 0$ reflect mainly the dependence on E_r. With decreasing collisionality, $\nu^* \propto n/T^2$, the electrons enter deeply the $1/\nu$ regime since the effect of the "ion root" E_r on the D^e_{jk} is small. The ion coefficients are mainly determined by the $\sqrt{\nu}$ regime which is very pronounced for the high-mirror advanced stellarator configuration under consideration [3].

For the typical LMFP conditions, $\delta^e_{\rm off} > \delta^i_{\rm off}$ holds with the tendency of driving the density profile hollow. For the case of no central particle refuelling, i.e., $\Gamma_{\rm ex} = 0$, eq. (2) gives for the pressure gradient

$$p' = -\frac{1}{2}\,(\delta^e_{\rm off} + \delta^i_{\rm off} - 2)\,nT', \tag{3}$$

with $p' > 0$ (for $T' < 0$) if $\delta^e_{\rm off} + \delta^i_{\rm off} > 2$ which holds in the stellarator LMFP regime. Please note in this context, that $\delta^e_{\rm off} + \delta^i_{\rm off} \approx 0$ (or even negative) in the deep tokamak banana regime. An inverted pressure profile, i.e., $p' > 0$, is in strong conflict with the MHD stability condition based on magnetic well, $V'' < 0$. Then, the typically stabilizing term, $p'V''$, becomes destabilizing. Consequently, the condition $p' < 0$ is mandatory leading to the requirement of an active density profile control, i.e., sufficiently large $\Gamma_{\rm ex}$.

Quantitative Estimates

An estimate of the necessary particle refuelling rate can be obtained in a local (i.e., at a fixed radius, r) solution of the ambipolarity condition for a required n'/n, for given density and "heating power". Here, the "heating power", or more precisely, the total heat flux density, $q_t = q_e + q_i$, over the flux surface of effective radius, r, given by

$$q_{e,i} = -nT\,\{D^{e,i}_{21}\,(\frac{n'}{n} \pm \frac{E_r}{T}) + D^{e,i}_{22}\,\frac{T'}{T}\}, \tag{4}$$

is used. For the example shown in Fig. 1, a W7-X configuration with high toroidal mirror ($\simeq 10\,\%$) [4] was selected. At about half the plasma radius ($r = 0.27$ m, $R = 5.5$ m), a (normalized) temperature gradient $rT'/T = 1$ was assumed which corresponds to a fairly peaked T profile. The physical dependence of all the results on the local T'/T, however, turned out to be fairly small (e.g., a steeper T'/T decreases T at given heat flux Q_t). Full density control with $n' = 0$ was assumed. The necessary refuelling rate, i.e., the total ambipolar particle flux, scales roughly with the heating power for the 3 densities in Fig. 1. Please note in this context, that full NBI heating with a mean energy of 60 keV just fulfils this request ($\simeq 10^{20}$ particles/s and per MW heating power). The temperature increases only slightly with Q_t in the LMFP regime, nearly independent on density. At the high Q_t values, the electron transport coefficients deeply within the $1/\nu$ regime exceed the ion ones, and, consequently, the E_r of the "ion root" ($E_r < 0$) decreases since only a small E_r is sufficient to reduce the D^i_{jk} to the electron level ($D^i_{22} \simeq D^e_{22}$ at the low collisionbaities, ν^*). Consequently, the density dependence of the heat flux disappears in the deep $1/\nu$ regime. For both the normalized quantities, $\delta^e_{\rm off}$ and $\delta^i_{\rm off}$, as well as for the "convective" term, $T\Gamma_{\rm ex}/q_t$, versus the collisionality ν^*, nearly no additional density dependence is found with the ambipolar E_r taken into account.

"Electron Root"

Only for the lower densities, an "electron root" was found from the ambipolarity condition. In order to decide if this root can be realized, additional thermodynamic arguments have to be considered. On the basis of the poloidal force balance with a shear viscosity term included, a generalized heat production which has to be minimized is derived [5]. The Euler-Lagrange form of this variational principle leads to a diffusion equation for the radial

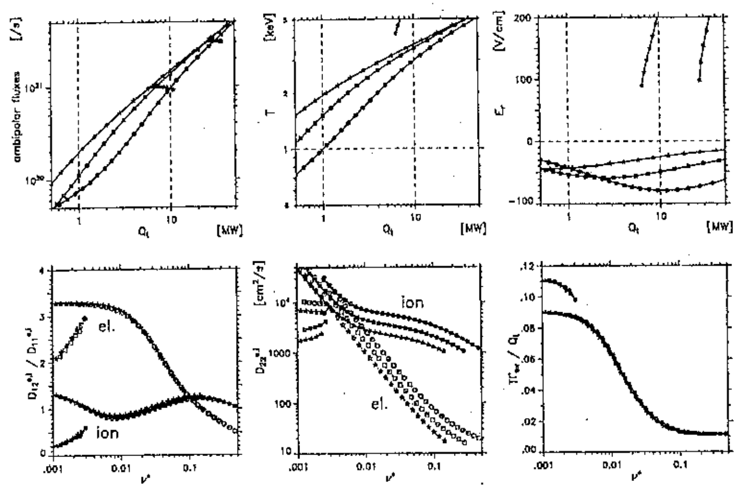

Fig. 1: *Ambipolar particle flux ($n' = 0$ assumed), temperature ($T_e = T_i$), ambipolar E_r vs. heat flux, Q_t (upper plots, from left to right), the norm. off-diagonal transport matrix terms, δ_{off}^{e} and δ_{off}^{i}, the heat diffusivities, D_{22}^{e} and D_{22}^{i}, and (for reference) the norm. "convective" heat flux $T\Gamma_{\text{ex}}/Q_t$ vs. collisionality, ν^* (lower plots, from left to right) for the high mirror W7-X configuration at an effective radius of $r = 27$ cm.*
Densities ($n_e = n_i$): $5 \cdot 10^{19}\ m^{-3}$ ($\star$), $1 \cdot 10^{20}\ m^{-3}$ ($\blacksquare$), and $2 \cdot 10^{20}\ m^{-3}$ ($\bullet$).

electric field which is suited for integration in a predictive neoclassical code. For a "local" analysis, i.e., at one radius, however, the minimization of the generalized heat production with respect to the shear layer position (the width is assumed to be sufficiently narrow) leads to the condition

$$\int_{E_r^i}^{E_r^e} (\Gamma_i - \Gamma_e)\, dE_r = 0, \tag{5}$$

with E_r^i (E_r^e) being the "ion" ("electron") roots. On the other hand, with the integral of eq. (5) being positive, the "ion root" will be realized. From this argument, the "electron root" solutions of Fig. 1 cannot be expected. In the integral of eq. (5), however, the assumptions for the "local diffusive ansatz" in the neoclassical theory are violated at least for the ions at $E_r \simeq 0$, and direct losses have to be taken into consideration. From this point of view, the usual neoclassical ansatz may lead to an underestimation of Γ_i for very small E_r supporting the prediction of the "ion root". Furthermore, a Kelvin-Helmholtz-like instability may be driven at the highly localized poloidal shear layer and may suppress the "electron root" feature. As a consequence, neoclassical transport predictions should not rely only on the optimistic prospects of the "electron root".

"Ion Root"

For the "ion root" in the deep LMFP regime (with $D_{jk}^{i} \simeq D_{jk}^{e}$), the total heat flux scales with $T^{9/2}$, comp. Fig. 1. This unfavourable scaling makes stellarator optimization with respect to neoclassical confinement mandatory. For reactor scenarios based on the "ion root", very high densities at moderate temperatures are favourable. However, the negative E_r may lead to the problem of impurity accumulation. Assuming full density profile control with $n' = 0$, the "ion root" E_r can be estimated from $\Gamma_i \simeq 0$ (since $\Gamma_e(E_r) = \Gamma_i(E_r) \ll \Gamma_i(0)$): $E_r \simeq \delta^i_{\text{off}} T'$. For the impurities, Z, stationary conditions with $\Gamma_Z = 0$ (no inner sources) leads to highly peaked profiles

$$\frac{n'_Z}{n_Z} \simeq (Z\,\delta^i_{\text{off}} - \delta^Z_{\text{off}})\,\frac{T'}{T}, \tag{6}$$

since $Z\,\delta^i_{\text{off}} - \delta^Z_{\text{off}} \gg 1$ for high Z. It seems to be unlikely that $Z\,\delta^i_{\text{off}} - \delta^Z_{\text{off}} < 0$ (which is the case in the deep banana regime in tokamaks) can be achieved by stellarator optimization. The negative radial electric fields of the "ion root" result in a strong inward term for the high Z impurities. For the "electron root", on the contrary, no accumulation problems are expected.

Predictive Neoclassical Transport Codes

Neoclassical theory for fairly general stellarator configurations seems to be sufficiently developed, so that the predictive neoclassical transport modelling is the natural next step. A first attempt was done by implementing the ASTRA code [6] in a stellarator specific version [7]. The neoclassical transport matrix with the analytical respresentation [3] is used. The ambipolar E_r is obtained by direct iteration which is only stable for the "ion root". So far, the diffusion equation for E_r (corresponding to the poloidal force balance with the shear viscosity included) is not implemented. This problem is being treated by an other code which is still under development.

This stellerator specific ASTRA code version will be used to describe the transient phenomena in case of pellet injection used for the necessary active density profile control. In particular, the refuelling rate required to control the density profile in the bulk part of the plasma may be in conflict with the global density control if a transport barrier develops at the outer radii. Finally, on the basis of self-consistent density and temperature profiles together with the ambipolar radial electric field, the severe problem of impurity transport has to be treated.

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