Runaway Generation during Disruptions in ITER Taking Account of Particle Trapping

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Introduction: In a fusion reactor relevant plasma, like in ITER, a noticeable part of the electromagnetic energy of the plasma can be transferred to kinetic energy of runaway electrons which are generated during disruptions. In ITER, the dominating process is the runaway generation by close collisions (secondary or avalanche like generation): A runaway is scattered by a thermal electron and transfers a small part of its kinetic energy to this electron. If the transferred energy exceeds the critical energy $W_C = \frac{e^3 n_e \ln\Lambda}{4\pi\varepsilon_0^2 E}\sqrt{Z_{eff}+5}$ for runaway generation, the thermal electron will become a new runaway [1]. In general, an additional condition for the occurrence of runaways in tokamak is that the toroidal electrical field is larger than the critical field [2, 3] $E_C = \frac{e^3 n_e \ln\Lambda}{4\pi\varepsilon_0^2 m_0 c^2} \approx 0.05\ \mathrm{V/m}$ for typical parameters in ITER. After a thermal quench in ITER, the electric field exceeds this critical field by two or three orders.

Secondary runaway generation: The rate for secondary generation is given by

$$\left(\frac{dN_r}{dt}\right)_{2.\ \mathrm{Gen.}} = N_r v_r n_e \sigma(W_c < \Delta W < E_{in} - W_c)$$

with the velocity v_r of the runaways (with the kinetic energy $E_{in} = (\gamma-1)m_0c^2$) and the electron scattering cross-section σ for an energy transfer $\Delta\, m_0c^2$ is given by $v_r = c\sqrt{\left(1+\left(\gamma^2-1\right)^{-1}\right)^{-1}}$

and $\sigma\left(\Delta_1<\Delta<\Delta_2\right) = \frac{2\pi r_e^2}{\gamma^2-1}\left[\frac{2\gamma^2}{\gamma-1-\Delta} - \frac{(\gamma-1)\gamma^2}{\Delta(\gamma-1-\Delta)} + \frac{2\gamma-1}{\gamma-1}\ln\frac{\gamma-1-\Delta}{\Delta} + \Delta\right]_{\Delta_1}^{\Delta_2}.$

"Runaway trapping": Since the necessary energy transfer for runaway generation (some keV) is small compared with the kinetic energy of the scattering runaway (some 10 MeV), the scattering angle for the thermal electron in the labour frame is large and the thermal electron would become a new runaway with a perpendicular momentum much higher than its parallel momentum. Considering the formula for the scattering angle θ_2, the ratio between the perpendicular and parallel velocity is given by $\frac{v_\perp}{v_\parallel} = \tan\theta_2 = \tan\arccos\frac{(\gamma+1)\Delta}{\sqrt{\gamma^2-1}\sqrt{(\Delta+1)^2-1}}$.

Many of these new runaways are trapped and are hence not accelerated further in spite of the high loop voltage after the thermal quench. The condition for particle trapping is [4]

$$\left|v_\parallel - \frac{v_E}{\Theta}\right|^2 < 2\frac{r}{R}\left(v_\perp^2 + 2\left(\frac{v_E}{\Theta}\right)^2\right)\left(1-\frac{1}{c^2}\left(\frac{v_E}{\Theta}\right)^2\right) \text{ with } v_E = \frac{E}{B} \text{ and } \Theta = \frac{B_p}{B_t}.$$

Due to the relatively high electron velocity ($v_r > 0.01c$), the term v_E/Θ is very small compared to the electron velocity ($v_E/\Theta < 10^{-6}c$) and can be neglected. Hence the limit for particle trapping simplifies to the well-known result: $\frac{v_\parallel}{v_\perp} < \sqrt{2\frac{r}{R}}$.

The lowest energy transfer at which the new runaway is not trapped depends on the energy of the scattering runaway and the considered position in the plasma due to the spatial variation of the strength of the magnetic mirror.

Starting with $\frac{v_\perp}{v_\parallel} = \tan\arccos\theta_2 = \tan\arccos\frac{(\gamma+1)\Delta}{\sqrt{\gamma^2-1}\sqrt{(\Delta+1)^2-1}} = \sqrt{\frac{1}{2}\frac{R}{r}}$, the equation for the lowest energy transfer is: $\Delta_{tr} = -2 \Big/ \left(1-\left(\frac{\sqrt{\gamma^2-1}}{(\gamma+1)}\cos\arctan\sqrt{\frac{1}{2}\frac{R}{r}}\right)^{-2}\right)$.

There are new two critical energies for the secondary generation that must be considered for the calculation of the cross-section. To get the rate for secondary runaway generation, the differential cross-section must be integrated between the lowest and the highest energy transfer that results in a new runaway. The lower boundary is the maximum of the two critical energies $\Delta_1 = \max\left(\Delta_{tr}, W_c/m_0c^2\right)$. Due to the scattering with identical particles and the resulting symmetry in the formulas, the upper boundary is the kinetic energy of the scattering electron minus the maximum of the two critical energies $\Delta_2 = \gamma - 1 - \max\left(\Delta_{tr}, W_c/m_0c^2\right)$.

The secondary generation rate at the plasma radius r is:

$$\left(\frac{dN_r}{dt}\right)_{2.\,\mathrm{Gen.}} = N_r n_e c\sqrt{\left(1+\left(\gamma^2-1\right)^{-1}\right)^{-1}}\, n_e \frac{2\pi r_e^2}{\gamma^2-1}\left[\frac{2\gamma^2}{\gamma-1-\Delta} - \frac{(\gamma-1)\gamma^2}{\Delta(\gamma-1-\Delta)} + \frac{2\gamma-1}{\gamma-1}\ln\frac{\gamma-1-\Delta}{\Delta} + \Delta\right]_{\Delta_1}^{\Delta_2}$$

To get the growth rate of the total runaway population, it is a good approximation to assume that the energy spectrum of the runaways decreases exponentially [1] like $\exp(-E_{kin}/15\,\mathrm{MeV})$ After the integration of the generation rate over the runaways energies with this assumption the resulting growth rate only depends on the electric field and the strength of the magnetic mirror. In Fig. 1 the results are plotted versus the minor plasma radius for the electric fields 2, 10, 40 [V/m] (dotted lines). Due to the large cross-sections for low energy transfer, large scattering angles are dominating and many of the generated runaways are trapped and are not considered in the plotted rates that strongly decrease outside the plasma centre.

The scattering of the trapped runaways with the thermal background plasma results in both a momentum loss (thermalization) and a momentum transfer from their high perpendicular momentum into parallel momentum (detrapping). If the trapped runaways leave the area of trapping, before they slow down below the critical energy, they are accelerated and become free runaways again. Under the assumption that momentum transfer to the parallel momentum is a factor two faster compared to the momentum loss to the background plasma, the generation rates are in good agreement (better than 5%) with Monte-Carlo and Fokker-Planck calculations presented in [5].The resulting growth rate is shown in Fig. 1 (solid lines). In spite of the strong detrapping the resulting growth rate shows a radial dependence and is approximately a factor two smaller at the plasma boundary compared to the centre.

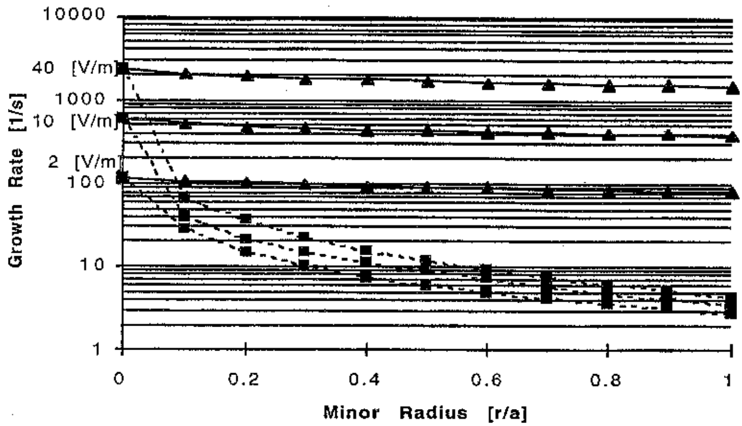

Fig. 1: The secondary generation rate vs. the normalised plasma radius. The generation rates are shown for three different electric fields. Since most runaways are generated as trapped particle, the generation rate strongly decreases outside the plasma centre (dashed line). Due to thermal scattering, most trapped runaways change their perpendicular momentum to parallel momentum before they thermalize and become free runaways again. The effective generation rate (solid line) shows an only weak radial dependence

Calculation of the resulting runaway current: For calculating the growth of the runaway population during the current quench, the following model is considered. The initial temperature profiles are fixed during the calculation. No power balance equation is considered. The initial plasma current decays according to the induction law. The initial number of runaways is zero. The Dreicer and the secondary runaway generation (presented above) are considered in the calculation. The generated runaways approximately move with the velocity of light and carry the resulting runaway current virtually without resistivity. Due to the low plasma temperature, flux diffusion must be taken into account. The numerical calculations were done with the ASTRA Code [6]. Starting with the fixed profiles of temperature and density of the background plasma as well as the initial value for the plasma current, the plasma current density, the plasma equilibrium, the resulting electric field and runaway population are calculated self-consistently. No plasma movement is considered in the calculations.

Results: The resistivity of the plasma strongly depends on the temperature and the Z_{eff}. At low resistivity the current decay needs some seconds and the resulting loop voltage is so low that the current resistively decays before a large number of runaways are generated. Considering a fast current decay due to disruption mitigation, a huge runaway population ($>10^{19}$) builds up and carries up to 100% of the initial current. A typical example is shown in Fig. 2. In case of disruption mitigation by killer pellets, a low plasma temperature of 10 eV and Z_{eff} = 5 could be assumed. In spite of the weak radial variation of the generation rate, the current density carried by the runaways after many e-foldings becomes comparable with the resistive current density in the centre although the total number of runaways is small. The loop voltage in the centre starts to break down. Due to the large flux diffusion at low temperature, the runaway current density can exceed the current density of the initial plasma current by a factor of 3 or more in the centre (Fig. 2 b). The current density in the outer regions decreases further until the complete plasma current is carried by the runaways and the electric field breaks down in the entire plasma. Due to the strong current peaking, the central q-value quickly falls below one and reaches the stationary value of 0.23, when the current peaking saturates. The radius of the q=1 surface extends to 0.5 m (r/a=0.2). Due to the large area with q<1 (compared to the gyro-radius of the runaways), the development of instabilities should be considered in further investigations.

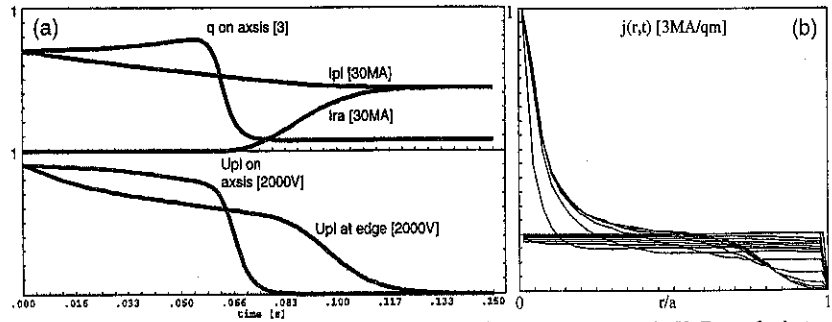

Fig. 2: Development of the runaway current I_{ra}. Electron and ion temperature are 10 eV, Z_{eff} = 5, electron density is 1.5 10^{20} m^3. The initial current is 21 MA, the resulting loop voltage is 1813 V. (a) The time histories show the total plasma current, the runaway current, the resulting q-value on the axis and the loop voltage in the centre and at the edge. (b) The total current density is plotted versus the normalised plasma radius every 10 ms. The initial profile is given by the top flat line at 0.6 MA/m^2 (in the model the current density vanishes at the limiter). The flat profiles slowly decrease like the total plasma current. After 60 ms, the strong current peaking starts for 20 ms. After 110 ms the profiles do not further change.

For an assessment of the possibility of damages to in-vessel components, the energy per thickness of the plasma layer which is scraped off and the energy spectrum of the runaway are important. The maximum of energy per layer thickness (33 MJ/m) is reached at r/a=0.7. Due to the lower volume in the centre and the small runaway density at the edge, the energy per thickness vanishes in both directions virtually linear. The maximum of the kinetic energy of a runaway is given by the balance between the accelerating electric field and the synchrotron radiation. Only a few runaways reach high energies such as up to 500 MeV. More than 75% of the runaways have energy below 30 MeV. There is a small (<10%) increase in the total energy of the runaway beam (here 45 MJ) compared to the calculation neglecting the radial dependence of the generation rate.

For a more detailed analysis of the heat load to the plasma facing components by the runaway impact, the movement and shrinking of the plasma column will be included in further the considerations.

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