THE ALFVEN DRIFT-WAVE INSTABILITY AND THE SCALING OF THE EDGE TEMPERATURE AT THE L-H TRANSITION

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ABSTRACT

The stability analysis of Alfven drift type modes leads to a threshold condition for the L-H transition. This condition yields the scaling for the edge temperature, which is in agreement with the experimental results.

1. INTRODUCTION

The measurement of the electron edge temperature T_0 and density n_0 on JET and other tokamaks suggest that there is an ideal MHD beta threshold for the onset of Giant ELM's. Furthermore, the data reveal also the existence of a second beta threshold (below the ideal ballooning limit) for the L-H transition and a third threshold for the onset of type III ELMs. These findings suggest that the Alfven drift-wave instability (ADW) plays an essential role in the edge plasma dynamics. There is strong experimental evidence that the main plasma instabilities change from ion temperature gradient modes in the plasma core to electron drift modes near the plasma edge. The stability theory gives the result that with increasing plasma pressure, the Alfven waves mix with electron drift waves and suppress the unstable long wavelength perturbations, which are dominant in the transport.

2. DISPERSION RELATION AND DIMENSIONLESS PARAMETERS

The expression for the ion density perturbation follows from the kinetic equation:

$$n_i' = \frac{e\varphi'}{T_{0i}}\left\langle\left(-1+\frac{\omega-\hat{\omega}_{*i}}{\omega-k_{\parallel}v_{\parallel}}\exp(-z)I_0(z)\right)f_{0i}(v_{\parallel})\right\rangle \Rightarrow n_i' = \frac{e\varphi'}{T_{0i}}\left(-1+\frac{\omega-\hat{\omega}_{*i}}{\omega}\cdot\frac{1}{1+z}\right)n_0$$

The perturbed electron density follows from the electron hydrodynamic equations:

a) the linearised electron density equation is:

$$\omega\frac{n'_e}{n_0}-\omega_*\frac{e\varphi'}{T_{0e}} = k_{\parallel}v'_{e\parallel};$$

b) the parallel component of the electron motion equation yields:

$$\frac{T'}{T_{0e}}+\frac{n'_e}{n_0} = \frac{e\varphi'}{T_{0e}}-\frac{\omega-\omega_{*p}}{k_{\parallel}c}\frac{eA'_{\parallel}}{T_{0e}}+R\frac{v'_{\parallel e}}{v_{Te}};$$

where R are the dissipative terms $R = \frac{\omega+i\nu_e}{k_{\parallel}v_{Te}}$;

c) Ampere's law gives:

$$k_\perp^2 \delta^2 \frac{eA'_\parallel}{T_{0e}} = -\frac{cv'_{e\parallel}}{v_{Te}^2};$$

d) The temperature perturbation follows from the energy equation:

$$\left(\omega - \frac{k_\parallel^2 v_{Te}^2}{\omega + i\nu_e}\right)\frac{T'}{T_{0e}} - \omega_{*T}\frac{e\varphi'}{T_{0e}} = \frac{2}{3}k_\parallel v'_{\parallel e}.$$

From these equations the expression for the perturbed electron density is derived:

$$n'_e = n_0 \frac{\left\{\omega_* - \frac{(k_\parallel^2 v_{Te}^2 k_\perp^2 \delta^2)}{\left[\omega - \omega_{*p} + \tilde{\omega}\cdot k_\perp^2\delta^2\right]}\cdot\left[1 + \frac{2}{3}\frac{(\omega_* - (3/2)\omega_{*T})\cdot\tilde{\omega}}{\omega\tilde{\omega} - k_\parallel^2 v_{Te}^2}\right]\right\}}{\left\{\omega - \frac{(k_\parallel^2 v_{Te}^2 k_\perp^2 \delta^2)}{\left[\omega - \omega_{*p} + \tilde{\omega}\cdot k_\perp^2\delta^2\right]}\cdot\left[1 + \frac{2}{3}\frac{\omega\cdot\tilde{\omega}}{\omega\tilde{\omega} - k_\parallel^2 v_{Te}^2}\right]\right\}}\frac{e\varphi'}{T_{0e}}$$

Here $\omega_* = -(ck_y T_{0e}/eB_0)\cdot\partial\ln(n_0)/\partial x$ is the electron drift frequency for the density, $k_\parallel$ is the longitudinal wave number, $v_{Te} = (T_{0e}/m_e)^{1/2}$ is the thermal electron velocity; ν_e is the electron collision frequency and $\delta = c/\omega_{pe}$ is the collisionless skin length, $\tilde{\omega} = \omega + i\nu_e$. We define the dimensionless frequency and transverse wave number as $\Omega = \omega\cdot x_{0p}/c_s$, $k = k_\perp\rho_s$, where $c_s = (T_e/M)^{1/2}$ is the sound speed; $\rho_s = c_s/\omega_{Bi}$ is the ion Larmor radius and T_e the electron temperature. The relevent dimensionless parameters in the problem are:

$$\beta_0 = 4\pi n_0 T_{0e}/B_0^2\cdot(M/m_e), \qquad \nu = \nu_e x_0/c_s$$

$$\mu = k_\parallel v_e x_{0p}/c_s, \quad \eta_e = d\ln(T_{0e})/d\ln(n_0), \quad \tau = T_{0e}/T_{0i},$$

The quasineutrality condition $n'_e = n'_i$ yields the dispersion relation for the Alfven drift wave instability. It is an algebraic equation of fifth order with complex coefficients containing five independent parameters: $\beta_0, \nu, \mu, \eta_e, \tau$. A more detailed analysis shows that after additional renormalization:

$$\Omega_n = \Omega/\sqrt{\mu}, \; k_n = k/\sqrt{\mu}, \; \beta_n = \beta_0/\mu, \; \nu_n = \nu/\sqrt{\mu}.$$

the ADW instability can be characterised by only **two dimensionless parameters** the normalised plasma beta and the normalised collision frequency

$$\beta_n = \frac{\beta_0}{\mu} = \left(\frac{M_i}{m_e}\right)^{1/2}\frac{4\pi n_0 T_{0e}}{B_e^2}\frac{1}{k_\parallel x_{0p}}, \qquad \nu_n = \frac{\nu}{\sqrt{\mu}} = \left(\frac{M_i}{m_e}\right)^{1/4}\frac{x_{0p}^{1/2}}{\lambda_e k_\parallel^{1/2}}$$

Here x_0 characterises the pressure scale length, λ is the mean free path and $k_\parallel$ is the parallel wave number. All quantities are taken near the separatrix.

3. TRANSPORT COEFFICIENTS

The diffusion and thermalconduction coefficients for the ADW instability are evaluated as:

$$D_\perp = \frac{\chi_{GB}}{\sqrt{\mu}}\cdot\overline{\overline{\chi}}_\perp(\beta_n, \nu_{ne}), \qquad \kappa_{e\perp} = \frac{3}{2}n_e\frac{\chi_{GB}}{\sqrt{\mu}}\overline{\overline{\chi}}_\perp(\beta_n, \nu_{ne}).$$

$$\overline{\overline{\chi}} = \nu_{cr}^{1/3} \frac{\left[1 + (\nu_n / \nu_{cr})^2\right]^{1/2}}{\left[1 / \nu_{cr}^2 + (\nu_n / \nu_{cr})^{4/3}\right]^{1/2}},$$

where $\nu_{cr} = 1 / (1 + \beta_n^2)^{3/2}$ (See Fig. 1 - Fig. 2).

4. L-H THRESHOLD CRITERION

The analysis of the turbulent coefficients shows that the transport decreases for

$$\beta_n > 1 + \nu_n^{2/3}.$$

This condition yields the scaling for the edge temperature at the L -H transition. The estimate of x_0 (inside the separatrix) is derived from the assumption that the turbulent transport coefficients are continuous across the separatrix and that the convection (collisionless) or conduction (collisional) model in the SOL applies.

5. COMPARISON WITH THE EXPERIMENTS

The temperature inside the separatrix is approximated by: $T(a - \Delta x) = T_0 \cdot (1 + \Delta x / x_0)$. Then we obtain for $\nu_n > 1$:

$$T_{eV}(a - \Delta x) = 32.6 \cdot A^{-1/5} s^{3/5} n_{019}^{-3/10} B_{0T}^{3/5} I_{MA}^{3/5} a_M^{-6/5} \cdot \Delta x_{cm} \quad \text{(theory)}$$

$$T_e^{crit}(a - 2\,cm) = 145\, n_e^{-0.3^{+0.2}_{-0.1}} B_t^{0.8^{+0.2}_{-0.1}} I_p^{0.5^{+0.3}_{-0.3}} \,[eV] \quad \text{(ASDEX UP)}$$

and for $\nu_n < 1$:

$$T_{eV}(a - \Delta x) = 23.3 \cdot A^{-1/2} s\, n_{019}^{-1} B_{0T} I_{MA} a_M^{-2} \cdot \Delta x_{cm}$$

The general expression for T_{eV} reads:

$$T_{eV}(a - \Delta x) = c_T \frac{f3 \cdot A^{-0.5 f1}}{n_{019}^{f3}} \left(\frac{s \cdot B_{0T} \cdot I_{MA}}{a_M^2} \right)^{f4} \cdot \Delta x_{cm},$$

where $f1 = (1 + 2/5 \cdot \nu 1)/(1 + \nu 1)$, $f2 = (1 + 3/10 \cdot \nu 1)/(1 + \nu 1)$, $f3 = (1 + \nu 1)/(1 + 7/10 \cdot \nu 1)$, $f4 = (1 + 3/5 \cdot \nu 1)/(1 + \nu 1)$ and $\nu 1 = (n_{019} / n_{cr})^{21/10}$, $n_{cr} = (B_{0T} \cdot I_{MA} / BI)^{4/7}$, $BI = c_{BI} A^{3/7} s^{-4/7} a_{0M}^{8/7} Z_{eff}^{2/7}$. This formula contains two parameters c_T, c_{BI}. The theory gives c_T=23.3, c_{BI}=1.4. The fitting with the ASDEX experiment gives c_T=10, c_{BI}=0.7. The same values are used for all the other tokamaks too. The comparison theoretical prediction with the experimental data [ITER Internal Memo, Yu.Igitchanov at. al., No. J17MD82970423F1 (1997)] is given on Figs. 3 - 6.

6. CONCLUSIONS

The main conclusions are: **1)** the Alfven drift model predicts that the turbulent transport is suppressed when the condition: $\beta_n > 1 + \nu_n^{2/3}$ is satisfied; **2)** the transport coefficients change their dependence on plasma parameters from $\chi \sim T_0^{3/2}/n_0^{1/5}$ (for $\beta_n < \nu_n^{2/3}$) to $\chi \sim n_0^{4/13}/T_0^{20/13}$ (for $\beta_n > \nu_n^{2/3}$); **3)** the Alfven drift model predicts the edge temperature scaling in agreement with experimental findings. Furthermore, the qualitative character of the scaling depends only weakly on the SOL transport.

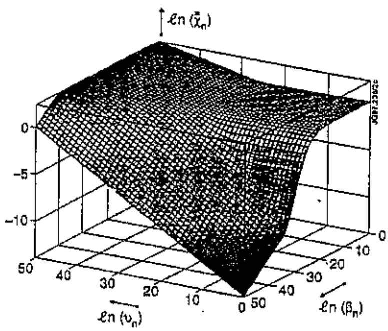

Fig. 1

Fig. 1. The dimensionless transport coefficient $\bar{\bar{\chi}}$.

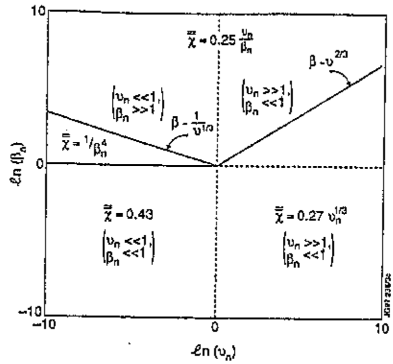

Fig. 2

Fig. 2 The asymptotic properties of $\bar{\bar{\chi}}$.

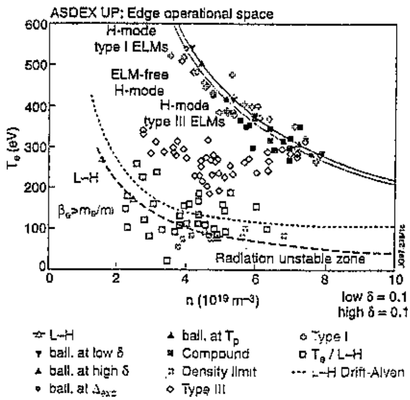

Fig. 3

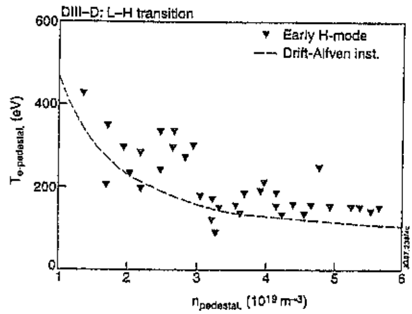

Fig. 4

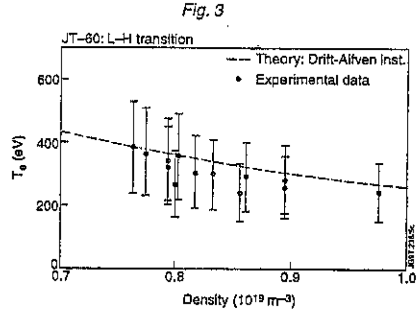

Fig. 5

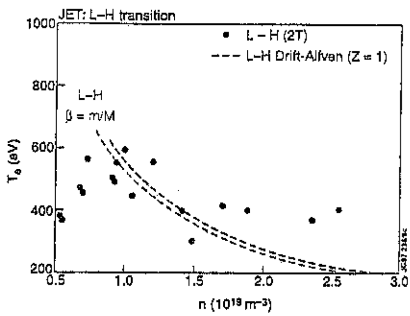

Fig. 6

Fig. 3 -6. The theoretical (dashed lines) and experimental (points) scalings of the edge temperature over edge density for different tokamaks.