Viscous Damping and Plasma Rotation in Stellarators

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Poloidal rotation with shear flow is one of the key elements in the theory of H-mode confinement in toroidal systems. There are various driving forces which may excite poloidal and toroidal rotation: Stringer spin-up, turbulent Reynolds stresses and lost orbits. Viscous damping is the main candidate to retard the rotation. In a collision dominated plasma viscous damping is provided by the magnetic pumping effect which arises from the variation of the magnetic field strength along the stream lines of rotation. It only depends on the Fourier spectrum of B on magnetic surfaces and not on the details of particle orbits. A comparison of viscous damping rates of various toroidal configurations has been given in [1]. The non-axisymmetric components in the Fourier spectrum of B lead to extra maxima in the poloidal force which complicates the bifurcation problem as compared to axisymmetric tokamaks. Enhanced poloidal damping also occurs if magnetic islands exist in the confinement region. In stellarators islands occur on rational magnetic surfaces if there exists a resonant field perturbation. Such perturbations always exist on "natural surfaces with the rotational transform $\iota = M/k$, k=1,2,3,..M is the number of field periods. The denominator indicates the number of islands. Symmetry breaking field perturbations introduce another class of islands which can easily dominate over the natural islands.

In general there is a combination of natural islands and symmetry breaking islands. Examples are given by the Wendelstein stellarators W 7-A and W 7-AS [2] Islands lead to enhanced radial transport and therefore also to enhanced poloidal damping. In the neighbourhood of islands magnetic surfaces are modified and the Fourier spectrum of B exhibits harmonics with the periodicity of the island. Due to toroidal curvature the field variation on magnetic surfaces is roughly $\delta B/B = r/R$.

Even if no islands are present in the plasma region magnetic surfaces at the plasma edge are modified by resonant Fourier harmonics. This in particular occurs in the boundary region of Wendelstein 7-AS, where the last magnetic surfaces exhibit the M/k structure of the natural islands.

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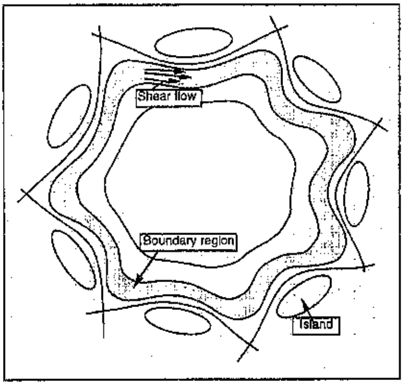
[1] J. Kisslinger, H. Wobig , " On rotation of collisional plasmas in toroidal systems ", Plasma Phys. Contr. Fusion 37 (1995) 893 - 922

[2] R. Jaenicke, K. Schwoerer, E. Ascasibar, P. Grigull, I. Lakicevic, M. Zippe, Proc. of 16th EPS-Conf. on Contr. Fus. and Plasma Phys. Venice 1989, Pt II, 627

Fig. 1: Magnetic islands in the boundary region of a stellarator.



In the following we start from a magnetic field configuration with nested magnetic surfaces which satisfies the condition of ideal equilibrium. On these nested magnetic surfaces the Hamada coordinate system is introduced which is characterised by straight magnetic field and a Jacobian equal to unity. In this coordinate system the base vectors are defined by $\mathbf{e_p} = \nabla s \times \nabla\varphi$, $\mathbf{e_t} = -\nabla s \times \nabla\theta$. $\mathbf{ep}$ is the poloidal base vector and $\mathbf{e_t}$ the toroidal base vector. s is the volume of the magnetic surface. The lowest order rotation $\mathbf{v_O} = - E(\psi)\mathbf{e_p} + \Lambda(\psi)\mathbf{B}$ stays on magnetic surfaces and satisfies the equation $\nabla \mathbf{v_O}=0$. The two flux functions E and Λ describe the poloidal and the parallel motion on magnetic surfaces. E is the radial electric field.

In a toroidal plasma the forces by magnetic pumping inhibit the rotation of the plasma, these forces are

$$<\mathbf{e_p}\bullet\nabla\bullet\pi> = <(p_{\parallel} - p_{\perp})\mathbf{e_p}\bullet\frac{\nabla B}{B}> \quad ; \quad <\mathbf{B}\bullet\nabla\bullet\pi> = <(p_{\parallel} - p_{\perp})\mathbf{B}\bullet\frac{\nabla B}{B}>$$

In a collisional plasma these equations reduce to (see ref. 1)

$$\begin{pmatrix} -<\mathbf{e_p}\bullet\nabla\bullet\pi> \\ <\mathbf{B}\bullet\nabla\bullet\pi> \end{pmatrix} = 3\tau P \begin{pmatrix} C_p & C_b \\ C_b & C_t \end{pmatrix} \begin{pmatrix} E \\ \Lambda \end{pmatrix}$$

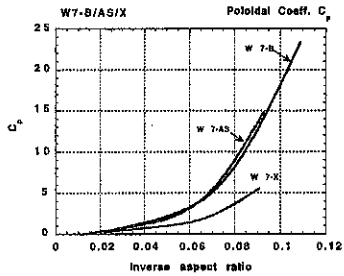
P = plasma pressure, τ = collision time. The coefficients are

$$C_p = <\left(\mathbf{e_p}\bullet\frac{\nabla B}{B}\right)^2> \; ; \; C_t = <\left(\mathbf{B}\bullet\frac{\nabla B}{B}\right)^2> \; ; \; C_b = <\left(\mathbf{e_p}\bullet\frac{\nabla B}{B}\right)\left(\mathbf{B}\bullet\frac{\nabla B}{B}\right)>$$

Thus the damping forces are the product of plasma parameters and geometrical coefficients. In the following numerical computations of these coefficients C_p,C_t, and C_b are shown. The poloidal coefficient C_p is of particular interest, since a small poloidal damping facilitates poloidal rotation and poloidal shear flow. The results of numerical calculations show large damping coefficients in standard l=2 stellarators and torsatrons in contrast to optimised stellarators of the Helias type.

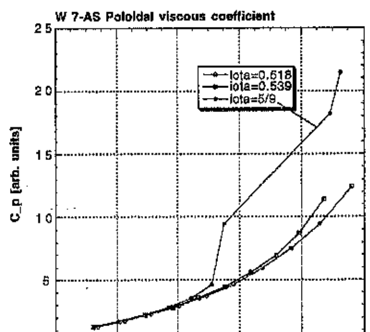
Fig. 2: Poloidal damping C_p coefficient in l=2 stellarators and the optimised stellarator W 7-X. In W 7-X the coefficient is roughly a factor 2 smaller than in W 7-AS and standard stellarators. A similar result exists for the threshold $D=C_p - C_b*C_b/C_t$ of poloidal spin-up. These coefficients increase towards the plasma boundary. However, since these coefficients are multiplied by τP which decreases strongly towards the boundary the viscous forces are small in the boundary region.

This may explain why plasma rotation is mainly observed in the boundary regions.



Magnetic islands may arise in the boundary region of some stellarators like Wendelstein 7-AS. These islands distort the magnetic surfaces in the neighbourhood and lead to an enhanced viscous damping (or magnetic pumping). Inside the island region the plasma equilibrium is not represented by the ideal MHD-theory, therefore the coefficients given here are only relevant in the region outside magnetic islands.

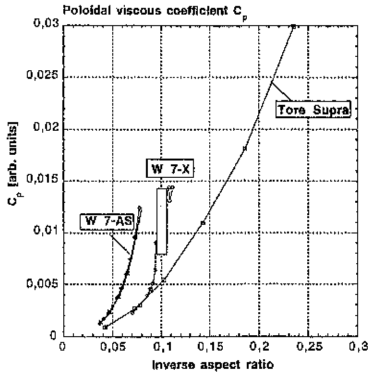
Fig. 3: Poloidal viscous damping rate C_p (collisional regime) in Wendelstein 7-AS. The regime of the rotational transform is between 5/10 and 5/9. The absissa is the average radius of the magnetic surface. Close to the island the damping rate is en-hanced by a factor of 2. The coefficient C_b exhibits a similar enhancement. Little effect of magnetic islands is seen in the toroidal coefficients C_t. In conclusion, mainly poloidal rotation will be inhibited by magnetic islands.



The standard case of Wendelstein 7-X has 5 islands at the boundary. These islands are the base of the divertor concept and the issue arises whether these islands suppress plasma rotation in this region. The following figure shows that only in a small neighbourhood of these islands enhancement of the poloidal damping coefficient occurs.

Fig. 4: Comparison with tokamak. Poloidal viscous coefficient. The gray rectangular region is the island at the boundary of W 7-X. Enhancement of viscous damping in W 7-X

occurs in a few cm distance from the islands. The two curves of W 7-AS differ by the rotational transform. Poloidal damping in W 7-X and a tokamak (Tore Supra) are nearly equal at equal plasma radii.



Discussion

The viscous damping of poloidal and toroidal plasma rotation depends on two factors, one factor is τ P - collision time times plasma pressure - and the other factors are geometrical factors C_p, C_t and C_b. With respect to the geometrical factor significant differences among the various stellarators exist. C_p is the relevant geometrical factor of poloidal rotation. In axisymmetric devices only the toroidal curvature effect gives rise to magnetic pumping and a finite coefficient C_p. In stellarators, however, additional helical harmonics lead to increased poloidal viscous damping . Reducing the poloidal variation of B - as has been done in optimised configurations of the Helias type - also reduces the poloidal viscous damping. In the neighbourhood of magnetic islands enhanced viscous damping arises due to the distortion of magnetic surfaces. This effect may be of importance in Wendelstein 7-AS, where magnetic islands in the boundary region strongly corrugate the surfaces. The viscous damping coefficients C_p and C_b are enhanced in the neighbourhood of the islands. In comparison to axisymmetric configurations standard stellarators exhibit a more complex Fourier spectrum of B and therefore magnetic pumping effect is larger. The optimisation scheme realised in Wendelstein 7-X, however, reduces the poloidal damping rate to the level of axisymmetric configurations. As seen in Fig. 4 the coefficient C_p is nearly the same in both configurations The present analysis is based on a collisional plasma model. Such a model is applicable to a plasma boundary, where the temperatures are below 100 eV and the densities above 10^{19} m^{-3}. At higher temperatures the plateau regime is reached and the viscous damping must be computed from a kinetic equation. This will be done in a subsequent paper.