BEAM TRACING DESCRIPTION OF EC WAVE BEAMS IN TOKAMAK PLASMAS

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1. Introduction

The problem of propagation of electron cyclotron waves in plasmas is usually solved within the framework of geometric optics or ray tracing. This technique describes correctly the wave refraction, but does not take into account the diffraction phenomena, which can become significant if one is concerned with highly collimated or focused microwave beams. These are of great importance for fusion devices, since they are employed for heating and current drive as well as for diagnostic purposes, where a strong wave collimation is desirable in order to improve the localization of the absorbed power and to increase the spatial resolution (e.g. in scattering experiments).

In order to include diffraction effects, the beam tracing technique [1] is employed in this paper. This approach includes the same physics as other methods, such as the parabolic equation [2] and the complex eikonal [3], but it greatly simplifies the general problem, reducing the full wave equation to a set of coupled ordinary differential equations. The beam features are determined by a central ray, which obeys the geometric optics equations, and by a set of parameters, calculated along it, which account for the electric field amplitude profile and the curvature of the phase front.

Three-dimensional Gaussian wave beam propagation in an anisotropic inhomogeneous tokamak plasma is studied. Numerical calculations are made for ASDEX-Upgrade-like parameters. It is shown that the effect of diffraction is of importance for the typical parameters of the tokamak and that the power deposition profile can be considerably broader than that given by the ray tracing approach.

2. The Beam Tracing equations

In the beam tracing approach, the geometric optics *ansatz* for the wave electric field $\mathbf{E}(\mathbf{r})$ is replaced by

$$\mathbf{E}(\mathbf{r}) = \mathbf{a}(\mathbf{r}) e^{i\kappa[s(\mathbf{r})+i\phi(\mathbf{r})]}, \tag{1}$$

where $\kappa = L\omega/c \gg 1$ (L is the inhomogeneity scale, ω is the frequency and c the speed of light). The function $s(\mathbf{r})$ is the *eikonal* function of geometric optics, while the *attenuation*

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function $\phi(\mathbf{r}) \geq 0$ is responsible for the description of the transverse structure of the beam. The beam tracing equations are obtained by substituting Eq.(1) into Maxwell's equation

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2} \hat{\varepsilon}\, \mathbf{E} = 0, \tag{2}$$

where $\hat{\varepsilon}$ is the (cold) plasma dielectric tensor. It can be shown that Eq.(2) can be consistently reduced, by taking the smallness of the parameter κ^{-1} into account, to a set of ordinary differential equations for the quantities q^α, N_α, $N_{\alpha\beta}$ and $\Phi_{\alpha\beta}$, which allow to express the unknown functions $s(\mathbf{r})$ and $\phi(\mathbf{r})$ as

$$s(\mathbf{r}) = N_\alpha(\tau)\,[x^\alpha - q^\alpha(\tau)] + \frac{1}{2} N_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)], \tag{3}$$

$$\phi(\mathbf{r}) = \frac{1}{2} \Phi_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)]; \tag{4}$$

x^α are the components of the position vector $\mathbf{r}$, N_α those of the refractive index $\mathbf{N} = c\mathbf{k}/\omega$ and Einstein's summation convention is adopted. The beam tracing equations are [4]

$$\frac{dq^\alpha}{d\tau} = \frac{\partial H}{\partial s_\alpha}, \qquad \frac{dN_\alpha}{d\tau} = -\frac{\partial H}{\partial x^\alpha}, \tag{5}$$

$$\frac{dN_{\alpha\beta}}{d\tau} + \frac{\partial^2 H}{\partial x^\alpha \partial x^\beta} + \frac{\partial^2 H}{\partial x^\beta \partial s_\gamma} N_{\alpha\gamma} + \frac{\partial^2 H}{\partial x^\alpha \partial s_\gamma} N_{\beta\gamma} + \frac{\partial^2 H}{\partial s_\gamma \partial s_\delta} N_{\alpha\gamma} N_{\beta\delta} = \frac{\partial^2 H}{\partial s_\gamma \partial s_\delta} \Phi_{\alpha\gamma} \Phi_{\beta\delta}, \tag{6}$$

$$\frac{d\Phi_{\alpha\beta}}{d\tau} + \left(\frac{\partial^2 H}{\partial x^\alpha \partial s_\gamma} + \frac{\partial^2 H}{\partial s_\gamma \partial s_\delta} N_{\alpha\delta} \right) \Phi_{\beta\gamma} + \left(\frac{\partial^2 H}{\partial x^\beta \partial s_\gamma} + \frac{\partial^2 H}{\partial s_\gamma \partial s_\delta} N_{\beta\delta} \right) \Phi_{\alpha\gamma} = 0, \tag{7}$$

where $H(\mathbf{N}, \mathbf{r})$ is an eigenvalue of the problem

$$\hat{L}\mathbf{e} \equiv \mathbf{N} \times (\mathbf{N} \times \mathbf{e}) + \hat{\varepsilon}\mathbf{e} = H\mathbf{e}. \tag{8}$$

In the previous equations, all the derivatives of H have to be calculated at $x^\alpha = q^\alpha(\tau)$ and $s_\alpha = N_\alpha(\tau)$. The first two Eqs. (5) describe a geometric optics ray, on wich $\phi = 0$ [see Eq.(4)]. This ray constitutes then the "backbone" of the beam and will be called *reference* ray and denoted by $\Re$. The remaining functions $N_{\alpha\beta}$ and $\Phi_{\alpha\beta}$ are connected with the curvature of the wave front and the width of the wave packet, respectively. The set of Eqs.(5-7) involves six equations for the central ray $\Re$ and twelve for the symmetric quantities $N_{\alpha\beta}, \Phi_{\alpha\beta}$, which must also satisfy the six constraints

$$N_{\alpha\beta} \frac{dq^\beta}{d\tau} + \frac{\partial H}{\partial x^\alpha} = 0, \tag{9}$$

$$\Phi_{\alpha\beta} \frac{dq^\beta}{d\tau} = 0. \tag{10}$$

Eqs.(9-10) are employed to check the accuracy of the solution.

Eqs. (5) give immediately the coordinates $q^1(\tau), q^2(\tau), q^3(\tau)$ of the reference ray $\Re$, together with the wave vector $\omega \mathbf{N}(\tau)/c$ at each point along it. In order to clarify the physical meaning of the functions $N_{\alpha\beta}, \Phi_{\alpha\beta}$, solutions of Eqs. (6-7), a beam propagating *in vacuo* along the x^1-axis can be considered. In this case, from the constraints (9-10) it follows $N_{1\alpha} = \Phi_{1\alpha} = 0$. The terms $N_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)]$, $\Phi_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)]$

in Eqs. (3-4) can then be regarded as (positive definite) quadratic forms in the x^2, x^3 -plane, whose contour-levels are ellipses. If the off-diagonal terms N_{23}, Φ_{23} vanish, the axes of these ellipses coincide with those of the laboratory. In this case, the *radii of curvature* and *beam widths* along the x^α -direction $(\alpha = 2, 3)$ can be introduced simply as

$$N_{\alpha\alpha} = \frac{1}{LR_\alpha}, \qquad \Phi_{\alpha\alpha} = \frac{2}{\kappa w_\alpha^2} \qquad \text{(no sum on } \alpha \text{)}. \tag{11}$$

In the general case, it is then clear that the quantites $N_{\alpha\beta}, \Phi_{\alpha\beta}$ contain the informations about curvature and width of the beam.

In order to calculate deposition profiles, a further equation for the power $P(\tau)$ has obviously to be added:

$$\frac{dP}{d\tau} = -2\gamma P, \tag{12}$$

where γ is the absorption coefficient, evaluated using the weakly relativistic approximation [5].

3. The numerical solution

The solution of Eqs. (5-7) in a general tokamak geometry for arbitrary launching conditions requires a numerical treatment. This can in principle be performed in a straightforward way, since we have to do with a set of ordinary differential equations, as already stressed. It can be shown that the Hamiltonian function H can be chosen such that $H = \det \hat{L}$ as in the usual geometric optics. On the reference ray it is then $H = 0$, and this condition can be used to control the accuracy of the solution along with Eqs.(9-10).

The beam tracing equations are solved for plasma and beam parameters in the range of interest of ASDEX-Upgrade tokamak. An elongated geometry with a Shafranov shift is assumed. This means that Cartesian coordinates in the poloidal plane can be introduced as

$$\left\{ \begin{array}{l} x = R_0 + a\cos\chi - \Delta(a) \\ z = a\lambda(a)\sin\chi, \end{array} \right. \tag{13}$$

where R_0 is the major radius, Δ is the Shafranov shift and λ is the elongation. The density is supposed to be constant on flux surfaces labeled by the radial coordinate a ($0 \le a \le a_M$). Typical values are $R_0 = 165$ cm, $a_M = 60$ cm, $1 \le \lambda(a) \le 1.5$, $-10 \le \Delta(a) \le 5$ cm. The frequency is $\omega/2\pi =$ 140 GHz, corresponding to the second-harmonic electron cyclotron resonance. The initial wave front has a circular symmetry, and width and curvature [in the sense of Eqs.(11)] are $w_0 =$ 3.8 cm and $R_0 =$ 142 cm.

As an example, in Fig. 1 the X -mode beam propagation in the poloidal plane is plotted, along with the corresponding deposition profile. The launching angle is 20° with respect to the equatorial plane and central heating is considered. Dashed lines are the geometric-optics calculations. The central density is $n_{e0} = 11 \cdot 10^{13}$ cm $^{-3}$.

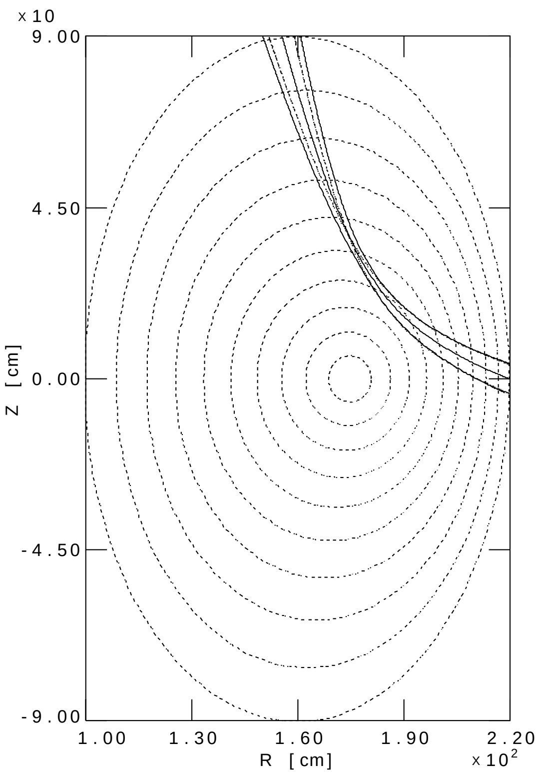

Figure 1 (a). X-mode wave beam propagation.

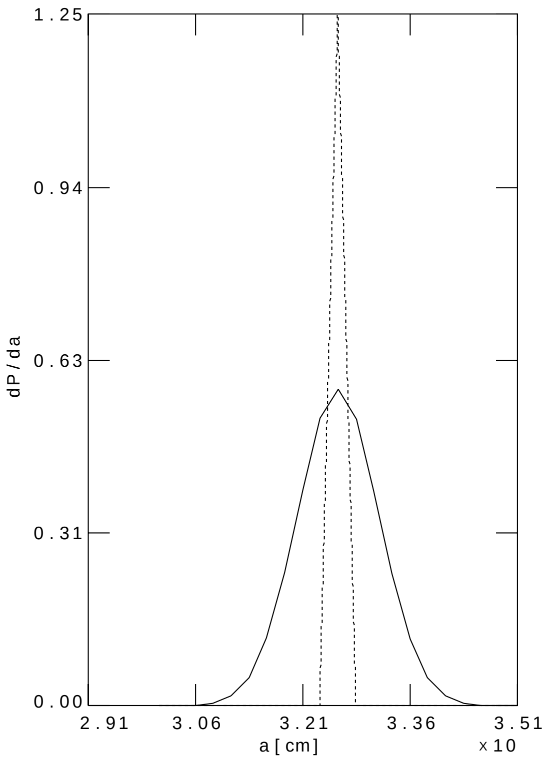

Figure 1 (b). Beam tracing and ray tracing deposition profiles.

Since geometric optics predicts a focus in the absorbtion region (replaced by a finite *waist* in the beam tracing description), it is clear that power deposition is confined in this approach in a much smaller zone.

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