Frustration of decoherence in Y-shaped superconducting Josephson networks

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We show that Y-shaped superconducting Josephson networks allow for engineering two-level quantum systems, which may be operated at points where their entanglement with the environment is frustrated. This happens since Y-shaped networks exhibit a new, finite coupling, infrared fixed point in their phase diagram. Our approach uses boundary conformal field theory, which naturally allows for a full field-theoretical treatment of the phase slips, describing quantum tunneling between the two degenerate levels.

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Superconducting Josephson devices are promising candidates for realizing quantum coherent two-level systems, interesting for their potential relevance to quantum information processing [1]. For the engineering of realistic devices, one should be able to tame the decoherence, unavoidably arising from the interaction of the two-level system with both the control circuitry and the plasmon modes lying outside the subspace spanned by the two low-lying operating states.

Decoherence in a physical system arises since the total state of the two-level system and of its environment evolves towards an entangled state; thus, it is a quantum phenomenon intrinsically different from dissipation, which relies on the transfer of energy from a subsystem to an environment [2]. When a system is such that it is coupled to more than one bath, and its entanglement with each one of the baths is suppressed by the other(s), the decoherence phenomenon is frustrated [3]. In the following we evidence that quantum frustration of decoherence may be induced in a superconducting Josephson device if its phase diagram admits a finite coupling infrared (IR) fixed point (FFP).

Josephson superconducting devices provide remarkable realizations of quantum systems with impurities, whose phase diagrams, in simple cases, admit only two fixed points: an unstable weak coupling fixed point (WFP), and a stable one at strong coupling (SFP), [4]. Neither one of the above fixed points yields frustration of decoherence, since, at weak coupling, there is not even quantum tunneling between the states while, at strong coupling, there is full entanglement between the two degenerate states and the plasmon modes [4]. In this letter we show that a FFP emerges in a Y-shaped Josephson junction network and that its existence is sufficient for inducing quantum frustration of decoherence.

For our analysis it is most convenient to use a description of 1d-superconducting devices in terms of 1+1-boundary conformal field theories (BCFT)'s, in which the plasmon modes are described as bosonic conformal fields, whose boundary conditions are set by the strength of the *impurity* Josephson junction(s) [4–6]. First of all, BCFT 's enable to account for the phase slip (instanton) trajectories between the minima of the boundary potential in terms of dual vertex operators and, thus, allow for a quantitative analysis of the decoherence arising from the interaction of the instantons with the modes of the plasmon field. Furthermore, BCFT 's provide us with reliable tools not only for analyzing the phases accessible to junctions of three (or more) quantum wires [7] and to models of quantum Brownian motion on frustrated planar lattices [8, 9], but also for predicting the response of quantum wires [10] or spin chains [11] to pertinent constrictions. Recently, ultra-cold atoms in Y-shaped potentials have been analyzed in [12], evidencing remarkable analogies of these systems with the corresponding fermionic systems.

A Y- shaped Josephson junction network (see Fig.1) is realized by weakly connecting three equal superconducting grains forming a "central region" **C**, pierced by a magnetic flux $\bar{\Phi}$, with the endpoints of three chains of Josephson junctions of length L, with lattice step a. The "outer" endpoint of each chain is connected to a bulk superconductor, at a fixed phase φ_i $(i = 1, 2, 3)$. These phases act as control parameters of the device, which may be regarded as a pertinent planar realization of the tetrahedral qubit proposed in Ref.[13]. The central triangular region is regarded as the quantum system, while the three chains provide the environment of plasmon modes. For the sake of simplicity, we assume that all the junctions are identical, except for the ones joining the endpoints of the chains to the vertices of the triangle. Defining E_c as the charging energy of each grain, and E_J as the Josephson energy of the junctions (throughout all the paper, we set $c = \hbar = 1$; the electric charge is measured in units of the Cooper pair charge e^*), the Hamiltonian describing

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the central triangular array $\mathbf{C}$ is given by

$$H_{\mathbf{C}} = \frac{E_c}{2}\sum_{i=1}^{3}\left[-i\frac{\partial}{\partial\phi_0^{(i)}} - W_g\right]^2 - 2E_J\sum_{i=1}^{3}\cos\left[\Delta\phi_0^{(i)} + \frac{\bar{\varphi}}{3}\right] \quad (1)$$

where $\Delta\phi_0^{(i)} = \phi_0^{(i)} - \phi_0^{(i+1)}$, $\phi_0^{(i)}$ is the phase of the superconducting order parameter at grain i, $\bar{\varphi}$ is the flux $\bar{\Phi}$ in units of the quantum of flux Φ_0^*, and W_g is a gate voltage. If $E_J/E_c \ll 1$, $W_g^{(i)} = N + \frac{1}{2} + h$, with integer N and $0 < h < 1/2$, the low-energy dynamics is governed only by the two states with total charge equal to N and to $N+1$; the effective Hamitonian may be expressed in terms of spin-1/2 operators and is given by $H_{\mathbf{C}}^{\rm Eff} = -E_c h\sum_{i=1}^{3}(S_0^{(i)})^z - E_J\sum_{i=1}^{3}[e^{i\frac{\bar{\varphi}}{3}}(S_0^{(i)})^+(S_0^{(i+1)})^- + {\rm h.c.}]$, with $(S_0^{(i)})^\pm = \mathbf{P_C}e^{\pm i\phi_0^{(i)}}\mathbf{P_C}$, $(S_0^{(i)})^z = \mathbf{P_C}(-i\frac{\partial}{\partial\phi_0^{(i)}} - N - \frac{1}{2})\mathbf{P_C}$, and $\mathbf{P_C}$ being an operator projecting onto the subspace with total charge N or $N+1$ at each grain. At low energy and long wavelengths, the three chains may be described by the Luttinger Liquid Hamiltonian [4]

$$H_0 = \frac{g}{4\pi}\sum_{i=1}^{3}\int_0^L dx\left[\frac{1}{v}\left(\frac{\partial\Phi_i}{\partial t}\right)^2 + v\left(\frac{\partial\Phi_i}{\partial x}\right)^2\right] \quad , \quad (2)$$

where $g = \sqrt{\frac{v_0}{v_0 + 4\pi a\Delta[1-\cos(2ak_f)]}}$, $v = v_0\sqrt{1 + 2\frac{4\pi a\Delta[1-\cos(2ak_f)]}{v_0}}$, with $\Delta = E^z - \frac{3}{16}\frac{E_J^2}{E_c}$ and E^z being the interaction energy of charges on neighboring junctions. The Fermi momentum and the bare Fermi velocity are given by $k_f = \arccos(\frac{hE_c}{E_J})$ and $v_0 = 2\pi E_J\sin(ak_f)$ [4, 6]. Fixing the phase at the outer boundary of the chains, implies Dirichlet boundary conditions on $\Phi_i(x)$ at $x = L$: $\Phi_i(L) = \varphi_i$. Furthermore, the charge tunneling between the triangular region $\mathbf{C}$ and the inner boundary of the three chains is described by a Josephson-like interaction, with nominal strength $\lambda \ll E_J$. As a result, using Neumann boundary conditions at the inner boundary, i.e. $\frac{\partial\Phi_i(0)}{\partial x} = 0\ \forall i$, allows to write the Josephson boundary Hamiltonian as $H_T = -\lambda\sum_{i=1}^{3}\cos[\Phi_i(0) - \phi_0^{(i)}]$.
A boundary field theory approach allows to trade the interaction Hamiltonian $H_{\mathbf{C}}^{\rm Eff} + H_T$ with an effective boundary Hamiltonian, H_b, involving only $\Phi_i(0)$, and given by

$$H_b = -2\bar{E}_W\sum_{i=1}^{3} : \cos[\vec{\alpha}_i\cdot\vec{\chi}(0) + \chi] : \quad , \quad (3)$$

with $\chi_1(x) = \frac{1}{\sqrt{2}}[\Phi_1(x) - \Phi_2(x)]$, $\chi_2(x) = \frac{1}{\sqrt{6}}[\Phi_1(x) + \Phi_2(x) - 2\Phi_3(x)]$, $\vec{\alpha}_1 = (1,0)$, $\vec{\alpha}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$, $\vec{\alpha}_3 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, $\chi = {\rm atan}[3\tan(\frac{\bar{\varphi}}{3})]$, and $\bar{E}_W = \left(\frac{a}{L}\right)^{\frac{1}{g}}E_W$, with $E_W \approx \frac{\lambda^2 E_J}{24(E_c)^2\tilde{h}^2}\sqrt{1 + 2\sin^2(\frac{\bar{\varphi}}{3})}$. The colons : : denote normal ordering with respect to the ground state of the plasmon modes, $|\{0\}\rangle$. H_b is the Bosonic version of the Y-junction Hamiltonian for three quantum wires [7]. Despite the fact that it does not contain the Klein factors, due to the bosonic nature of Cooper pairs, we shall show that there is still a range of values of g and χ where the phase diagram exhibits a FFP. This happens at $\chi = \pi$, as for the quantum brownian motion on a planar frustrated lattice [8], since, in a suitable range of values of g, neither the WFP, or the SFP, is stable. The second-order Renormalization

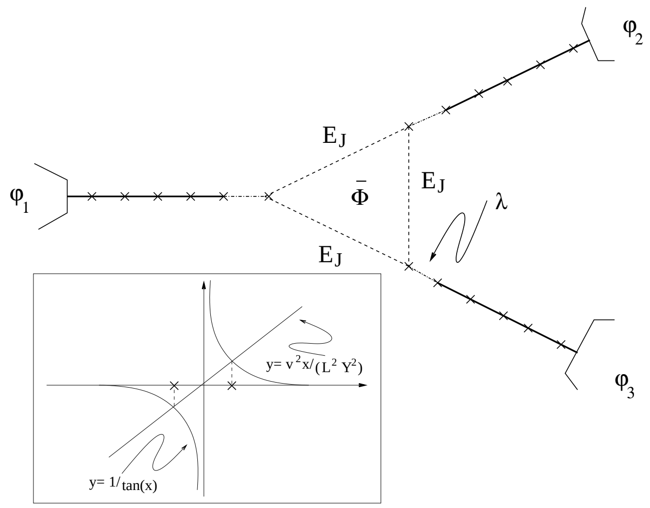

FIG. 1: Sketch of the Y-shaped Josephson network; **Inset**: graphical exact solutions for the energy levels at $g = 9/8$.

Group (RG) equation for the running coupling strength $G = L\bar{E}_W$ near the weakly coupled fixed point is given by $\frac{dG}{d\ln(\frac{L}{L_0})} = \left(1 - \frac{1}{g}\right)G - 2G^2$ (L_0 is a reference length scale), and shows that H_b is a relevant perturbation for $g > 1$, while it is irrelevant for $g < 1$. Since at the SFP, the fields $\chi_j(x)$ obey Dirichlet boundary conditions at both boundaries, the strong coupling effective boundary Hamiltonian depends only on the dual fields $\Theta_j(x)$, defined by $\frac{\partial\Theta_j}{\partial x} = \frac{1}{v}\frac{\partial\chi_j}{\partial t}$ and $\frac{1}{v}\frac{\partial\Theta_j}{\partial t} = \frac{\partial\chi_j}{\partial x}$, with a normal mode expansion given by $\Theta_j(x,t) = \sqrt{2g}\left\{\theta_{0,j} + \frac{\pi P_j}{L}vt + i\sum_{n\neq 0}\cos\left[\frac{\pi n x}{L}\right]\frac{\alpha_{j,n}}{n}e^{-i\frac{\pi n}{L}vt}\right\}$, with $[\theta_{0,i}, P_j] = i\delta_{i,j}$ and $[\alpha_{i,n}, \alpha_{j,m}] = n\delta_{n+m,0}\delta_{i,j}$. At $\chi = \pi$, the eigenvalues p_1, p_2 of the zero-mode operators P_1, P_2 span the honeycomb lattice defined by $(p_1, p_2) = \sqrt{\frac{g}{2}}(n_1 + \sqrt{2}\beta_1, \frac{1}{\sqrt{3}}(2n_2 + n_1 + 2\sqrt{2}\beta_2))$, and by $(p_1, p_2) = \sqrt{\frac{g}{2}}(n_1 - \frac{1}{3} + \sqrt{2}\beta_1, \frac{1}{\sqrt{3}}(2n_2 + n_1 - 1 + 2\sqrt{2}\beta_2))$, with integer (n_1, n_2), $\beta_1 = (\varphi_1 - \varphi_2)/(2\pi\sqrt{2})$, and $\beta_2 = (\varphi_1 + \varphi_2 - 2\varphi_3)/(2\pi\sqrt{6})$. Following the approach outlined in Ref.[8], the "dual" boundary Hamiltonian $\tilde{H}_b$ may be presented as $\tilde{H}_b = -Y\sum_{i=1}^{3}\{T^- V_i(0) + T^+\bar{V}_i(0)\}$, with $V_i(\bar{V}_i) =: \exp\left[-(+)\frac{2}{3}\vec{\alpha}_i\cdot\vec{\Theta}\right]$:, and $\vec{T}$ being an effective isospin operator, connecting the minima of the honeycomb lattice of the zero-mode eigenvalues. Y is an effective coupling strength defined as $Y = E_J - \bar{E}_W$ [14]. From the O.P.E. of the vertex operators entering $\tilde{H}_b$, the RG equation for the running coupling strength $y = LY$ can be derived as $\frac{dy}{d\ln\left(\frac{L}{L_0}\right)} = \left(1 - \frac{4g}{9}\right) y - \frac{2g}{3} y^3$. Thus, one sees that, for $\chi = \pi$, and $1 < g < \frac{9}{4}$, neither the WFP, or the SFP, are stable. Accordingly, a minimal hypothesis for the phase diagram requires a FFP at $y = y_*$, with y_* finite. For instance, for $g = \frac{9}{4} - \gamma$, with $\gamma \ll 1$, one obtains $y^* \approx \left(\frac{2}{3}\right)^{\frac{3}{2}} \sqrt{\gamma}$.

For $y \ll 1$ near the SFP, the low-energy spectrum is given by $E = \frac{\pi v}{2L}[\vec{p}]^2 + E'$, where $\vec{p} = (p_1, p_2)$ labels the zero-mode contribution, while E' comes from the plasmon modes. At particular values of β_1 and β_2, the zero-mode contributions to the total energy coming from two nearest neighboring sites on the honeycomb lattice, may become degenerate with each other: this happens, for instance, if $\beta_1 = 1/3\sqrt{2}, \beta_2 = 0$. The two degenerate quantum states $|\uparrow\rangle$ and $|\downarrow\rangle$ -labelled by $(n_1, n_2) = (0, 0)$ on sublattice A and by $(n_1, n_2) = (1, 0)$ on sublattice B- become degenerate, and are characterized by opposite values of the Josephson current flowing across chain-1 and chain-2: $I_1 = -I_2 = \pm\frac{\pi g v e^*}{3L}$, while $I_3 = 0$.

Quantum tunneling between the two degenerate states is induced by $\tilde{H}_b$, with matrix element $-Y$. Setting $\beta_2 = 0$, and $\beta_1 = 1/3\sqrt{2} + \delta/(2\pi)$, with $\delta/2\pi \ll 1$, one easily gets the effective Hamiltonian of a two-level quantum system as

$$H_2 = \epsilon_0(\delta)\mathbf{I} + \epsilon(\delta)\sigma^z - Y\sigma^x \quad , \qquad (4)$$

where $\epsilon_0(\delta) = \frac{g}{2}\left(\frac{1}{9} + \frac{\delta^2}{4\pi^2}\right)$, $\epsilon(\delta) = \frac{g}{3}\frac{\delta}{\sqrt{2}\pi}$, the σ^a's are the Pauli matrices, and δ is a control parameter determined by the phases $\{\varphi_i\}$.

To provide an estimate of the entanglement between the two-level system described by the Hamiltonian in Eq.(4) and the environment realized by the plasmon modes of the three chains, one should compute $\chi^{''}(\omega)/\omega$ [3], where $\chi^{''}(\omega)$ is the imaginary part of the Fourier transform of the "transverse" dynamical spin susceptibility given by $\chi_\perp(\omega) = -i\int_{-\infty}^{\infty} \frac{dz}{2\pi}\{g^*_{\uparrow,\uparrow}(-z)g_{\downarrow,\downarrow}(z+\omega) - g^*_{\downarrow,\downarrow}(-z)g_{\uparrow,\uparrow}(z+\omega)\}$, with $g_{\sigma,\sigma'}(t) = \langle\sigma|e^{-i(H_0+\tilde{H}_b)t}|\sigma'\rangle$. Use of the dilute instanton gas approximation, yields an expression for the Schwinger-Dyson equation for $g_{\sigma,\sigma'}(\omega)$ given by

$$g_{\sigma,\sigma'}(\omega) = \frac{\delta_{\sigma,\sigma'}[[g^{(0)}_{\bar{\sigma}}]^{-1}(\omega) + Y^2\Gamma_\sigma(\omega)] + iY\delta_{\sigma,\bar{\sigma}'}}{\mathcal{D}(\omega)} \quad , \qquad (5)$$

with $\bar{\sigma} = -\sigma$, $g^{(0)}_\sigma(t) = \langle\sigma|e^{-iH_0t}|\sigma\rangle$, and $\mathcal{D}(\omega) = \{[g^{(0)}_\uparrow]^{-1}(\omega) + Y^2\Gamma_\downarrow(\omega)\}\{[g^{(0)}_\downarrow]^{-1}(\omega) + Y^2\Gamma_\uparrow(\omega)\} + Y^2$. The "self-energy" $\Gamma_\sigma(\omega)$ is the Fourier transform of $\Gamma(\tau_1 - \tau_2) = \langle\{0\}| : e^{\pm i\frac{2}{3}\Theta(\tau_1)} :: e^{\mp i\frac{2}{3}\Theta(\tau_1)} : |\{0\}\rangle = [e^{\frac{\pi i}{L}v\tau_1} - e^{\frac{\pi i}{L}v(\tau_2+i\eta)}]^{-\frac{8}{9}g}$ at frequency $\omega - \epsilon_\gamma$, where $\epsilon_{\uparrow(\downarrow)} = \pm\epsilon(\delta)$. At low frequencies, one has that $\Gamma_\gamma(\omega) \approx e^{\frac{8}{9}\pi i g}\Gamma[1 - \frac{8}{9}g]\left(\frac{L}{\pi v}\right)^{\frac{8}{9}g}(\omega - \epsilon_\gamma)^{\frac{8}{9}g-1}$.

When the SFP is stable, that is, for $g > 9/4$, the boundary interaction is irrelevant, and one may neglect corrections to the amplitudes of order Y^2, getting $\frac{\chi^{''}(\omega)}{\omega} \propto [\delta(\omega + 2\epsilon(\delta)) - \delta(\omega - 2\epsilon(\delta))]/\omega$. Thus, for $g > 9/4$, there is no entanglement between the two-level quantum system and the plasmon modes. However, one cannot conclude that the decoherence is frustrated, since, in this range of g, there is no tunnel splitting of the two degenerate states. At variance, when $g < 1$, the running coupling constant y becomes relevant and, by keeping only the leading contributions in y, one gets $\frac{\chi^{''}(\omega)}{\omega} \propto [|2\epsilon(\delta) + \omega|^{3-\frac{16}{9}g} - |2\epsilon(\delta) - \omega|^{3-\frac{16}{9}g}]/\omega$. It should be noticed that, for a finite-size Y-shaped network, the removal of the degeneracy between the two zero eigenmodes, allows for an extra renormalization of the anomalous dimension of the boundary operator, α, [3]. To the zeroth order, one gets $\alpha = \frac{4}{9}g$, while third order contributions renormalize α according to $\frac{d\alpha}{d\ln\left(\frac{L}{L_0}\right)} = -\alpha y^3$. In the following, we neglect third order contributions since they do not affect the phase diagram of the device. In Fig.2, it is presented a plot of $\chi^{''}(\omega)/\omega$ *vs.* ω near the SFP, the WFP, and the FFP. One sees that, near the WFP, a large part of the spectral weight has moved from the side peaks towards $\omega = 0$, thus signaling [3] a strong decoherence of the two-level system described by Eq.(4). Operating the device near the attractive FFP, not only enables one to avoid a strong entanglement with the environment, but also to achieve a good tunnel splitting between the two degenerate levels. To verify this, one may compute $\chi^{''}(\omega)/\omega$ to the leading order in γ, in the limit for $g = \frac{9}{4} - \gamma$: as a result one has two peaks centered around $\pm\sqrt{[\epsilon(\delta)]^2 + (\frac{\pi v}{L}y_*)^2}$, where y_* is the fixed point value of the running coupling constant. This renormalization of the energies of the two-level quantum system clearly signals a quantum coherent tunneling due to the frustration of decoherence induced by the weak entanglement of the system with the environment. However, it should be noticed that the two peaks now have a finite width $\propto \frac{\pi v}{L}(y_*)^{1+\frac{8}{9}g}$. The results for $\chi^{''}(\omega)/\omega$ are reported in Fig.2.

Following the procedure outlined in Ref.[15], for $g = 9/8$ one finds that the exact low-lying energy levels are given by $\frac{v}{L\tan\left(\frac{LE}{v} + \frac{\delta}{2}\right)} + \frac{Y^2}{E - \frac{\pi v}{2L} - \frac{v\delta}{2L}} = 0$. In the inset of Fig.1, it is reported the graphical solution for $\delta = 0$. The two levels define a two-level quantum system, controllable by tuning δ.

For $g = 1 + \gamma'$, with $\gamma' \ll 1$, the existence of a FFP can be derived also from the RG equation at the WFP, leading to $G = G^* \approx \gamma'/2$. However, a finite-size device operating near G^* exhibits a nondegenerate minimum. Only when the FFP is close to the SFP, a finite size, Y-shaped Josephson junction network supports a two-level quantum system. Furthermore, for a device of finite size L, the FFP is stable also against small fluctuations of the flux $\bar{\Phi}$, provided that v/L is sufficiently big. In fact, if, as a result of the fluctuation in $\bar{\Phi}$, the degeneracy point

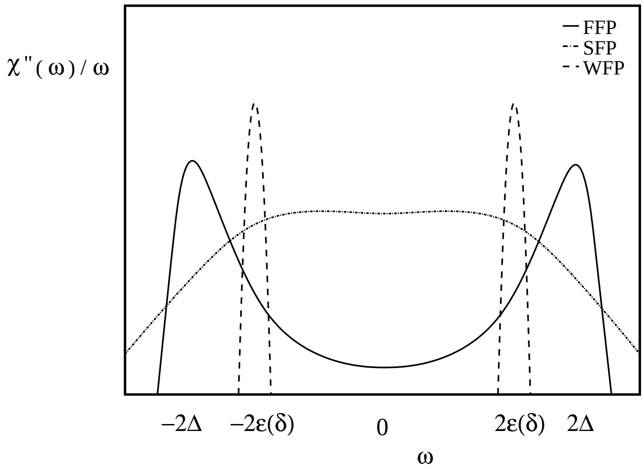

FIG. 2: Qualitative behavior of $\chi^{"}(\omega)/\omega$ in the various regimes.

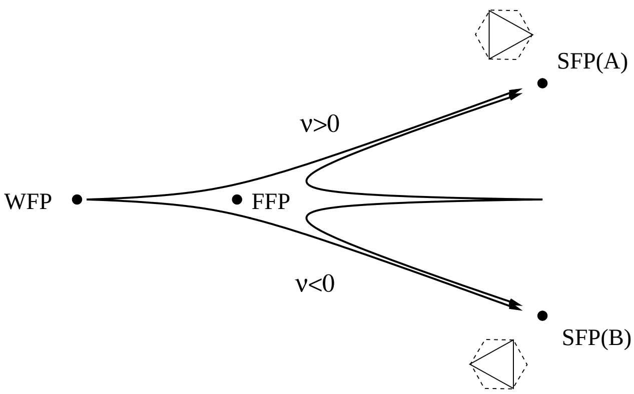

FIG. 3: Sketch of the RG flow for $\chi = \pi + \nu$ ($\nu/\pi \ll 1$).

$\chi = \pi$ is displaced by a small amount ν (i.e., $\chi = \pi + \nu$, with $\nu/\pi \ll 1$), one has that $v/L > \bar{E}_W \sin(\nu)$, where $\bar{E}_W \sin(\nu)$ is the energy splitting between the minima of the two triangular sublattices forming the honeycomb lattice. A sketch of the RG flow diagram is reported in Fig.3. One sees that, for $v/L < \bar{E}_W \sin(\nu)$, the system flows towards the SFP and that, depending on $\mathrm{sgn}(\nu)$, the minima of the boundary potential lie on either one of the triangular sub-lattices forming the honeycomb lattice Near the stable FFP, the quantum coherence properties are universal, since they are independent of the precise values of the bare parameters. In fact, for $g > 1$, inhomogeneities in the outer chains, induced by differences in the Josephson energies of the junctions, provide an irrelevant perturbation, since the pertinent operator scales as $\left(\frac{L}{L_0}\right)^{1-g}$ [4]: thus, inhomogeneities in the device's fabrication should not alter the main results of our analysis. Today 's technology allows to fabricate superconducting devices [16] whose parameters E_J, E_c, E^z, a, L, safely yield values of g ranging from $g < 1$, to $g \sim 2$. The relevant control parameter δ of the effective two-level system supported by the Y- shaped Josephson network is determined by the phase differences β_1, β_2, ultimately dependent on the phase differences between the bulk superconductors ending the chains. In a realistic setting, δ may be acted upon if one regards the Y-shaped Josephson network as a pertinent planar realization of the superconducting tetrahedral qubit, proposed in Ref.[13].

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