

# Phonon background versus analogue Hawking radiation in Bose-Einstein condensates

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We determine the feasibility of detecting analogue Hawking radiation in a Bose-Einstein condensate in the presence of atom loss induced heating. We find that phonons created by three-body losses overshadow those due to analogue Hawking radiation. To overcome this problem, three-body losses may have to be suppressed, for example as proposed by Search *et al.* [Phys. Rev. Lett. **92** 140401 (2004)]. The reduction of losses to a few percent of their normal rate is typically sufficient to suppress the creation of loss phonons on the time scale of a fast analogue Hawking phonon detection.

PACS numbers: 03.75.Nt, 03.75.Gg, 04.80.-y,

*Introduction:* An analogue of the Hawking effect [1, 2] should in principle exist in cold moving fluids [3]. A fluid whose velocity profile contains a transition from subsonic to supersonic flow will emit a thermal radiation of quantized sound waves (phonons). Gaseous Bose-Einstein condensates (BECs) were often considered most promising for the observation of this phenomenon [4–10]. However, for accessible Hawking temperatures the condensate must have densities for which three-body losses become relevant [11]. These strongly constrain the time available for phonon detection already on the level of the mean-field. Here, we consider limitations arising from three-body loss in a quantum treatment [12] and show that they are even more severe.

The primary consequence of three-body loss is a reduction of the condensate density in time, resulting in a decrease of the Hawking temperature. Three-body losses also heat the condensate by driving the many-body quantum state away from the Bogoliubov vacuum [13]. This creates phonons, which contribute in general to a background indistinguishable from analogue Hawking radiation [14]. Here we compare these two effects and show that in equilibrium the loss-phonons always overshadow those created by the Hawking effect. For the densities required to observe analogue Hawking radiation [11] the loss-phonons are created on the same time-scale as the Hawking phonons.

Our findings indicate that it may be necessary to suppress three-body losses in a BEC in order to observe analogue Hawking radiation. Fortunately, suppression schemes exist [15, 16]. A suppression will have a two-fold benefit: Firstly it increases the time-scale for the loss-heating to reach equilibrium, making it possible to conduct an experiment before they become relevant. Secondly, we can employ higher density condensates and obtain a stronger Hawking signal, due to the reduced effect of loss on the mean-field.

The surface at which the normal component of the condensate flow exceeds the local speed of sound is termed

*sonic horizon* [4]. As consequence of the causal disconnection of the supersonic and subsonic regions, quantum field theory predicts particle creation [17]. The *analogue Hawking temperature* that characterizes their thermal spectrum is given by [5]

$$T_H = \frac{\hbar g_h}{2\pi k_B c_h}, \quad c_h = c(x_h), \quad (1)$$

$$g_h = \frac{1}{2} |\hat{n} \cdot \nabla (c^2 - v^2)|_{x=x_h} = c_h^2 |\hat{n} \cdot \nabla M|_{x=x_h}. \quad (2)$$

Here  $v$  is the flow speed,  $c$  the speed of sound,  $M = v/c$  the Mach number,  $x_h$  denotes a position on the horizon and  $\hat{n}$  a normal vector to it. The underlying correspondence between the equations of motion for a scalar quantum field in curved space-time and for quantum phonons in a fluid can provide further analogies between cosmological effects and phenomena in a fluid [4].

Values of  $T_H$  for typical fluids are very small. Hence Bose-Einstein condensates have been considered as prime candidates for an observation of the analogue Hawking effect, owing to their low temperatures. We have shown in Ref. [11] however, that to reach temperatures even of the order of 10 nK, condensates have to be typically driven into a regime where three-body recombination significantly affects the *mean-field* on a time scale of 50 ms. Here we employ Bogoliubov theory used in Ref. [13] to determine the consequences for the *quantum field*.

*Bogoliubov-de Gennes equations:* We split the field operator for bosonic atoms  $\hat{\Psi}(\mathbf{x}) = \phi(\mathbf{x}) + \hat{\chi}(\mathbf{x})$  into a condensate wavefunction  $\phi(\mathbf{x}) = \langle \hat{\Psi}(\mathbf{x}) \rangle$  and its quantum fluctuations  $\hat{\chi}(\mathbf{x})$ . The condensate wave function obeys the Gross-Pitaevskii equation

$$i\hbar \frac{\partial \phi}{\partial t} = \mathcal{L}\phi \equiv \left( -\frac{\hbar^2}{2m} \nabla^2 + W + g|\phi|^2 \right) \phi, \quad (3)$$

where  $m$  is the atomic mass and  $g$  the interaction strength related to the scattering length  $a_s$  by  $g = 4\pi\hbar^2 a_s/m$ , while  $W$  denotes an external potential. We use  $\int d^3\mathbf{x} |\phi(\mathbf{x})|^2 = N_{\text{cond}}$ , the number of condensate atoms. Where required, we use the notation  $\rho = |\phi|^2$  for the condensate density,  $v = i\hbar/(2m)[(\nabla\phi^*)\phi - \phi^*\nabla\phi]$  for its velocity and  $c = \sqrt{g\rho/m}$  for the speed of sound.

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We decompose the fluctuating component as

$$\hat{\chi}(\mathbf{x}) = \sum_n [u_n(\mathbf{x})\hat{\alpha}_n + v_n^*(\mathbf{x})\hat{\alpha}_n^\dagger]. \quad (4)$$

The presence of a subscript distinguishes references to the condensate velocity  $v$  from those to the mode  $v_n$ . The functions  $u_n, v_n$  obey the Bogoliubov-de Gennes (BdG) equations [18–20]:  $\mathcal{L}_{\text{BdG}}[u_n(\mathbf{x}), v_n(\mathbf{x})]^T = \epsilon_n[u_n(\mathbf{x}), v_n(\mathbf{x})]^T$ , with

$$\mathcal{L}_{\text{BdG}} = \begin{bmatrix} \mathcal{L} - \mu + g\hat{Q}|\phi|^2\hat{Q} & g\hat{Q}\phi^2\hat{Q}^* \\ -g\hat{Q}^*\phi^{*2}\hat{Q} & -(\mathcal{L} - \mu + g\hat{Q}|\phi|^2\hat{Q})^* \end{bmatrix}, \quad (5)$$

where  $\hat{Q} = 1 - |\phi\rangle\langle\phi|/N_{\text{cond}}$  projects onto the function space orthogonal to the condensate mode. The modes also must be normalized according to  $\int d^3\mathbf{x}[|u_n(\mathbf{x})|^2 - |v_n(\mathbf{x})|^2] = 1$ , for the fluctuations to obey bosonic commutation relations  $[\hat{\alpha}_n, \hat{\alpha}_m^\dagger] = \delta_{n,m}$ .

For a homogeneous condensate in a quantization volume  $V$  we have  $W = 0$ ,  $\phi = \mu/g$ . The BdG equations are then solved by [21, 22]

$$\begin{aligned} u_{\mathbf{q}}(x) &= \frac{1}{\sqrt{V}}\tilde{u}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{x}}, \quad \tilde{u}_{\mathbf{q}} = \frac{\epsilon_{\mathbf{q}} + \epsilon_0}{2\sqrt{\epsilon_{\mathbf{q}}\epsilon_0}}, \\ v_{\mathbf{q}}(x) &= \frac{1}{\sqrt{V}}\tilde{v}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{x}}, \quad \tilde{v}_{\mathbf{q}} = -\frac{\epsilon_{\mathbf{q}} - \epsilon_0}{2\sqrt{\epsilon_{\mathbf{q}}\epsilon_0}}, \\ \epsilon_0 &= \hbar^2\mathbf{q}^2/2m, \quad \epsilon_{\mathbf{q}} = \sqrt{\hbar^2\mathbf{q}^2c^2 + \epsilon_0^2}. \end{aligned} \quad (6)$$

In the following we will assume that a sonic horizon is present in the condensate such that analogue Hawking radiation is created, but that the bulk condensate can still be considered as a homogeneous reservoir. No details regarding how to achieve this situation are required here, but can be found in [11].

*Loss induced phonons:* Dziarmaga and Sacha have shown in Ref. [13] that besides a reduction of the condensate population, atom losses also result in creation of phonons since the many-body quantum state is driven away from the Bogoliubov vacuum  $|0\rangle$ , defined by  $\alpha_n|0\rangle = 0$ . These phonons will make a detection of analogue Hawking radiation more difficult. Thus the relative strength of the phonon sources has to be determined. We focus on three-body losses in what follows, since they are most prominent in BECs at high densities.

Three-body recombination results in a molecule and an energetic atom. The excess energy due to molecular binding,  $E_b = \hbar^2/ma_s^2$ , is split between the kinetic energies of molecule and fast atom in the ratio 1 : 2. When these kinetic energies suffice for both particles to leave the trap, their corresponding quantum fields can be eliminated from the picture [23]. One obtains an effective master equation that describes the effect of the loss process on the quantum state of the remaining trapped atoms. Similar steps are possible for  $l$ -body loss and one obtains [13, 23]

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \sum_l \gamma_l \int d\mathbf{x} \mathcal{D}[\hat{\Psi}^l]\hat{\rho}, \quad (7)$$

where  $\mathcal{D}[\hat{a}]\hat{\rho} \equiv \hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}/2 - \hat{\rho}\hat{a}^\dagger\hat{a}/2$ .  $\hat{H}$  is the usual Hamiltonian describing the conservative dynamics of the remaining trapped atoms. Importantly,  $\gamma_l$  is the *event-rate* for a given loss process. Thus for example  $\gamma_3 = K_3/3$  [24], where  $K_3$  is the usual number loss rate for a condensate [25].

Inserting the expansion of  $\hat{\Psi}$ , Eq. (7) can be rewritten in terms of quasi-particle operators  $\hat{\alpha}_m, \hat{\alpha}_m^\dagger$ . One then recognizes that each quasi-particle mode  $m$  is coupled to a heat reservoir and its occupation  $n_m(t)$  will relax towards a thermal state [13]. For the high density condensates that we consider, the three-body loss channel is strongly dominant [11]. If a single channel dominates, we can write [13]

$$dn_m(t)/dt = -l\gamma_l\alpha_{lm}N^{l-1}(t)[n_m(t) - n_{lm}], \quad (8)$$

where  $N(t)$  is the number of condensate atoms. The coefficients  $\alpha_{lm}$  and  $n_{lm}$  are determined by [13]

$$\begin{aligned} \int d\mathbf{x} |\phi_0|^{2(l-1)}|u_m|^2 &= \alpha_{lm}(1 + n_{lm}), \\ \int d\mathbf{x} |\phi_0|^{2(l-1)}|v_m|^2 &= \alpha_{lm}n_{lm}, \end{aligned} \quad (9)$$

with the condensate mode  $\phi_0$  defined by  $\phi = \sqrt{N(t)}\phi_0$ . Eq. (8) evolves each occupation number towards the equilibrium value  $n_{lm}$ . This value itself is time dependent in general, but can be assumed to vary slowly if losses are not too strong [13].

Before we proceed we specify the considered geometry in more detail: It appears that analogue Hawking studies are simplest in reduced dimensionality, that is in quasi 1D or 2D condensates [11]. Let the condensate be  $d$ -dimensional. We then have  $D = 3 - d$  transverse dimensions in which we assume the atoms to be tightly confined. In the longitudinal dimensions we imagine a homogenous condensate over a volume  $V$ . The three-dimensional vector  $\mathbf{x}$  is decomposed as  $\mathbf{x} = \mathbf{z} + \mathbf{r}_\perp$ , where  $\mathbf{z}$  is longitudinal and  $\mathbf{r}_\perp$  transverse. With this splitting, we can write the condensate and Bogoliubov modes:  $\phi_0 = Ae^{-r_\perp^2/2\sigma^2}/\sqrt{V}$ ,  $u_{\mathbf{q}}(\mathbf{x}) = \tilde{u}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{z}}Ae^{-r_\perp^2/2\sigma^2}/\sqrt{V}$ ,  $v_{\mathbf{q}}(\mathbf{x}) = \tilde{v}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{z}}Ae^{-r_\perp^2/2\sigma^2}/\sqrt{V}$ . Here  $\sigma$  is the ground state width of the transverse harmonic confinement and  $r_\perp = |\mathbf{r}_\perp|$ . We fix  $A$  by  $\int d^D\mathbf{r}_\perp A^2 \exp[-r_\perp^2/\sigma^2] = 1$ , which gives  $A = \pi^{-(3-d)/4}\sigma^{-(3-d)/2}$ . After replacing the discrete index  $m$  by the continuous label  $q$ , Eq. (9) can be solved by

$$\begin{aligned} n_{lq} &= |\tilde{v}_q|^2, \\ \alpha_{lq} &= \pi^{(3-d)(1-l)/2}V^{1-L}l^{(d-3)/2}\sigma^{(3-d)(1-l)}, \end{aligned} \quad (10)$$

using  $|\tilde{u}_q|^2 - |\tilde{v}_q|^2 = 1$ .

Let us denote the “ $l$ -body loss temperature” of this thermal state by  $T_{Ll}$ . We have  $n_{lq} = (e^{\epsilon_q/k_B T_{Ll}} - 1)^{-1}$ , hence  $k_B T_{Ll} = \epsilon_q / \log[1/n_{lq} + 1]$ . Since  $n_{lq} = |\tilde{v}_q|^2 \approx$

$mc/2\hbar q \approx 1/\xi q \gg 1$  for phonons, we can finally approximate  $k_B T_{Ll} = \epsilon_q n_{lq}$ . Using  $\tilde{v}_q = \sqrt{mc/(2\hbar q)}$  and  $\epsilon_q = \hbar qc$  for phononic wave numbers gives

$$T_{Ll} = \frac{mc^2}{2k_B}. \quad (11)$$

The Hawking temperature is limited to

$$T_H \lesssim \frac{mc^2}{\sqrt{2\pi} D k_B}, \quad (12)$$

with  $D \gg 1$  to avoid too rapidly varying Mach numbers at the horizon [11]. We see that with both effects in equilibrium the loss-temperature is always greater than  $T_H$ . In a situation where atoms are continuously lost from the condensate, a rigorous equilibrium in which to interpret Eq. (11) does not exist. Nonetheless, when the loss is not too strong, we expect a quasi-equilibrium to apply. It is found that the actual heating can even slightly exceed Eq. (11) [13].

*Heating time scale:* Note, that the equilibrium temperature associated with any loss process is independent of the corresponding loss rate, while the evolution towards equilibrium, Eq. (8), is not. Inserting the expression for  $\alpha_{lm}$  one obtains

$$\frac{dn_m(t)}{dt} = -l^{(d-1)/2} \tilde{\gamma}_l^d \rho_{\text{cond}}^{l-1}(t) [n_m(t) - n_{lm}], \quad (13)$$

where we use the condensate density in reduced dimensions  $\rho_{\text{cond}} = N(t)/V$  and the effective loss rate

$$\tilde{\gamma}_l^d = \pi^{(3-d)(1-l)/2} \sigma^{(3-d)(1-l)} \gamma_l. \quad (14)$$

If we consider short time scales on which  $\rho_{\text{cond}}^{l-1}(t)$  can be treated as constant, the solution of Eq. (13) is

$$n_m(t) = n_{lm} \left[ 1 - \exp(-l^{(d-1)/2} \tilde{\gamma}_l^d \rho_{\text{cond}}^{l-1} t) \right]. \quad (15)$$

The time-scale on which the phonon population reaches its equilibrium value is therefore  $\tau = l^{(1-d)/2} \tilde{\gamma}_l^{-d} \rho_{\text{cond}}^{1-l}$ . Since the reduced-dimensional density is related to the three-dimensional peak density by  $\rho_{3D, \text{peak}} = \rho_{\text{cond}} \pi^{-(3-d)/2} \sigma^{-(3-d)}$ , we can estimate  $\tau$  using 3D quantities as

$$\tau = l^{(1-d)/2} \gamma_l^{-1} \rho_{3D, \text{peak}}^{1-l}. \quad (16)$$

For usual condensate densities  $\tau$  can be quite large. One obtains  $\tau = 141\text{s}$  for  $^{23}\text{Na}$  at  $\rho_{3D} \sim 10^{20} \text{ m}^{-3}$  ( $d = 1$ ). However, the densities required for reasonable analogue Hawking temperatures are much higher [11] with consequently faster loss phonon creation. Let us parametrize the density as  $\rho = \sqrt{3f/K_3 \Delta t}$ , which implies that within a time  $\Delta t$  a fraction  $f$  of this density will be lost due to three-body recombination. We now choose  $\Delta t$  as the time which is required to detect the Hawking effect. We imply  $\Delta t \sim 50 \text{ ms}$  in what follows, about the time required for quick spectroscopic phonon detection [26]. We

atom	$^4\text{He}$	$^{23}\text{Na}$	$^{87}\text{Rb}$	$^{137}\text{Cs}$
$g \times 10^{50} [\text{Jm}^3]$	15.7	1	0.5	0.66
$K_3 \times 10^{42} [\text{m}^6/\text{s}]$	9000	2.12	32	130
$\rho_{\text{max}} \times 10^{-19} [\text{m}^{-3}]$	3.0	194	50	25
$\xi [\mu\text{m}]$	0.42	0.086	0.12	0.12
$T_H [\text{nK}]$	3.8	16	2.2	1.4
$\eta_0 [\%]$	0.7	13	0.2	0.09
$mc_{\text{max}}^2/k_B [\mu\text{K}]$	0.34	1.4	0.2	0.25
$\hbar\omega_{\perp}/k_B [\mu\text{K}]$	6	30	4	5
$\omega_{\perp}/2\pi [\text{kHz}]$	125	625	83	104
$E_b/k_B [\mu\text{K}]$	2100	2700	190	29
$r_0/a_{\perp}$	27	14	9.7	3.4

TABLE I: Comparison of common BEC species regarding analogue Hawking radiation and loss heating. The upper 5 rows are reproduced from Ref. [11]. Nextly we indicate  $\eta_0$ , defined as fraction of the original three-body loss rate that allows  $T_H = 30\text{nK}$  for  $\Delta t = 50 \text{ ms}$ , while delaying the time scale for loss phonon production to  $\tau \sim 20\Delta t$ , see Eq. (18). The lower 5 rows show the hierarchy of energy scales  $mc^2 \ll \hbar\omega_{\perp} \ll E_b$  required to achieve quasi 1D or 2D trapping, while still allowing three-body loss products to escape. The  $\omega_{\perp}$  are selected to allow this. The mean field energies use  $\rho = \rho_{\text{max}}$ . We define  $r_0$  by  $W(r_0) = E_b$ , where  $W$  is the transverse harmonic trap. The radius  $r_0$  should be large compared to  $a_{\perp}$ . This allows a tightly harmonic trap in the condensate region and a smooth transition to an asymptotic value  $W(r \gg a_{\perp}) < E_b$ .

arrive at a heating time scale under these conditions of

$$\tau = \Delta t/3f. \quad (17)$$

This is of the order of the proposed measurement time  $\Delta t$  and hence too short, unless we have  $f \ll 1/3$ . However as was found in [11], in the regime of such small loss the Hawking temperatures become problematically low. Hence, we usually have  $\tau \sim \Delta t$ . Alternatively we can calculate the time to create one phonon in a given spectral region, by the Hawking effect or by the loss. For the same parameter regime as above, one finds that the time scales are comparable.

*Suppression of three-body loss:* It has been proposed to inhibit three-body loss processes in BECs by periodically flipping the phase of the weakly bound molecular state that causes the loss [15]. The phase flip can be achieved using repeated  $2\pi$  pulses of laser light resonant on an electronic excited state transition of the bound state. Destructive interference is then responsible for a reduction in three-body loss rates to only a few percent of their usual value. We now investigate whether this is sufficient to overcome three-body loss related obstacles to the creation of analogue Hawking radiation in BECs.

By Eq. (17), we require a loss suppression that enables  $f \ll 1/3$ . We pick a specified target temperature and

assume three-body loss was reduced to  $\tilde{K}_3 = \eta K_3$ , with  $0 < \eta < 1$ . The fractional loss within the measurement time  $\Delta t$  will then be

$$f = \frac{2\pi^2(k_B T_H)^2 D^2 K_3 \Delta t}{3g^2} \eta \equiv f_0 \eta. \quad (18)$$

We have used  $\rho = \sqrt{3f/K_3 \Delta t}$  and Eq. (12). See table I for values of  $\eta$  that allow analogue Hawking radiation at  $T_H = 30$  nK while separating the time-scales for phonon measurement and loss heating ( $\tau \gg \Delta t$ ).

A crucial indicator for the efficiency of the scheme presented in [15] is the number of laser pulses that fit into the average life-time of a molecule, before its quantum state is perturbed by a collision with a condensate atom. This life-time can be estimated as  $\tau = (\kappa\rho)^{-1}$ , where  $\kappa \sim 10^{-16} m^3 s^{-1}$  for  $^{87}\text{Rb}$  [27] and  $^{23}\text{Na}$  [28]. For densities  $\rho_{\text{Rb}} = 5 \times 10^{20} m^{-3}$ ,  $\rho_{\text{Na}} = 2 \times 10^{21} m^{-3}$ , we obtain  $\tau \sim 10 \mu\text{s}$ . According to Ref. [15] this allows a loss reduction to a few percent.

A side effect of the laser pulses that suppress three-body losses is one-body loss due to Rayleigh scattering of laser photons [15]. The increased one-body loss rate has been estimated as  $\gamma_1 \sim 0.1 \text{ s}^{-1}$ . We then see from Eq. (16) that the time scale for one-body loss induced phonon creation is about 10 s and hence unproblematic. *Reaction products of loss process:* Given the importance of three-body losses, we have to address the evolution of the molecules and fast atoms created in the recombination process. For Eq. (7) to be valid, it is required that

they are energetic enough to leave the trap [23]. Also for the cosmological analogy to hold, we wish to avoid the complications of a coupled atom-molecular condensate. Finally, collisions between the loss products and remaining atoms would induce further unwanted heating if the loss products remained in the trap [29].

To ensure that the molecules and fast atoms can leave the trap, we require the trap depth characterized by  $\hbar\omega_\perp$  to satisfy  $\hbar\omega_\perp \ll E_b$ . If we further wish to study the sonic horizon in a quasi one or two dimensional setup, the strength of transverse confinement is constrained by  $mc^2 \ll \hbar\omega_\perp \ll E_b$ , where  $mc^2$  is the interaction energy of the confined condensate. Exemplary numbers for these energies are shown in table I, which demonstrate that this hierarchy can just be fulfilled.

*Conclusions:* We have shown that loss induced phonons are an overwhelming background for analogue Hawking radiation in a Bose-Einstein condensate. To overcome this problem we suggest a moderate suppression of three-body losses. This can make the time-scale of loss induced phonon creation sufficiently long for a fast detection of the analogue Hawking effect.

### Acknowledgments

We thank R. Schützhold for drawing loss heating to our attention and further fruitful discussions. It is also a pleasure to acknowledge discussions with C. Savage.

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