Non-Linear Surface Waves At a Single Interface of Semimagnetic Semiconductor – Left Handed Materials (LHM)

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***Abstract*-The nonlinear characteristics of TE surface waves at microwave frequencies in a layered structure of non-linear semimagnetic-semiconductor cover and left-handed material substrate have been investigated. Numerical analysis and derivation were carried out for the dispersion relation in its general form. The power flow has also been studied as a function of wave number for different frequencies and magnetic fields. It is shown that the proposed waveguide structure depends significantly on the operating wave frequency and can be efficiently controlled by varying the frequency.**

***Index Terms*- Left-handed materials, Dispersion relation, Power flow.**

I-INTRODUCTION

Recently, there have been considerable research studies in left-handed materials (LHM), which it is also termed as metamaterials or backward-wave materials) that have a negative refractive index. These materials, theoretically discussed first by Veselago [1], have simultaneously negative electrical permittivity ε , and negative magnetic permeability μ . L.H. material has also exhibited unique properties, such as the reversals of Snell Law, the Doppler Shift, and the Cherenkov radiation. Pendry et al [2-3] in 1996 were the first to predict structure of the above characteristics, in split-ring-resonators with (SSRs) continuous wires. They also showed that a left-handed material slab could focus evanescent modes and resolve objects only a few nanometers wide in the optical domain. Smith et al. in 2000 [4] were the first to compose and experimentally chech the Pendry's ideas. Later, in 2001 Shelby et al. [5] have demonstrated a two dimensionally isotropic left-handed material which consists of two dimensionally periodic array of copper split ring resonators and wires. Ramakrishna et al. [6] showed that a left-handed material slab bounded by different dielectric slabs amplifies evanescent waves. Ziolkowsky et al. [7] studied left-handed materials both analytically and numerically. Engheta [8] made a theoretical analysis of thin subwavelengh cavity resonators containing left-handed materials. Zang et al. [9] studied electromagnetic fields propagating through left-handed material slabs. Kong et al. [10] provided a general formation for the electromagnetic wave interaction with stratified left-handed material structures. Hamada et al. [11] investigated nonlinear TM surface waves in an interface of antiferromagnet and left-handed material.

In this paper, we investigate the TE electromagnetic wave propagation characteristics of microwave frequencies in a layered structure of non-linear semimagnetic-semiconductor cover and a left-handed material cladding as shown in figure (1). Each layer extends to infinity in the xz plane. An external static magnetic field H_0 is applied to the material in the z-direction. For simplicity, the wave propagation is considered to be in the positive x-direction and all the field components are independent in z-direction, i. e.

$\frac{\partial}{\partial z}=0$.

The most interesting properties of the nonlinear electromagnetic surface waves are the dependence of their dispersion on the value of the electric field on the interface and the non monotonic fall of the electromagnetic field in the nonlinear medium. These features are promising from the point of view of their practical usage in microelectronics. The electromagnetic surface waves have been studied previously in semimagnetic semiconductor [12]. It was found that the surface waves in semimagnetic semiconductor possess a much wider existence frequency region than linear surface waves, and the value of the existence frequency region depends on the concentration of current carriers essentially.

The obtained numerical results of the dispersion relation and the power flow are also presented and then discussed. As far as we know the surface waves in semimagnetic-semiconductor structure with LHM have not been studied until now.

II-Theoretical Model of TE Surface Waves

The semi-infinite semimagnetic semiconductor in an antiferromagnetic phase is described by a uniaxial two-sublattice conductive antiferromagnet [12]. It is assumed that the antiferromagnet occupies the half-space y > 0, whereas a linear medium with dielectric constant ε occupies the half-space y < 0. Easy axis and the external static magnetic field H_0 (axis z) are parallel to each other and are directed along semimagnetic semiconductor surface. It is assumed the longitudinal component ε_{zz} of the permittivity tensor of the conductive antiferromagnet has the form $\varepsilon_{zz}(\omega)=\varepsilon_{zz}^{o}(\omega)+\delta\left|Ez\right|^2$ and the plasma formula for linear term $\varepsilon_{zz}^{o}(\omega)$ is suitable for description of the high-frequency properties of the conductive antiferromagnet at low temperatures, i.e. $\varepsilon_{zz}^{o}(\omega) = \varepsilon_o(1-\frac{\omega_p^2}{\omega^2})$.

Here δ is a constant; $\omega_P = \sqrt{4\pi e^2 n_o/\varepsilon_o m^*}$ is the electron plasma frequency; e, n_o, and m* are the charge, the density and the effective mass of the electron, respectively. ε_0 is the dielectric constant of the semimagnetic-semiconductor, and c is the velocity of light.

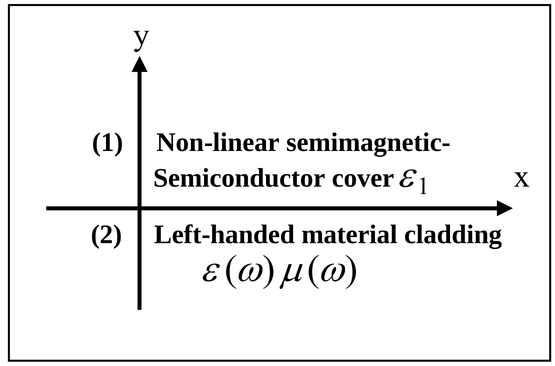

Fig.(1): Coordinate system for the single interface between non-linear semimagnetic-semiconductor cover and left-handed material cladding

By considering $[\mu]$ is as the magnetic permeability tensor of the nonlinear medium where:

$$[\mu]=\begin{bmatrix}\mu_1 & i\mu_2 & 0\\ -i\mu_2 & \mu_1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

The electromagnetic properties of the semimagnetic semiconductor are determined by the diagonal μ_1 and off-diagonal μ_2 components of the antiferromagnet magnetic permeability tensor μ_{ij}:

$\mu_1 = 1+\omega_a w_m (Y_+ + Y_-)$;

$\mu_2 = i\,\omega_a w_m (Y_- - Y_+)$;

Where

$Y_+ = 1\backslash(\omega_r^2-(\omega+\omega_o)^2)$;

$Y_- = 1\backslash(\omega_r^2-(\omega-\omega_o)^2)$.

The parameters of the magnetic subsystem in the semimagnetic semiconductor are determined by the exchange field H_e, the anisotropy field H_a, and the sublattice magnetization M. These parameters determine the frequency of the antiferomagnetic resonance, $\omega_r = \gamma(2H_e H_a + H_a^2)^{\frac{1}{2}}$ and also frequencies $\omega_a = \gamma H_a$ and $\omega_m = 4\pi\gamma M$ (γ is the gyromagnetic ratio). In our study the values used were: $\omega_a = 3.29 \times 10^{12}\,Hz$, $\omega_m = 11.75 \times 10^{12}\,Hz$ and $\omega_r = 8.779 \times 10^{11}\,Hz$. We characterize the intensity of the external magnetic field H_0 and $\omega_o = \gamma H_o$.

The components of electric and magnetic fields of TE surface waves propagating along x-axis in the xy–plane with wave number propagation constant k and an angular frequency ω have the following form:

$$\vec{E} = (0, 0, E_z)\, e^{i(kx - \omega t)}$$

$$\vec{H} = (H_x, H_y, 0)\, e^{i(kx - \omega t)}$$

We study in the next section the behavior of electromagnetic waves in both media as the following:

A- In a nonlinear semimagnetic-semiconductor medium

In the nonlinear semimagnetic-semiconductor medium, Maxwell's curl equations have the simple form:

$$\nabla \times \vec{E} = i\omega\mu_o\mu\vec{H} \qquad (1)$$

$$\nabla \times \vec{H} = -i\omega\mu_o\varepsilon\vec{E} \qquad (2)$$

Where ε is the relative dielectric constant of the nonlinear medium.

From the Maxwell equations for a nonlinear medium, one can get:

$$\frac{\partial^2 E_2}{\partial y^2} - k^2 E_z + \frac{w^2}{c^2}\mu_V \delta E_z^3 = 0 \qquad (3)$$

Where

$$\mu_V = \mu_1 + \frac{\mu_2^2}{\mu_1}, \quad k^2 = q^2 - \frac{\omega^2}{c^2}\mu_V\,\varepsilon_{zz}^o.$$

If it is assumed that E_z and dE_z / dy vanish in the limit $y \to \infty$, then the nonlinear second-order differential equation of μ_V have the following solutions:

$$E_z(y) = E_z(0)\begin{cases} ch^{-1}[k(y - y_o)] & if\ \mu_V\delta > 0 \\ sh^{-1}[k(y - y_0)] & if\ \mu_V\delta < 0 \end{cases} \qquad (4)$$

Here $E_z(0) = \frac{ck}{\omega}\sqrt{\frac{2}{|\mu_v\delta|}}$ is the maximum value of E_z (y) in the nonlinear medium, y_o is a constant of integration defining a position of the maximum magnitude of the electric field in the nonlinear medium. The magnetic field H_x is written as:

$$H_x = \frac{\mu_1\frac{\partial E_z}{\partial y} - \mu_2 k E_z}{i\omega\mu_0[\mu_1^2 - \mu_2^2]} \qquad (5)$$

B- In a Left-Handed Medium

From Maxwell's equation, the differential equation is obtained:

$$\frac{\partial^2 Ez}{\partial y^2} - (k_2^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\mu(\omega))E_z = 0 \qquad (6)$$

Where $k_0^2 = \frac{\omega^2}{c}$, $k_2^2 = (k^2 - \frac{\omega^2}{c}\varepsilon(\omega)\mu(\omega))^{\frac{1}{2}}$

and $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$, $\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_r^2}$

The losses are neglected, and the values of the parameters ω_p, ω_r and F are chosen to fit approximately to the experimental data [13, 14]: $\omega_p / 2\pi = 10\,GHz$, $\omega_r / 2\pi = 4\,GHz$, and F = 0.56. For this set of parameters, the region in which permeability and permittivity are simultaneounsly negative is from 4 GHz to 6 GHz.

The solution of the above equation is :

$$E_z = Ae^{k_2 y} \qquad (7)$$

$$H_x = \frac{-i}{\omega\mu_0\mu(\omega)}\frac{\partial E_z}{\partial y} \qquad (8)$$

The dispersion equation describing the propagation of surface waves can be obtained with the conditions of continuity of H (y) and E(y) at the boundary y = 0.

By requiring the tangential component of the electric and magnetic fields to be continuum at y = 0, we get the dispersion relation:

$$\upsilon = \tanh(k_{1}, y_0) = \frac{\mu_v}{\mu(\omega)} \cdot \frac{k_2}{k_1} + \frac{\mu_2}{\mu_1} \cdot \frac{k}{k_1} \quad (9)$$

Equation (9) is the dispersion relation which determines the wave propagation characteristics, and this equation leads to the values of the propagation constant which can be substituted in the power formula presented in equations 10, 11.

III-Power Flow of TE surface Waves

The total power flux (P) of TE surface waves propagation in the y-direction can be written as:

$$P = \frac{1}{2} \int (E \times H^{*})\, dy$$

A- In Left-Handed Substrate

The power flux (P) of the wave propagation in the y-direction is as the following:

$$P_{L.H.} = \frac{k}{\omega \mu_0\, \mu(\omega)} \int_{-\infty}^{0} E_z^2 \,.\, dy$$

$$= \frac{k k_1^2}{2 \mu_0\, \omega\, \mu(\omega) \mu_1\, \delta k_2\, k_0^2} \left[1 - \left(\frac{\mu_0\, k_2}{\mu(\omega) k_1} + \frac{\mu_2}{\mu_1} \frac{k}{k_1}\right)^2 \right] \quad (10)$$

B- In a non-linear semimagnetic-semiconductor cover

The power flux (P) of the wave propagation in the y-direction is as the following:

$$P_{NL} = \frac{1}{2 \omega \mu_0 (\mu_1^2\ \mu_2^2)} \int_0^{\infty} E_z \left(\mu_1 k E_z - \mu_2 \frac{\delta E_z}{\delta y}\right) dy$$

$$P = \frac{k k_S}{\omega \mu_0\, \mu_0^2\, \delta k_0^2} \left(1 + \frac{\mu_0 k_2}{\mu(\omega) k_1} + \frac{\mu_2 k}{\mu_1 k_1}\right) - \frac{\mu_2\, k_1^2}{\omega \mu_0\, \mu \delta k_0^2\, (\mu_1^2 - \mu_2^2)} \left[1 - \left(\frac{\mu_0\ k_2}{\mu(\omega)\,.\,k_1} + \frac{\mu_2\ k}{\mu_1 k_1} \right)^2 \right] \quad (11)$$

IV-Results & Discussion

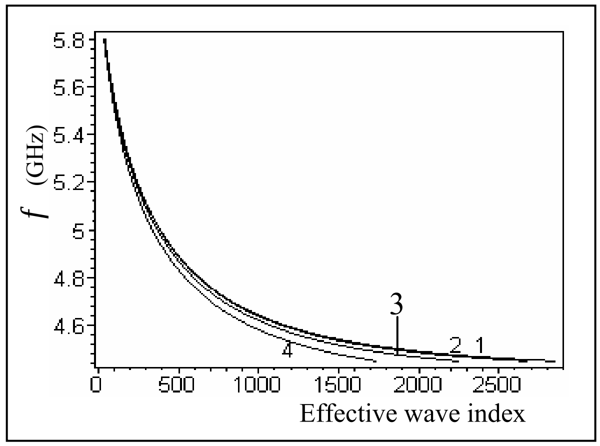

Fig.2. The Computed operating frequency (f) versus effective the wave index β versus for different values of the external applied magnetic field H_0: curve 1, H_0=0; curve 2, H_0=10T; curve 3, H_0= 20T; curve 4, H_0 = 30T. The interface nonlinearity, υ = 0.9 and the normalized plasma frequency $\frac{\omega_p}{\omega_r} = 1.2$.

Fig.2 presents the variation of the frequency with the effective wave index β where $\beta = \frac{k}{ko}$, for different external magnetic field H_0 (0T, 10T, 20T, 30T). This relation were carried out with at a fixed nonlinear term of υ = 0.9 and at fixed normalized plasma frequency of $\frac{\omega_p}{\omega_r}$ = 1.2. k_0 is the wave number of the free space. These curves show that the frequency is decreasing with increasing the wave index β and this happens only in the case of the propagation of TE-semimagnetic surface waves. It is also seen from fig.2 that by increasing the external applied magnetic field H_0 the surface waves operating frequency is decreasing more rapidly.

It's obvious from the graphs that the effective frequency of the surface wave propagation has the range of 4.0 GHz to 6.0 GHz; this frequency range is being affected by the external applied magnetic field H_o.

We characterize the intensity of the external magnetic field Ho by $\omega_o = \gamma Ho$ where γ is the gyromagnetic ratio.

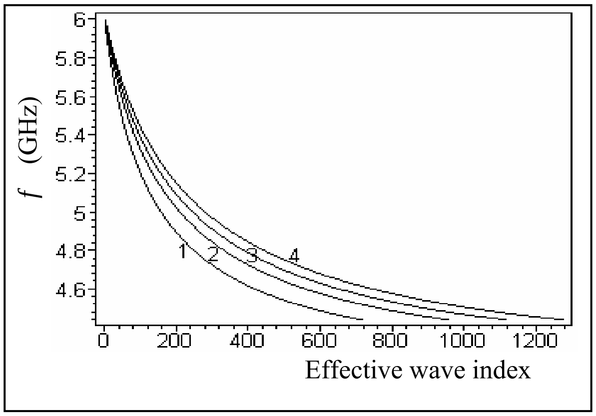

Fig.3. The Computed operating frequency (f) versus the effective wave index β for different values of the normalized plasma frequency ω_p / ω: curve 1, ω_p / ω= 0.9; curve 2, ω_p / ω= 1.2; curve 3, ω_p / ω= 1.4; curve 4, ω_p / ω= 1.6. The interface nonlinearity, υ = - 0.5 and the external applied magnetic field H_0 = 0.3T

The frequency characteristics versus the effective wave index β at different values of the normalized plasma frequency $\frac{\omega_p}{\omega_r}$ of 0.9, 1.2, 1.4, and 1.6 are shown in Fig.3. These results were obtained at fixed nonlinear term of υ = - 0.5 and fixed external applied magnetic field H_0 = 30T and fixed normalized plasma frequency $\frac{\omega_p}{\omega_r}$ = 1.2. Fig.4 presents the frequency variation versus the effective wave index β at different values of the nonlinear term υ. The curve denoted by 1 presents the frequency variation where υ = -1 i. e. where the behavior is pure linear, whereas the other three curves present the case where υ >-1, i. e. the nonlinear case; curve 4 is for υ = 0. Fig.4 shows the frequency variation curves versus β decreasing by decreasing υ which means that in the two layered structure of nonlinear semimagnetic-semiconductor and left-handed material the TE surface wave power propagates in the left-handed material substrate more strongly than in the semimagnetic-semiconductor cover.

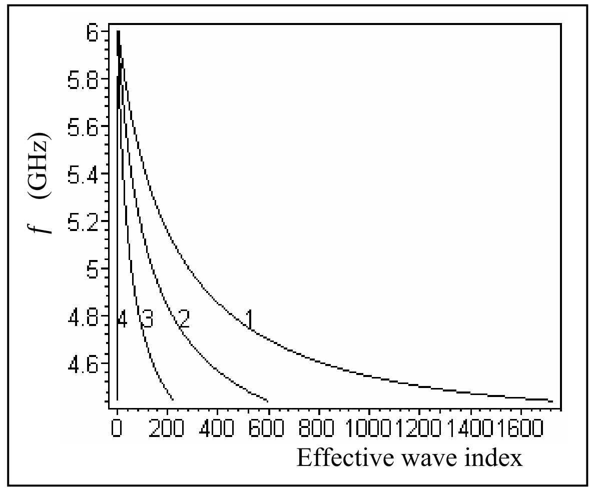

Fig.4. The Computed operating frequency (*f*) versus the effective wave index β for different values of The interface nonlinearity, υ: curve 1, μ = -1; curve 2, μ = -0.5; curve 3, μ = -0.2; curve 4, μ = 0. The external applied magnetic field H_0 = 30T and the normalized plasma frequency $\frac{\omega_p}{\omega_r}$ = 1.2.

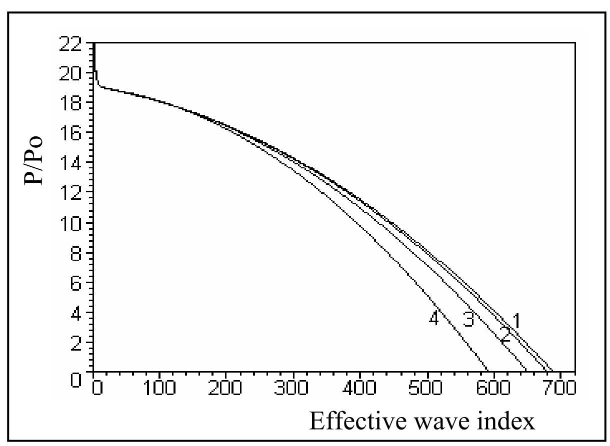

Fig.5. The Computed normalized total power p / p_o versus the effective wave index β for different values of the external applied magnetic field H_0: curve 1, H_0 = 0T; curve 2, H_0 = 10T; curve 3, H_0 = 20T; curve 4, H_0 = 30T. The normalized plasma frequency $\frac{\omega_p}{\omega_r}$ = 1.5 and the operating frequency f = 4.9 GHz.

The power has an opposite direction in the left-handed material to that of the semimagnetic-semiconductor medium. This is due to the fact that in a LHM medium which characterized by negative dielectric constant, $\varepsilon < 0$, the energy flux and wave vector have opposite directions [15]. This behavior is different than that observed for the structure of single interface of nonlinear layer and right-handed material (RHM) [16]. Fig.5. presented the computed effective wave index β, versus the normalized total power p / p_o for different values of H_o (0T, 10T, 20T, 30T).

The normalized power p / p_o, where P_o = $\frac{1}{2\omega\varepsilon_0\delta}$ is plotted versus the effective wave index β, as shown in Fig. 6, for different values of frequency, namely 4.8 GHz, 4.9 GHz, 5.0 GHz, and 5.1 GHz at fixed H_o = 5T and fixed $\frac{\omega_p}{\omega_r}$ = 1.5. It is seen from Fig. that p / p_o decreases with increasing β until it falls to zero.

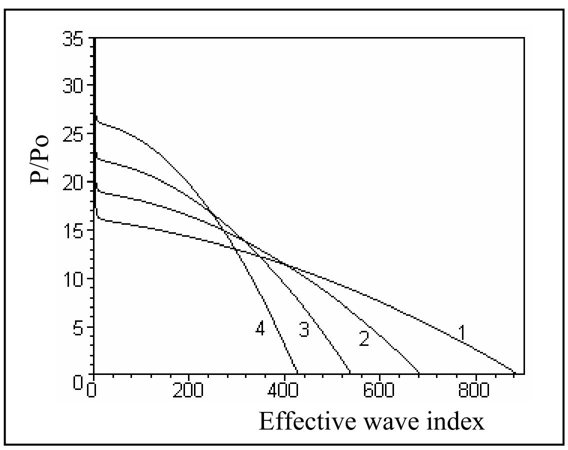

Fig.6. The Computed the normalized total power flow p / p_o versus the effective wave index β, for different operating frequency (*f*): curve 1, 4.8GHz; curve 2, 4.9GHz; curve 3, 5GHz; curve 4, 5.1GHz. The external applied magnetic field H_0 = 5T and the normalized plasma frequency $\frac{\omega_p}{\omega_r}$ = 1.5.

The behavior of p / p_o here is unlike what has been published previously by Hamada et al. [11] where p / p_o has been found to increase up to some critical value, and then falls to zero.

Hamada et al. also found that for a certain single normalized power lever, there are two electromagnetic waves of different velocities, which means that one power can produce two spatial TM waves. Such behavior, which presents a case of some optical bistability [17], has not been noticed in our current study.

V-Conclusion

We have presented an analytical study of TE surface waves supported by a single interface between a non-linear semimagnetic–semiconductor cover and a left-handed metamaterial cladding.
It has been found that the effective frequency of the surface wave propagation has the range of 4.0 GHz to 6.0 GHz and this operating wave frequency significantly affects the above waveguide structure.

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