

## Gossamer Superconductivity, New Paradigm?

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We shall review our recent works on d-wave density wave (dDW) and gossamer superconductivity (i.e. d-wave superconductivity in the presence of dDW) in high- $T_c$  cuprates and CeCoIn<sub>5</sub>. a) We show that both the giant Nernst effect and the angle dependent magnetoresistance (ADMR) in the pseudogap phases of the cuprates and CeCoIn<sub>5</sub> are manifestations of dDW. b) The phase diagram of high- $T_c$  cuprates is understood in terms of mean field theory, which includes two order parameters  $\Delta_1$  and  $\Delta_2$ , where one order parameter is from dDW and the other from d-wave superconductivity. c) In the optimally to the overdoped region we find the spatially periodic dDW, an analogue of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, becomes more stable. d) In the underdoped region where  $\Delta_2/\Delta_1 \ll 1$  the Uemera relation is obtained within the present model. We speculate that the gossamer superconductivity is at the heart of high- $T_c$  cuprate superconductors, the heavy-fermion superconductor CeCoIn<sub>5</sub> and the organic superconductors  $\kappa$ -(ET)<sub>2</sub>Cu(NCS)<sub>2</sub> and (TMTSF)<sub>2</sub>PF<sub>6</sub>.

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### 1 Introduction

**Question:** What does strong correlation mean?

**Answer:** First of all it means the Coulomb dominance; the Coulomb interaction is stronger than that due to phonon exchange. For superconductors this means unconventional order parameters: d-wave, f-wave, g-wave superconductors.

Since the discovery of high- $T_c$  cuprates La<sub>2-x</sub>Ba<sub>x</sub>CuO<sub>4</sub> by Bednorz and Müller [1] in 1986, it appears that the debate over the nature and mechanism of this unusual superconductivity continues. However, d-wave superconductivity as in BCS theory and arising due to anti-paramagnon exchange has been established, at least in the vicinity of the optimal doping.[2, 3] Also from the low temperature thermal conductivity May Chiao et al deduced  $\Delta/E_F = 1/10$  and  $1/14$  for Bi-2212 and YBCO respectively [4, 5]. From these we obtain  $\Delta = 500K$  and  $280K$  for Bi-2212 and YBCO respectively and  $E_F \simeq 5000K$ , which is almost universal. Here  $\Delta$  is the maximal gap of d-wave superconductivity at  $T = 0$ . Also recently the universality of the Fermi velocity  $v = 2.3 \times 10^7$  cm/sec has been established by the angle resolved photoemission spectrum [6].

From these we conclude that the cuprate superconductors are in the BCS limit, far away from the Bose-Einstein condensation limit, and that the superconducting fluctuation effect should be at most 10%.

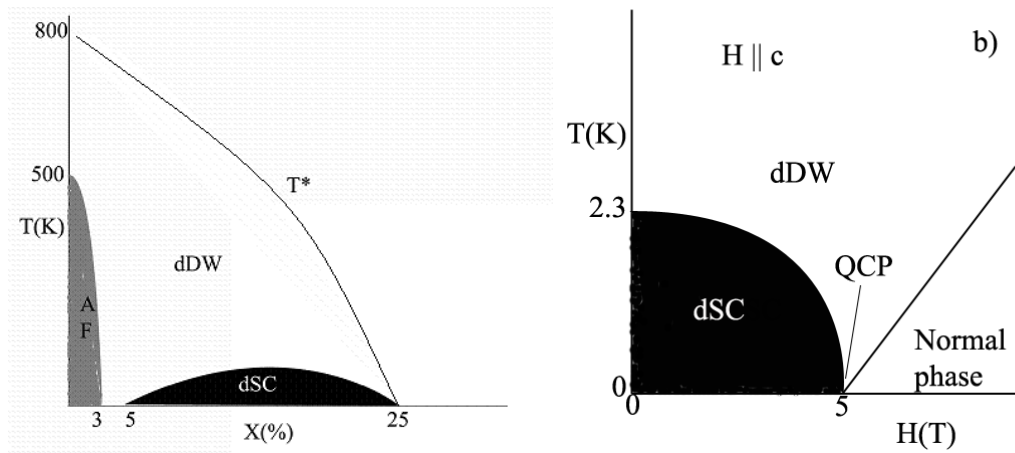
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Therefore theories based on the large superconducting fluctuations [7, 8] appear to be unrealistic. Also in many numerical computations on the cuprates, it was assumed that  $\Delta \sim E_F$ . It is clear that such approximations lead to rather unrealistic predictions. On the contrary, with  $\Delta = 0.1E_F$  Kato et al recently found hundreds of quasiparticle bound states around a vortex of f-wave superconductors, a model system for  $\text{Sr}_2\text{RuO}_4$  [9]. These bound states are the analogues of Caroli, de Gennes and Matricon bound states around a vortex in s-wave superconductivity [10, 11]. More recently the STM data around a vortex in  $\text{Sr}_2\text{RuO}_4$  has been reported by Lupien et al [12]. Indeed the observed quasiparticle spectrum is very consistent with the theoretical analysis in [9]. Of course in  $\text{Sr}_2\text{RuO}_4$   $\Delta$  should be less than  $0.01 E_F$ .

In 1993 Volovik [13] showed that the quasiparticle density of states in the vortex state in d-wave superconductors is calculable within a quasiclassical approximation. This work has been extended into several directions: a) thermodynamic functions; b) thermal conductivity; c) scaling relations; d) for arbitrary field orientation; and e) for a variety of gap functions  $\Delta(\mathbf{k})$ 's [14, 15, 16, 17, 18, 19]. As is well known the gap symmetry and the gap function has been the central issue since the discovery of the heavy-fermion superconductors [20]. Since 2001 Izawa et al have succeeded in determining the superconducting gap functions  $\Delta(\mathbf{k})$ 's in  $\text{Sr}_2\text{RuO}_4$  [21],  $\text{CeCoIn}_5$  [22],  $\kappa\text{-(ET)}_2\text{Cu(NCS)}_2$  [23],  $\text{YNi}_2\text{B}_2\text{C}$  [24],  $\text{PrOs}_4\text{Sb}_{12}$  [25, 26],  $\text{UPd}_2\text{Al}_3$  [27, 28] and  $\text{CePt}_3\text{Si}$  [29, 30] through the angle-dependent thermal conductivity. For a review of these aspects see [31].

The phase diagrams of high- $T_c$  cuprates and  $\text{CeCoIn}_5$  are shown in Fig. 1a) and Fig. 1b) respectively. As you may recognize, we have replaced the pseudogap phase with d-wave density wave, which is the



**Fig. 1** The phase diagrams for high- $T_c$  cuprates (left) and  $\text{CeCoIn}_5$

main topic in section 2. Also we have chosen  $T^*$  to vanish at the same point where the superconductivity vanishes, suggesting the system has a quantum critical point at  $x = 25\%$  [32]. D-wave density wave (dDW) for the pseudogap phase in high- $T_c$  cuprates has been proposed by several people. [33, 34, 35, 36]. However, unlike these authors we do not consider the commensurate dDW with  $Z_2$ - symmetry, which is a descendant of the flux phase [37]. Rather we limit ourselves to the incommensurate dDW with the  $U(1)$  gauge symmetry as in the conventional charge density wave [38, 39].

We shall see later that the incommensurate dDW is crucial to understanding the phase diagram in Fig. 1a). Also we shall discuss the recent angle dependent magnetoresistance (ADMR) data in  $\text{Y}_{0.68}\text{Pr}_{0.32}\text{CuO}_4$  [40, 41] and  $\text{CeCoIn}_5$  [42], which provides strong support for dDW in these systems. Once one accepts the phase diagrams in Fig. 1, the d-wave superconductivity in the high- $T_c$  cuprates and  $\text{CeCoIn}_5$  should arise in the presence of dDW. Borrowing the beautiful word from R.B. Laughlin [43] we call these superconductors ‘‘gossamer superconductors’’. Therefore the exploration of the gossamer superconductivity appears to be

the most urgent [44]. We shall interpret the Uemura relation in the vicinity of  $x = 5\%$  in terms of this gossamer superconductivity.

## 2 D-wave density waves

There are many parallels between the cuprates, the heavy-fermion superconductor CeCoIn<sub>5</sub> and the organic conductor  $\kappa$ -(ET)<sub>2</sub>Cu(NCS)<sub>2</sub>: the quasi-two dimensional Fermi surface, the proximity of the antiferromagnetic phase and d-wave superconductivity [22, 45]. In addition d-wave density wave in the pseudogap phase appears to be an additional common feature [41, 42, 46, 47]. In the absence of a magnetic field the Nambu Green function for dDW is given by [41]

$$G^{-1}(\omega, \mathbf{k}) = \omega - \xi(\mathbf{k})\rho_3 - \eta(\mathbf{k}) - \Delta(\mathbf{k})\rho_1 \quad (1)$$

where the  $\rho_i$ 's are the Pauli matrices operating on the spinor space. For d-wave charge density wave we can take either  $\Delta(\mathbf{k}) = \Delta \cos(2\phi)$  or  $\sin(2\phi)$  with  $\tan \phi = k_y/k_x$  and  $\eta(\mathbf{k}) = \mu$ , the chemical potential, which acts as the imperfect nesting. Further

$$\xi(\mathbf{k}) = v(k_{\parallel} - k_F) + \frac{v'}{c} \cos(ck_z) \quad (2)$$

where  $k_{\parallel}$  is the radial component in the x-y plane and  $v$  and  $v'$  are the Fermi velocities.

Then the quasiparticle density of states is given by

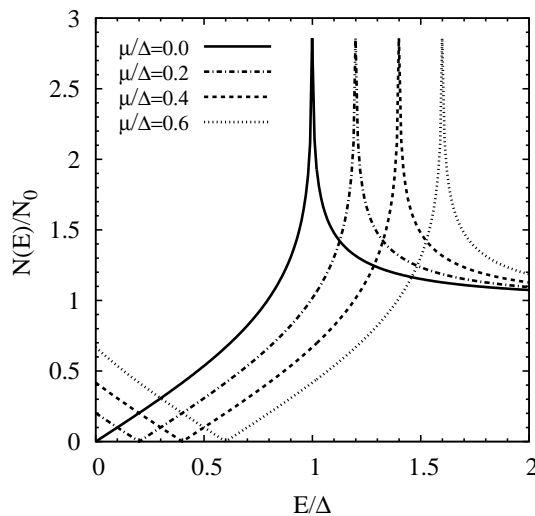
$$N(E)/N_0 = G(x - y) \quad (3)$$

where

$$G(x) = \frac{2x}{\pi} K(x) \text{ for } x \leq 1 \quad (4)$$

$$= \frac{2}{\pi} K(x^{-1}) \text{ for } x > 1. \quad (5)$$

and  $x = E/\Delta$ ,  $y = \mu/\Delta$  and  $K(x)$  is the complete elliptic integral.  $N(E)/N_0$  is shown in Fig. 2.



**Fig. 2** The quasiparticle density of states for a dDW superconductor

Note that  $N(0)/N_0 \simeq |\mu|/\Delta$  for  $\mu < \Delta$ . Therefore the chemical potential provides nonvanishing quasiparticle density of states at  $E=0$ . This gives rise to pockets on the Fermi surface at  $(\pi, \pi)$  direction in the high- $T_c$  cuprates [48]. Also as we shall see later  $\Delta_2 \leq \mu$  is crucial for the presence of d-wave superconductivity in the middle of dDW. Here  $\Delta_2$  is the maximum energy gap of a d-wave superconductor.

### 3 Landau Quantization

As noted by Nersesyan et al [49, 50] the quasiparticle spectrum is quantized in the presence of a magnetic field. Let us consider a magnetic field  $\mathbf{B}$  applied within the  $x' - z$  plane tilted by an angle  $\theta$  from the  $z$  axis. Also  $\hat{x}'$  is defined by  $\hat{x}' = \hat{x} \cos \phi + \hat{y} \sin \phi$ . Then the magnetic field is introduced by  $\mathbf{k} \rightarrow \mathbf{k} + e\mathbf{A}$  with

$$\mathbf{A} = B(y \cos \phi - x \sin \phi)(\hat{z} \sin \theta + (\hat{x} \cos \phi + \hat{y} \sin \phi) \cos \theta) \quad (6)$$

Then for  $d_{xy}$ -wave DW, the quasiparticle energies are given by

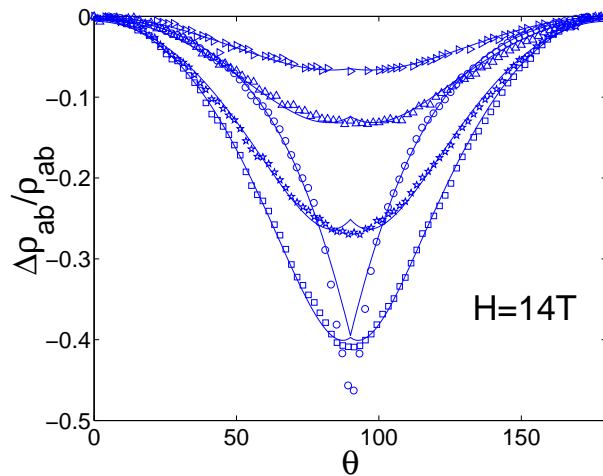
$$E_{1n}^{\pm} = \pm \sqrt{2neBv_2|(v \cos \theta \cos \phi - v' \sin \theta) \cos \phi| - \mu} \quad (7)$$

$$E_{2n}^{\pm} = \pm \sqrt{2neBv_2|(v \cos \theta \cos \phi + v' \sin \theta) \cos \phi| - \mu} \quad (8)$$

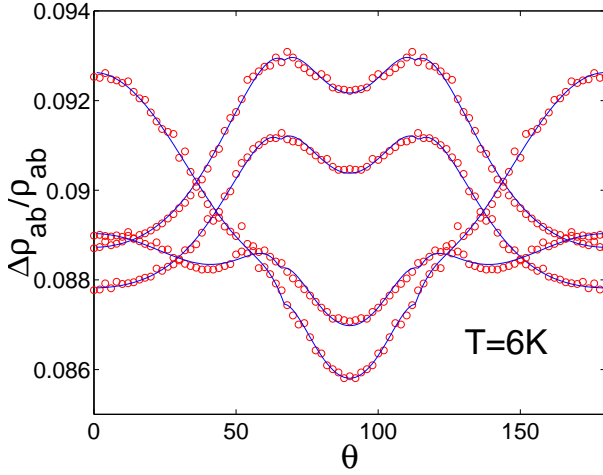
$$E_{3n}^{\pm} = \pm \sqrt{2neBv_2|(v \cos \theta \sin \phi - v' \sin \theta) \sin \phi| - \mu} \quad (9)$$

$$E_{4n}^{\pm} = \pm \sqrt{2neBv_2|(v \cos \theta \sin \phi + v' \sin \theta) \sin \phi| - \mu} \quad (10)$$

Here  $n=0,1,2,\dots$ . Except for the  $n=0$  Landau level they are doubly degenerate. Also unlike the quasi-one dimensional systems [39], there are 4 branches of the Landau levels [41]. As shown elsewhere these Landau spectra are most readily seen by angle dependent magnetoresistance (ADMR), the nonlinear Hall conductivity and the giant Nernst effect [39, 51]. Indeed ADMR appears to provide the most sensitive test of unconventional density wave (UDW) as seen in  $\alpha$ -(ET)<sub>2</sub>KHg(SCN)<sub>4</sub> and the Bechgaard salts (TMTSF)<sub>2</sub>X with X=PF<sub>6</sub> and ReO<sub>4</sub> [52, 53, 54]. Here we present such an analysis of ADMR data provided by C. Almasan, T. Hu and V. Sandu in the pseudogap region in Y<sub>0.68</sub>Pr<sub>0.32</sub>CuO<sub>4</sub> [40, 41] and CeCoIn<sub>5</sub>, which are shown in Fig. 3a and 3b. The electric conductivity is given by [41]



**Fig. 3** ADMR of pseudogap region of Y<sub>0.68</sub>Pr<sub>0.32</sub>CuO<sub>4</sub>. Curves are for T= 105 K, 75 K, 65 K, 60 K and 52 K from top to bottom. The curve for T =52 K is reduced by a factor of 10.



**Fig. 4** ADMR of pseudogap region of CeCoIn<sub>5</sub>, for H = 3 T, 5 T, 8 T and 14 T from top to bottom [42].

$$\sigma(B, \theta) = \sum_n \sigma_n \text{sech}^2(\beta E_n/2) \quad (11)$$

where  $\beta = 1/k_B T$  and the sum is over all the Landau levels. However, when  $\beta|E_1| \gg 1$ , the two lowest Landau levels suffice. Then we obtain

$$\sigma(B, \theta) = \sigma'_0 (1 + \cosh \zeta_0)^{-1} + \sigma'_1 \left[ \frac{1 + \cosh x_1 \cosh \zeta_0}{\cosh x_1 + \cosh \zeta_0} + \frac{1 + \cosh x_2 \cosh \zeta_0}{\cosh x_2 + \cosh \zeta_0} \right] \quad (12)$$

where  $\zeta_0 = \beta\mu$ ,  $x_1 = \beta\sqrt{2eBv_2|v \cos \theta - v' \sin \theta|}$  and  $x_2 = \beta\sqrt{2eBv_2|v \cos \theta + v' \sin \theta|}$ . Here we consider the case  $\phi = 0$  and  $\sigma'_0$  and  $\sigma'_1$  are  $\sigma_0$  and  $\sigma_1$  multiplied by some integer which accounts for the proper degeneracy. From the fitting of Fig. 3a) we obtain  $v = 2.3 \times 10^7$  cm/s,  $v'/v \leq 0.1$ ,  $E_F = 5000$  K,  $\Delta = 360$  K and  $\mu \simeq 40 - 60$  K for  $Y_{0.68}Pr_{0.32}CuO_4$  with  $T_c = 55$  K. Similarly the data from CeCoIn<sub>5</sub> is analyzed in [42]. We find  $v = 3.3 \times 10^6$  cm/s,  $v'/v \simeq 0.5$ ,  $E_F = 500$  K,  $\Delta = 45$  K and  $\mu = 8.4$  K for CeCoIn<sub>5</sub>. These values are consistent with other observations in CeCoIn<sub>5</sub> [55]. The Hall conductivity is given similarly by

$$\sigma_{xy} = -\frac{2e^2 \cos^2 \theta}{\pi} n(B, T) \quad (13)$$

with

$$n(B, T) = \tanh(\zeta_0/2) + \frac{\sinh(\zeta_0)}{\cosh x_1 + \cosh \zeta_0} + \frac{\sinh(\zeta_0)}{\cosh x_2 + \cosh \zeta_0} + \dots \quad (14)$$

A similar expression has been obtained in [54]. The giant Nernst effect in the pseudogap phase of the high- $T_c$  cuprates and CeCoIn<sub>5</sub> has already been discussed in [46, 47, 56]. In conclusion, ADMR, the non-linear Hall conductivity and the giant Nernst effect should provide a clear signature of UDW.

## 4 Gossamer Superconductivity

Let us consider a simplest coupled equation for  $\Delta_1$  (dDW) and  $\Delta_2$  (d-wave superconductivity) [44]:

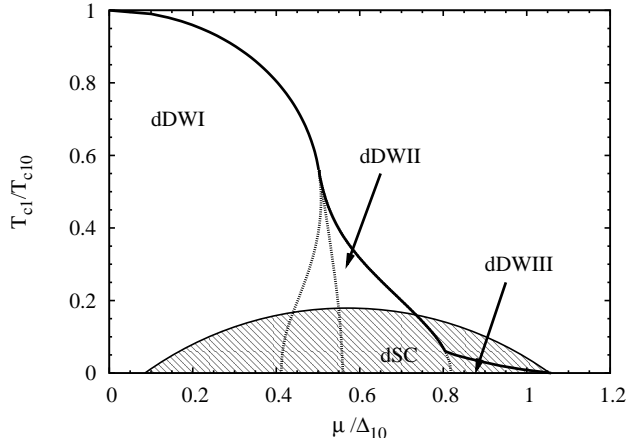
$$\lambda_1^{-1} = 4\pi T \sum_n \text{Re} \left\langle \frac{f^2}{[(\sqrt{\omega_n^2 + \Delta_2^2 f^2} - i\mu)^2 + \Delta_1^2 f^2]^{1/2}} \right\rangle \quad (15)$$

$$\lambda_2^{-1} = 4\pi T \sum_n \text{Re} \left\langle \frac{f^2(1 - \frac{i\mu}{\sqrt{\omega_n^2 + \Delta_2^2 f^2}})}{[(\sqrt{\omega_n^2 + \Delta_2^2 f^2} - i\mu)^2 + \Delta_1^2 f^2]^{1/2}} \right\rangle \quad (16)$$

where  $\lambda_1$  and  $\lambda_2$  are dimensionless coupling constants,  $f = \cos(2\phi)$  and  $\langle \dots \rangle$  means  $\int_0^{2\pi} \frac{d\phi}{2\pi}$ . A similar set of equations is considered in [36, 57]. First let us consider Eq. (14) for  $\Delta_2 = 0$ . Then we discover that the equation is the same as for a d-wave superconductor in the presence of the Pauli term [58]. Now if one puts  $\Delta_1 = 0$ , we obtain the equation for  $T_{c1}$  for dDW ( $=T^*$ ) as

$$\ln\left(\frac{T_{c1}}{T_{c10}}\right) = \text{Re}\Psi\left(\frac{1}{2} - \frac{i\mu}{2\pi T_{c1}}\right) - \Psi\left(\frac{1}{2}\right) \quad (17)$$

This is shown in Fig. 5. Here  $\Psi(z)$  is the digamma function and Eq.(17) is the same as in s-wave super-



**Fig. 5** Proposed phase diagram for high- $T_c$  cuprates

conductors [59, 60]. The figure for  $T_{c1}$  bends back and there will be no solution for  $\mu/\Delta_{10} > 0.57$ . On the other hand in the region  $0.41 < \mu/\Delta_{10} < 0.57$   $T_{c1}$  is double-valued. [58]. A similar phase boundary is obtained numerically in [36]. However, the phase boundary for dDW is extended if we allow the spatial variation of  $\Delta$ :  $\Delta(\mathbf{r}) \propto \cos(\mathbf{q} \cdot \mathbf{r})$ .

This is similar to the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in d-wave superconductors [61], with  $T_{c1}$  given by

$$-\ln\left(\frac{T_{c1}}{T_{c10}}\right) = \text{Re}\langle (1 \pm \cos(2\phi))\Psi\left(\frac{1}{2} - \frac{i\mu(1 - p \cos \phi)}{2\pi T_{c1}}\right) \rangle - \Psi\left(\frac{1}{2}\right) \quad (18)$$

where  $p$  is the new adjustable parameter. Then  $\pm$  in Eq. (18) corresponds to  $\mathbf{q} \parallel [100]$  and  $\mathbf{q} \parallel [110]$  and  $p = \frac{v|q|}{2\mu}$ . The extended solution is shown in Fig. 5 as well. Then it is more appropriate to split dDW in three separate regions as indicated in Fig. 5. Here we took  $\Delta_{10} = 1700$  K and  $T_{c1} = 800$  K in accordance with Ref.[62]. Finally we indicate the d-wave superconducting region by a shaded area, which should follow from the set of equations (15) and (16). Also it is possible that dDW III may be submerged under the d-wave superconductivity.

Since  $T_{c1} > T_{c2}$  in general, it is natural to assume  $\lambda_1 \geq \lambda_2$ . Then in the vicinity of  $x \simeq 0.5\%$ , where d-wave superconductivity begins to appear, we can assume  $\Delta_2/\Delta_1 \ll 1$ . Note that  $\mu \sim 2300(x - 0.0675)$  K [63] in the whole region. Then combining Eqs. (15) and (16) we find

$$\lambda_2^{-1} - \lambda_1^{-1} \simeq 4\pi T \mu^2 \sum_n \left\langle \frac{f^2}{[\omega_n^2 + \Delta^2 f^2]^{\frac{3}{2}}} \right\rangle \quad (19)$$

$$\simeq \frac{2\mu^2}{\Delta^2(T)} \left(1 - 2(\ln 2) \frac{T}{\Delta_0}\right) \quad (20)$$

where  $\Delta^2(T) = \Delta_1^2(T) + \Delta_2^2(T)$ . Then Eq.(20) is solved as

$$\left(\frac{\Delta_2(T)}{\Delta_2(0)}\right)^2 \simeq 1 - 2(\ln 2) \frac{T}{\Delta(0)} \left(\frac{\Delta_1(0)}{\Delta_2(0)}\right)^2 \quad (21)$$

or

$$T_{c2} \simeq \frac{1}{2(\ln 2)} \frac{\Delta_2^2(0)}{\Delta(0)} \quad (22)$$

On the other hand the superfluid density in the gossamer superconductivity is given by [44]

$$\rho_s(T) = 2\pi T \Delta_2^2(T) \sum_n \text{Re} \left\langle \frac{f^2}{[(\sqrt{\omega_n^2 + \Delta_2^2 f^2} - i\mu)^2 + \Delta_1^2 f^2]^{\frac{3}{2}}} \right\rangle \quad (23)$$

which gives

$$\rho_s(0) \simeq \frac{\Delta_2^2(0)}{\Delta^2(0)} \quad (24)$$

Finally we find

$$T_{c2} = \frac{1}{2(\ln 2)} \Delta(0) \rho_s(0) \quad (25)$$

Since  $\lambda^{-2}(0) = \frac{4\pi e^2}{m^*} n \rho_s(0)$  the above relation can be interpreted as the celebrated Uemura relation [64], which could not be obtained within the framework of the BCS theory.

Also the present phase diagram suggests that the optimally doped superconductor sits at the boundary of dDW I and dDW II. Of course the present analysis requires further elaboration. Nevertheless, the present model appears to describe qualitatively the phase diagram of high- $T_c$  cuprate superconductors. Also, Fig. 5 suggests naturally that both dDW III and d-wave superconductivity terminate at  $\mu/\Delta_{01} = 1.06$  (or  $x = 25\%$ ), implying the quantum critical point (QCP) at  $x = 25\%$ . We have mentioned previously that the d-wave superconductivity in CeCoIn<sub>5</sub> is also most likely ‘‘gossamer’’.

## 5 Conclusions

We have seen previously that most of the metallic ground states in high- $T_c$  cuprates, heavy-fermion conductors and organic conductors belong to one of the mean field ground states: a) unconventional superconductivity, b) unconventional density wave; or c) the coexistence of both unconventional superconductivity and UDW. The present study suggests that perhaps a) most of the superconducting phase or ‘‘non-Fermi liquid’’ behaviors are related to UDW; and b) the pseudogap phases in both high- $T_c$  cuprates and CeCoIn<sub>5</sub> are gossamer; and c) the superconductivity in  $\kappa$ -(ET)<sub>2</sub> salts and in Bechgaard salts (TMTSF)<sub>2</sub>PF<sub>6</sub> also appear to be gossamer [65, 66]. In the last system the superconductivity is expected to be triplet and should contain an unconventional spin density wave (USDW) [53, 54]. This suggests that there are a variety of gossamer superconductors, which await our exploration.

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