

## Multi-Device Scaling of Neoclassical Tearing Mode Onset with Beta

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The islands from tearing modes driven unstable and sustained by helically perturbed neoclassical bootstrap current at high beta often provide the practical limit to long-pulse, high confinement tokamak operation [1,2]. The discharges studied are ELMy H-mode single-null divertor (SND) at  $q_{95} \gtrsim 3$ . Periodic sawteeth with  $m/n=1/1$  and  $2/2$  are observed to induce  $m/n=3/2$  neoclassical tearing modes (NTMs) in the tokamaks Asdex-Upgrade [3], DIII-D [4] and JET [5]; confinement can drop by up to 30%, constituting a “soft” beta limit. Data for the onset of these modes was obtained by slowly raising beta on a time scale longer than the sawteeth period and observing the beta value at onset. Comparison of the measured critical beta to a model for the critical beta is made in terms of dimensionless parameters. This modeling is then used for extrapolation/prediction to a reactor-grade tokamak.

For a classically stable tearing mode,  $\Delta' < 0$ , the perturbed neoclassical bootstrap current, proportional to beta poloidal  $\beta_\theta$ , can induce destabilization if  $\beta_\theta$  exceeds a critical value [1,2]

$$\beta_{\theta c} \approx \frac{(-\Delta' r)(w_s/r) / (\epsilon^{1/2} L_q / L_p)}{\frac{w_s^2}{w_s^2 + w_d^2} - \frac{w_{pol}^2}{w_s^2}} \quad (1)$$

Here  $w_s$  is the “seed” island width which must exceed a “threshold” island width  $w_{th}$ , i.e. the NTM will grow if  $w_s > w_{th}$  and  $\beta_\theta > \beta_{\theta c}$ . Experiments indicate [3–6] that  $w_{th} \approx w_{pol}$ , the predicted polarization/inertial threshold, which scales [7] as  $w_{pol}/r \propto \rho_{i*} g^{1/2}(\epsilon, \nu)$ . Here  $g$  is a function of the collisionality  $\nu = \nu_i / \epsilon \omega_{e*}$ , with  $g = 1$  at  $\nu \ll 1$  and  $g \approx \epsilon^{-3/2} \gg 1$  for  $\nu \gg 1$ . The smaller relative threshold  $w_d/r$  predicted due to incomplete flattening of the pressure within an island, scales as  $(\chi_\perp / \chi_\parallel)^{1/4} \propto (\chi_{BOHM} / C_s w^{-1})^{1/4} \propto \rho_{i*}^{1/3}$  and is evaluated to have  $w_d^2 \ll w_{pol}^2$ .

Scaling or extrapolation to a reactor grade tokamak also requires understanding of how the relative seed island  $w_s/r$  scales. For example, if  $w_s/r$  decreases faster than  $w_{pol}/r$  decreases, with  $\rho_{i*}$  for example, then  $\beta_{\theta c}$  can increase as the denominator in Eq. (1) becomes smaller. On the other hand,  $\beta_{\theta c}$  decreases  $\propto w_{pol}/r$  if  $w_{pol}^2/w_s^2 < 1$  is constant, i.e.  $\beta_{\theta c} \propto \rho_{i*} g^{1/2}(\nu, \epsilon)$ , an unfavorable scaling for a reactor grade tokamak which will have low  $\rho_{i*}$ . The relative seed island scaling is taken as  $w_s/r \propto \beta_\theta^\gamma S^{-\alpha} \propto \rho_{i*}^{3\alpha} \nu^\alpha$  (with  $\gamma = \alpha/2$  for simplicity and proper limit at zero beta and  $\alpha$  determined from fits to the experimental data) where  $S \propto \beta^{1/2} / \nu \rho_{i*}^3$  is the magnetic Reynold’s number; this allows for the effects of geometrically coupled perturbations of the periodic sawtooth instability to the  $q=3/2$  surface [8]. Note that in this form,  $w_s/w_{pol} \propto \rho_{i*}^{3\alpha-1}$ , at constant collisionality, and the eventual stabilization of a reactor grade tokamak depends on whether  $\alpha \gtrsim 1/3$ . For  $\alpha = 1/3$ ,  $\beta_{\theta c}/\rho_{i*}$  is a function of collisionality only and thus  $\beta_{\theta c}$  decreases with lower  $\rho_{i*}$ . However, if  $\alpha > 1/3$ ,  $\beta_{\theta c}/\rho_{i*}$  will eventually tend to infinity, i.e.

indicate stability, at low enough  $\rho_{i*}$ , high  $S$  so that  $w_s^2 < w_{pol}^2$  and the seed island has become too small to excite the instability even though the threshold island width has also become small.

Data from Asdex-Upgrade (AUG), DIII-D and JET at the onset of sawtooth induced  $m/n=3/2$  neoclassical tearing mode is plotted in Fig. 1(a) and (b) with all values evaluated at the  $q=3/2$  surface on the outboard midplane and  $\beta_N = \beta(\%)/(I/aB)$  the global normalized beta. Figure 1 (a) shows that a single separable power law in  $v = v_i/\epsilon\omega_{e*}$  does not fit multiple-devices and that  $\alpha$  is thus  $\neq 1/3$ . Note the similarity in the  $v$  scaling (positive power) of the “medium-sized” tokamaks AUG and DIII-D and the opposite  $v$  scaling (negative power) of the “large-sized” JET. JET being larger and, typically, of lower  $\rho_{i*}$  and higher  $S$  may be entering the regime where, with  $\alpha > 1/3$ ,  $w_s$  is decreasing faster than  $w_{pol}$  with lower  $\rho_{i*}$ . Figure 1(b) shows the dependence of  $\beta_N/f(v)$  normalized by  $1/3$  [crossing of curves value in Fig 1(a)] versus  $\rho_{i*}$  where the three separate fitted  $f(v)$  from Fig. 1 (a) are used. The linear dependence on  $\rho_{i*}$  is a strong indication of the validity of the polarization/inertial threshold model and the value of multi-device dimensionless comparisons for extending the data range, here in  $\rho_{i*}$ .

As  $\beta_N/\rho_{i*}$  is not a function of  $v$  alone,  $\alpha \neq 1/3$  and the complete dimensionless model from Eq. (1) is fitted to the AUG/DIII-D/JET data as

$$\frac{\beta_{NC}}{C_0\rho_{i*}} = \frac{C_1\rho_{i*}^{3\alpha-1}v^\alpha}{\frac{1}{1+C_4\rho_{i*}^{2/3-6\alpha}v^{-2\alpha}} - \frac{(1+C_2v)/(1+C_3v)}{C_1\rho_{i*}^{2/3-6\alpha}v^{2\alpha}}} \quad (2)$$

with fitted constants  $C_0, \dots, C_4$  and  $\alpha$ . Common values are assumed for all three tokamaks. Note that: (1)  $C_0 \propto -\Delta r$  and is assumed constant for all devices and parameter regimes; (2)  $C_1$  and  $\alpha$  give the seed island amplitude and scaling; (3)  $C_2$  and  $C_3$  determine the “knee” in the transition from low to high collisionality regimes of the polarization threshold; and (4)  $C_4$  gives the amplitude of the transport threshold, nearly negligible for existing devices but retained as potentially significant for low  $\rho_{i*}$  reactor-grade tokamaks.

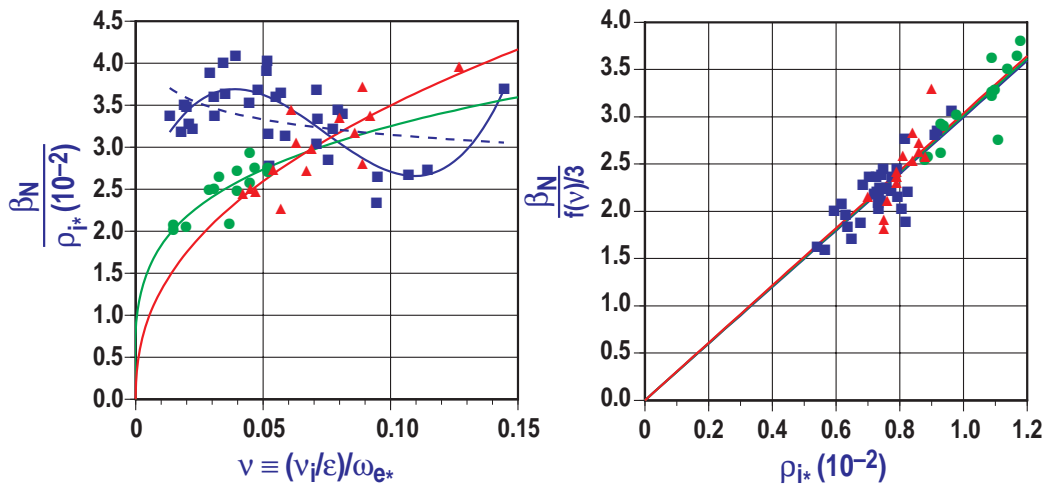


Fig. 1. (a) Critical  $\beta_N$  normalized by  $\rho_{i*}$  versus collisionality  $v$ . (b) Critical  $\beta_N$  normalized by the separate fitted functions of  $v$  [from Fig. 1(a)] versus  $\rho_{i*}$ . ■ JET, ● AUG, ▲ DIII-D.

In fitting, the constants are adjusted to minimize the sum of the square of the difference in the fitted critical  $\beta_N$  and the experimental critical  $\beta_N$ . A test is made to ensure that all data points are on the “unstable” branch of Eq. (1) and not on the “saturated” branch. The unstable branch occurs for seed islands less than a minimum value  $w_{\min}$  for which  $\beta_{\theta c}$  in Eq. (1) is a minimum, i.e.,

$$w_{\min} \approx (3w_{\text{pol}}^2 + 3w_d^2)^{1/2}, \quad (3a)$$

assuming  $w_{\min}^2 \gg w_d^2$  and

$$\beta_{\theta c, \min} \approx -\Delta' \frac{3\sqrt{3}}{2} (w_{\text{pol}}^2 + w_d^2)^{1/2} / (\epsilon^{1/2} L_q / L_p). \quad (3b)$$

This is readily adapted for Eq. (2). The best fit contour plot of the combined data of the critical  $\beta_N$  in  $\rho_{i^*} - v_i/\epsilon\omega_{e^*}$  space is shown in Fig. 2. Note that at fixed  $\rho_{i^*} = 7.0 \times 10^{-3}$ , for example, the horizontal dashed line typical of existing data, the minimum in  $\beta_N$  with  $v$  reproduces the change in sign of  $d\beta_N/dv$  from AUG/DIII-D to JET. High critical  $\beta_N$  occurs at either: (1) high  $\rho_{i^*}$ -high  $v$  (larger  $w_{\text{pol}}$ , upper right of Fig. 2, AUG/DIII-D) or (2) low  $\rho_{i^*}$  and/or low  $v$  — high  $S$  (smaller  $w_s$ , lower left of Fig. 2, JET). At fixed  $v$ , the vertical dashed line in Fig. 2,  $\beta_{NC}$  decreases with  $\rho_{i^*}$  until a minimum is predicted to be reached then rises rapidly with yet lower  $\rho_{i^*}$  as  $w_s^2/w_{\text{pol}}^2 \rightarrow 1$ . The nominal ITER/EDA operating point ( $\beta_N = 2.5$ ,  $\rho_{i^*} = 1.2 \times 10^{-3}$  and  $v_i/\epsilon\omega_{e^*} = 0.13$  at Greenwald density  $G = 1.35$ ) is predicted to be just stable with  $w_s < w_{\text{threshold}}$  and the sawteeth would be unable to excite the 3/2 neoclassical

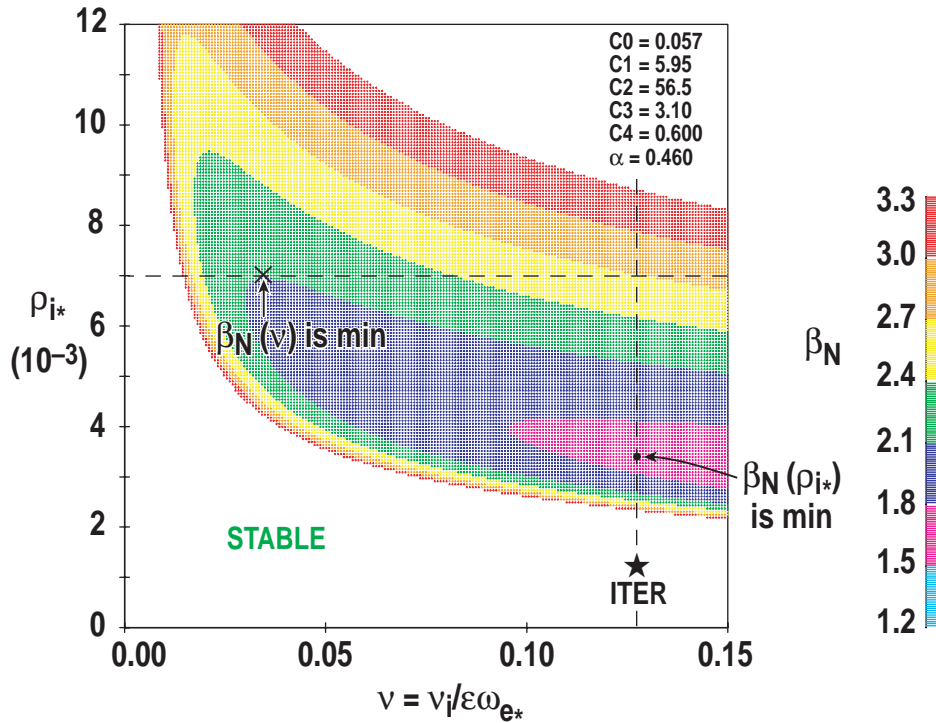


Fig. 2. Contour plot of best fit of combined data to the NTM dimensionless scaling model. The minimum in  $\beta_N$  ( $\times$ ) with  $v$  at a fixed  $\rho_{i^*}$  is shown along with the predicted minimum with  $\rho_{i^*}$  at a fixed  $v$  ( $\bullet$ ). The nominal operating point for the ITER/EDA ( $\star$ ) is just in the stable region.

tearing mode. Thus, the model suggests that a reactor-grade tokamak could operate in a regime where stability improves with decreasing normalized ion gyroradius, a favorable result.

Future work is to: (1) include data from other devices, particularly the high field Alcator-C-Mod and the large JT-60U and (2) to directly confirm the  $w_s/r \propto S^{-\alpha}$  scaling. Data from Alcator C-Mod at higher field and smaller size but with similar  $\rho_{i*}$  and  $v$  as existing devices would lend confidence in the dimensionless scaling approach. Data from the larger size JT-60U could access lower  $\rho_{i*}$ . The directly measured scaling of  $w_s/r$  is needed to refine and confirm the model.

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