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Gaussian Wave Beam Tracing in Tokamak Plasmas

E. Poli [1] , A. G. Peeters [1] , G. V. Pereverzev [1] and M. Bornatici [2]

[1] *Max-Planck-Institut für Plasmaphysik, EURATOM Association, D-85748 Garching, Germany*
[2] *INFM, Department of Physics "A. Volta", University of Pavia, I-27100 Pavia, Italy*

1. The Beam Tracing technique

Microwave beams play an important role in the physics of fusion devices; namely, they are utilized for resonant heating and non-inductive current drive experiments, and in diagnostic applications. For all these goals, a high collimation or focusing of the electromagnetic beam can be very useful, because it allows to improve the localization of power deposition.

The beam tracing technique [1,2] is employed in this paper to obtain numerical predictions on the propagation and absorption (and consequent current drive) of short-wavelength electromagnetic beams in tokamak plasmas.

The limit of weak inhomogeneity, i.e., the so-called short-wavelength limit, demands $\lambda \ll L$, where λ is the wavelength and L the inhomogeneity scale of the medium: the problem can then be characterized by a dimensionless small quantity $1/\kappa$, with $\kappa \equiv \omega L/c$ ($\omega/2\pi$ is the wave frequency). In the widespread ray tracing method, based on the theory of geometrical optics [3,4], Maxwell equations are reduced, by means of an asymptotic expansion with respect to $1/\kappa$, to a set of ordinary differential equations. In this approach, the radiation propagates along Hamiltonian rays, which correctly account for wave refraction. Diffraction effects, on the contrary, are not included at this level, since the different rays do not "interact" with each other. In the aforementioned case of highly collimated or focused wave beams, these effects should not be neglected. Although very similar, from a physical point of view, to other analogous techniques (such as the parabolic equation [5] or the quasi-optical approximation [6,7]) that include diffraction effects in the treatment of short-wavelength wave propagation, the beam tracing approach results in an easier set of *ordinary* differential equations, thus retaining one of the most powerful features of the ray method. The beam is described by means of a central or *reference* ray, denoted by $\Re$, which satisfies the usual geometrical optics equations, and by a set of parameters (calculated along $\Re$) connected with the curvature of the wave front and the amplitude profile. Also in this respect, this method is very effective, since the calculation of a large amount of rays is no longer required.

The (complex) phase of the radiation field $\mathbf{E}(\mathbf{r}) = A(\mathbf{r})\mathbf{e}(\mathbf{r})\exp\{i\kappa[s(\mathbf{r}) + i\phi(\mathbf{r})]\}$, solution of Maxwell wave equation $(c^2/\omega^2)\nabla \times (\nabla \times \mathbf{E}) - \kappa^2 \boldsymbol{\varepsilon} \cdot \mathbf{E} = 0$, is written as (summation over repeated indices is adopted)

$$s(\mathbf{r}) = s_0(\mathbf{r}) + K_\alpha(\tau)\,[x^\alpha - q^\alpha(\tau)] + \frac{1}{2}S_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)], \tag{1}$$

$$\phi(\mathbf{r}) = \frac{1}{2}\phi_{\alpha\beta}(\tau)[x^\alpha - q^\alpha(\tau)][x^\beta - q^\beta(\tau)], \tag{2}$$

where $q^\alpha(\tau)$ and $K_\alpha(\tau)$ are, respectively, the components of the position vector $\{x^\alpha\} \equiv \mathbf{r}$ and the wave vector $\{k_\alpha\} \equiv \nabla s$, calculated on $\Re$. Such a Taylor expansion of s and ϕ around the central ray is possible because it is assumed that the amplitude profile, described by the function $\phi \geq 0$, is Gaussian, so that the value of the electric field has to be known only near the beam axis. It is supposed that the beam width W is such that $\lambda \ll W \ll L$.

The quantities $q^\alpha(\tau), K_\alpha(\tau), S_{\alpha\beta}(\tau), \phi_{\alpha\beta}(\tau)$, involved in Eqs. (1-2), can be found by solving the following set of *beam tracing equations* [1]

$$\frac{dq^\alpha}{d\tau} = \frac{\partial H}{\partial k_\alpha}, \qquad \frac{dK_\alpha}{d\tau} = -\frac{\partial H}{\partial x^\alpha}, \tag{3}$$

$$\frac{dS_{\alpha\beta}}{d\tau} + \frac{\partial^2 H}{\partial x^\alpha \partial x^\beta} + \frac{\partial^2 H}{\partial x^\beta \partial k_\gamma} S_{\alpha\gamma} + \frac{\partial^2 H}{\partial x^\alpha \partial k_\gamma} S_{\beta\gamma} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta} S_{\alpha\gamma} S_{\beta\delta} = \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta} \phi_{\alpha\gamma}\phi_{\beta\delta}, \tag{4}$$

$$\frac{d\phi_{\alpha\beta}}{d\tau} + \left(\frac{\partial^2 H}{\partial x^\alpha \partial k_\gamma} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta} S_{\alpha\delta}\right)\phi_{\beta\gamma} + \left(\frac{\partial^2 H}{\partial x^\beta \partial k_\gamma} + \frac{\partial^2 H}{\partial k_\gamma \partial k_\delta} S_{\beta\delta}\right)\phi_{\alpha\gamma} = 0, \tag{5}$$

where $H(\mathbf{k};\mathbf{r})$ plays the role of a Hamiltonian function and can be taken as the (real) determinant of the dispersion tensor $\mathbf{\Lambda} = (c^2/\omega^2)(-k^2\mathbf{I} + \mathbf{kk}) + \boldsymbol{\varepsilon}^{\,h}$, where $\boldsymbol{\varepsilon}^{\,h}$ is the Hermitian part of the dielectric tensor (it is supposed $|\boldsymbol{\varepsilon}^{\,a}| \ll |\boldsymbol{\varepsilon}^{\,h}|$), just as in the case of usual geometrical optics. On the reference ray, it is $H|_\Re = 0$. All the derivatives of H in Eqs. (3-5) are evaluated on $\Re$.

The Hamiltonian Eqs. (3) give the position of the central ray $\Re$ and the wave vector along it. Eqs. (4-5) allow to calculate the second-order terms in expansion (1-2). It is easy to see [2] that the quantities $S_{\alpha\beta}$ are connected with the description of the phase-front curvature, i.e., with the convergence or divergence of the beam, whereas the unknowns $\phi_{\alpha\beta}$ are related to the width of the electric field profile.

In the beam tracing approach, the evolution of the power carried by the beam has to be calculated on the reference ray only. The following equation must then be integrated along with Eqs. (3-5)

$$\frac{d(P/P_0)}{d\tau} = -2\gamma \frac{P}{P_0}, \tag{6}$$

where P_0 is the input power and γ is the absorption coefficient, evaluated using the weakly relativistic approximation for the dielectric tensor [8]. In Eq. (6), $P(\tau)$ represents the total power of the electromagnetic beam.

2. Features of the numerical solution

The approach that has been outlined above is applied to the calculation of propagation and absorption of electron cyclotron (EC) wave beams in a tokamak plasma. A current drive routine [9], moreover, allows one to compute non-inductively generated current. The integration of Eqs. (3-6) is performed by means of a `FORTRAN` code. This code is modularly structured, in such a way that the user should be able to adapt it to his particular needs in the simplest way. For this reason, the beam tracing equations are written and solved in Cartesian coordinates, and the core of the program is therefore independent of the particular geometry of the problem. The dispersion function H and its derivatives have then also to be supplied to the core in Cartesian coordinates. A separate routine provides the interface between the core and the plasma parameters, which are expressed as functions of the appropriate curvilinear coordinates.

In this paper, the various plasma profiles needed in the calculations are supposed to be known analytically. Toroidal coordinates are linked to Cartesian ones as follows

$$x = [\mathcal{R}_0 + r\cos\chi - \Delta(r)]\cos\varphi \equiv \mathcal{R}\cos\varphi, \quad (7)$$

$$y = [\mathcal{R}_0 + r\cos\chi - \Delta(r)]\sin\varphi \equiv \mathcal{R}\sin\varphi, \quad (8)$$

$$z = r\lambda(r)\sin\chi, \quad (9)$$

where the radial coordinate r, the poloidal angle χ and the toroidal angle φ have been introduced. In Eqs. (7-9), $\mathcal{R}_0$ is the major radius; the Shafranov shift $\Delta(r)$ and the elongation $\lambda(r)$ are known functions. It is clearly $\mathcal{R}^2 = x^2 + y^2$; the z-axis coincides with the vertical axis of the torus. The toroidal and poloidal magnetic fields are respectively given by $\mathbf{B}_t = B_t\mathbf{e}_\varphi$ and $\mathbf{B}_p = B_p\mathbf{e}_\chi$, with $\mathbf{e}_{\varphi,\chi} = \partial\mathbf{r}/\partial(\varphi,\chi)$ and $B_t = B_t(\mathcal{R})$, $B_p = B_p(r,\chi)$. The electron density profile $n_e(r)$ has also to be assigned.

The component of the wave vector parallel to the magnetic field is calculated as $k_{||} \equiv \mathbf{k}\cdot\mathbf{B}/B$; it is then $k_\perp^2 = k^2 - k_{||}^2$. It has to be noticed that, since $\mathbf{B} = \mathbf{B}(\mathbf{r})$, the quantities $k_{||}$ and $k_\perp$ become also functions of $\mathbf{r}$. The derivatives of the dispersion function H that appear in Eqs. (3-5) are computed according to the equations given above. Finally, in order to calculate the absorption coefficient and the current-drive efficiency, the electron temperature and the effective charge $Z_{\rm eff} \equiv \sum_i n_i Z_i^2/n_e$ (where i runs over the ion species in the plasma) must also be prescribed.

3. Applications

Here, the beam and plasma parameters are chosen in order to match the experimental conditions typical of ASDEX-Upgrade. The major radius is $\mathcal{R}_0 = 165$ cm, the minor radius $a = 60$ cm. Second-harmonic extraordinary (X)-mode EC heating at $\omega/2\pi = 140$ GHz (with $B(\mathcal{R}_0) = 2.5$ T) is considered. The initial width of the wave beam is 3.8 cm; the input power is 1 MW. Parabolic density and temperature profiles (with $T_{max} = 1$ keV) are assumed.

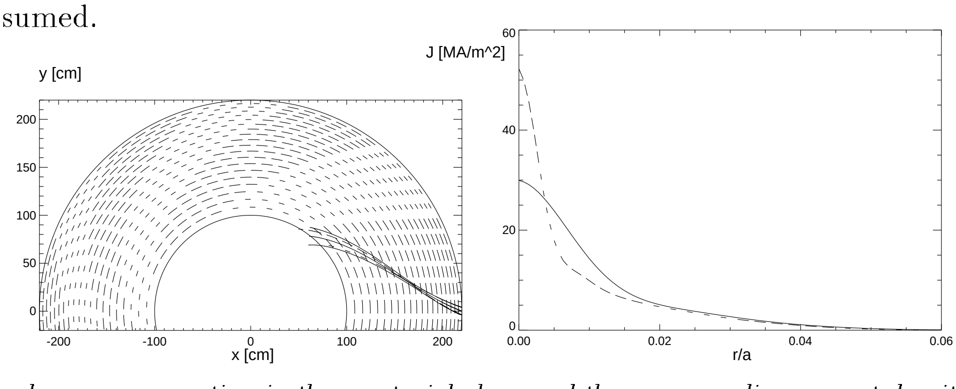

Fig. 1. X-mode wave propagation in the equatorial plane and the corresponding current density profile.

In Fig. 1, a beam propagating in the equatorial plane is considered. The toroidal launching angle is 20 °; the beam is focused with a front curvature $1/R = 1/100$ cm $^{-1}$.

The central electron density is $6 \cdot 10^{13}$ cm $^{-3}$; elongation and Shafranov shift are respectively $1 \leq \lambda(r) \leq 1.3$, $-8 \leq \Delta(r) \leq 5$ cm. Dashed lines represent geometrical-optics calculations. The current density profiles, calculated by means of beam tracing and ray tracing method, are both peaked around the magnetic axis (where $r = 0$); since in this region the beam width predicted by standard geometrical optics (i.e., neglecting beam diffraction) is considerably smaller than the beam-tracing one, the profile of the driven current is correspondingly narrower.

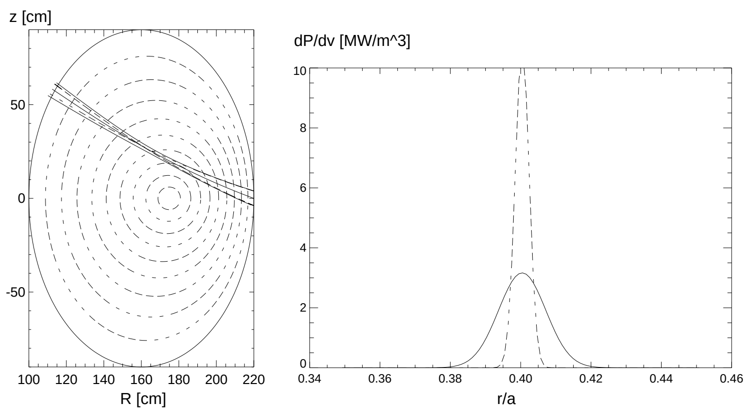

Fig. 2. X -mode poloidal wave propagation and the corresponding power absorption profile.

In Fig. 2 (same electron density as before, poloidal launching angle 20 °), beam-tracing and ray-tracing wave propagation in the poloidal plane are again compared. In this figure it is $1 \leq \lambda(r) \leq 1.5$ and $-10 \leq \Delta(r) \leq 5$ cm, and the beam is focused stronger than in Fig. 1, with $1/R = 1/82$ cm. Also in this case, the geometrical-optics beam width is almost zero in correspondence of the absorption layer (for $\mathcal{R} = \mathcal{R}_0$), while diffraction calculations show that the width remains finite. Since in the latter case a wider region is affected by absorption, the beam-tracing deposition profile is significantly broader and less peaked than the ray-tracing one.

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