Nonlinear Absorption of 2nd Harmonic X-Mode ECRH at W7-AS Stellarator*

Martin F. Heyn[1], Sergei V. Kasilov[2], Winfried Kernbichler[1], H. Maaßberg[3], M. Romé[4], U. Gasparino[3], N.B.Marushchenko[2]

[1] *Institut für Theoretische Physik, Technische Universität Graz, Austria*
[2] *Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine*
[3] *Institut für Plasma Physik, Max-Planck-Institut, Garching, Germany*
[4] *Dipartimento di Fisica, Università di Milano, Italy*

Introduction

In the high power experiments at W7-AS the ECRH beam is launched almost perpendicular to the magnetic field in such a way that the cyclotron resonance absorption zone is located very near to the magnetic field extremum along the magnetic field line. For this case, quasilinear theory predicts that a significant amount of the energy is absorbed by the trapped particles with small parallel velocities inside the resonance zone. However, for typical parameters of high power experiments, i.e., a beam with 400 kW, second harmonic X-mode, launched from the low field side, the assumptions of quasilinear theory are not valid any more and therefore both, the kinetic description of ECRH heating as well as the wave absorption mechanism itself should be modified to take into account nonlinear wave-particle interaction. Such effects have been extensively studied in the past, mainly in analytical form. On the other hand, in the typical range of experimental parameters, the expansion parameter used in analytical theory varies from small to even large values in velocity space. In the present contribution, the nonlinear particle dynamics in the electromagnetic field of the 2nd harmonic ECRH (X-mode) in W7-AS is studied numerically.

The Model

For the numerical treatment of the problem a simplified model is used in which the main magnetic field near the extremum is assumed to be uniform along the z-axis, $\mathbf{B} = \mathbf{e}_z B_0$, and the radiation beam with Gaussian shape propagates along the x-axis, i.e., the corresponding X-mode electric field is $\mathbf{E} = \mathbf{e}_y E \cos(kx - \omega t) \exp\{-\alpha z^2/2\}$. The Hamiltonian for this problem is expanded up to linear order in the wave field vector potential, and, in Bessel expansion, only the resonant term is retained which corresponds to the second harmonic resonance, $\omega \approx 2|\omega_c|$, (see, e.g., [1])

$$H = m_0 c^2 \gamma + \frac{1}{2} k v_E J_\perp e^{-\alpha z^2/2} \cos\psi, \tag{1}$$

where

$$\gamma = \sqrt{1 + \frac{1}{m_0^2 c^2}\left(p_z^2 - 2 m_0 \omega_{c0} J_\perp\right)}, \qquad \psi \equiv 2\phi - \omega t + \psi_0, \qquad \omega_{c0} = eB_0/(m_0 c) < 0,$$

and $v_E = eE/(m_0\omega)$. The positive perpendicular invariant $J_\perp$ is related to the perpendicular kinematic momentum $p_\perp$ through $J_\perp = -p_\perp^2/(2m_0\omega_{c0})$ and ϕ is the gyrophase. The canonical conjugate variables are (ϕ, $J_\perp$) and (z, p_z).

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The change of parallel energy $w_{||} = p_z^2/(2m_0)$ during the interaction with the beam is small and therefore one can neglect the variation of the parallel velocity during the interaction process and approximate with good accuracy $z = v_{||}t$ with $v_{||} = p_z/(\gamma m) = \mathrm{const}$. With this assumption and with the dimensionless time and energy variables,

$$\tau = \sqrt{\frac{\alpha}{2}}\,|v_{||}|t, \qquad w = \sqrt{\frac{2}{\alpha}}\frac{|\omega_{c0}|p_\perp^2}{|v_{||}|m_0^2c^2}, \tag{2}$$

only the following two equations remain,

$$\frac{\mathrm{d}w}{\mathrm{d}\tau} = \varepsilon\, w\, e^{-\tau^2}\sin\psi, \qquad \frac{\mathrm{d}\psi}{\mathrm{d}\tau} = \delta - w + \varepsilon\, e^{-\tau^2}\cos\psi, \tag{3}$$

with the parameters

$$\varepsilon = \sqrt{\frac{2}{\alpha}}\frac{N|\omega_{c0}|}{|v_{||}|}\frac{E}{B_0}, \qquad \delta = \sqrt{\frac{2}{\alpha}}\frac{|\omega_{c0}|}{|v_{||}|}\left(2 - \frac{\omega}{|\omega_{c0}|} - \frac{v_{||}^2}{c^2}\right), \tag{4}$$

where $N = kc/\omega$. This reduced set of two equations for the particle dynamics with just two free parameters allows for a parametric numerical study of the given problem. On the other hand, if the dimensionless parameter

$$\epsilon_{QL} = w\,\varepsilon = \frac{2N\omega_{c0}^2 v_\perp^2 E}{\alpha v_{||}^2 c^2 B_0} = \frac{2N\omega_{c0}^2}{\alpha c^2}\frac{E}{B_0}\tan^2\chi \tag{5}$$

is sufficiently small, these equation can be solved by an expansion of w in a series using the fact that $\varepsilon \ll 1$ (typically $w \geq 1$) and the results of quasilinear theory are recovered. Note that this parameter does not depend on the temperature and if this parameter becomes large, quasilinear theory will be invalid for all energies simultaneously.

In the derivation of the equation of the slow evolution of the electron distribution function, the parallel motion of electrons is assumed to be periodic in the z coordinate with a period L_z. For a localized beam considered here, $L_z^2\alpha \ll 1$. Also, the beam is centered at $z = L_z/2$ so that electrons pass the beam once after having traveled the distance L_z. At the periodic boundary $z = 0$ a Poincaré cut is introduced and the kinetic equation becomes an equation for the conservation of the particle flux through this cut. Since the wave-particle phase is randomized between successive passes of the particle through the beam by collisions, the particle flux density Γ does not dependent on ψ and is given by

$$\Gamma(t, w, v_{||}) = \frac{1}{2\pi}\int \mathrm{d}t_0 \int \mathrm{d}w_0 \int \mathrm{d}\psi' \delta(t - \tau_b - t_0)\delta(w - W(w_0, \psi'))\Gamma(t_0, w_0, v_{||}). \tag{6}$$

Here,

$$\Gamma(t, w, \psi, v_{||}) \equiv v_{||}Jf(t, w, \psi, v_{||}), \qquad J = \sqrt{\frac{\alpha}{2}}\frac{|v_{||}|m_0c^2}{4\omega_{c0}^2}, \tag{7}$$

are the particle phase space flux density trough the unit area of the Poincaré cut (note that this cut is a 5D hyper-surface $z = \mathrm{const}$ in 6D phase space) and the Jacobian of coordinates $(w, \psi, P_y, y, p_z, z)$ respectively, $\tau_b = L_z/|v_{||}|$ is the bounce time, $W(w_0, \psi')$ and $\Psi(w_0, \psi')$ are the values of w and ψ at the Poincaré cut given by the solution of the equations of motion with initial conditions w_0, ψ' on the previous intersection of the orbit with the cut. Equation

(6) is the desired mapping equation describing the electron distribution function in presence of the nonlinear wave-particle interaction in simple magnetic geometry. The generalization for the case of general magnetic geometry including the effects of Coulomb collisions and particle drift between the Poincaré cuts [2,3] follows in a straightforward way.

The absorbed power density integrated along the magnetic field line is obtained from the difference between the incoming and outgoing energy fluxes, (a similar expression for the absorbed power density is given in [4])

$$\langle P\rangle_z = \int \mathrm{d}z P_{\mathrm{abs}} \quad = \quad \frac{\pi\alpha c^4}{4\omega_{c0}^2}\int\limits_0^\infty \mathrm{d}w_0 \int\limits_{-\infty}^{\infty} \mathrm{d}p_z |p_z|^3 f^{(in)} \langle \Delta W(w_0, p_z)\rangle \tag{8}$$

$$\langle \Delta W(w_0, p_z)\rangle \quad \equiv \quad \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} \mathrm{d}\psi' \left[W(w_0,\psi') - w_0\right]. \tag{9}$$

Note that W depends on p_z through the parameters ε and δ as defined in (4). In order to estimate the effects of nonlinearity on the wave absorption in a rarefied plasma, the absorption coefficient is introduced as

$$\kappa = \frac{\int \mathrm{d}z\, P_{\mathrm{abs}}}{\int \mathrm{d}z\, S_x}, \qquad S_x = \frac{cE^2}{8\pi}\mathrm{e}^{-\alpha z^2}, \tag{10}$$

where S_x is the Poynting vector.

Results

In this report the mapping equation (6) has not been actually solved rather it has been assumed that the distribution of particles entering the beam is Maxwellian. For the computation, typical parameters of high power experiments are used, $B = 25$ kG, $T_e = 1$ keV, $n = 10^{13}$ cm^{-3}, $\alpha = 0.25$ cm^{-2}, and $R_0 = 120$ cm. The beam input power is taken to be $P_0 = 400$ kW.

In Figures 1, 2 and 3 a comparison between the quasilinear and nonlinear model is given for the case $\omega/\omega_{c0} = 1.995$. In this case, the resonant zone is located in the thermal region of velocity space at $v \approx 2v_T$. For small and medium pitch angles the energy absorption is well described by quasilinear theory while for higher pitch angles the nonlinear effects are dominant.

In Figure 4 the power flux in the beam is given as a function of the big radius R. The ECRH beam is launched from the low field side. The $1/R$ dependence in the magnetic field is taken into account, whereas n and T are kept constant. The resonance zone, $\omega = 2\omega_{c0}$, is located at $R_0 = 120$ cm. It can be seen that the single pass absorption in the nonlinear case is significantly lower than in the quasilinear case.

Conclusions

Nonlinear wave-particle interactions play an important role in high power ECRH scenarii with perpendicular injection typical for W7-AS. The role of those effects strongly differs in different regions of phase space. They are strong for particles with pitch angles close to 90 degrees representing trapped particles which are responsible for convective radial energy transport. At the same time, for passing particles the nonlinear effects are much smaller and the dynamics of such particles can be described by the quasilinear theory. Therfore, ECCD computations can be performed within the quasilinear theory using conventional Fokker-Planck solvers.

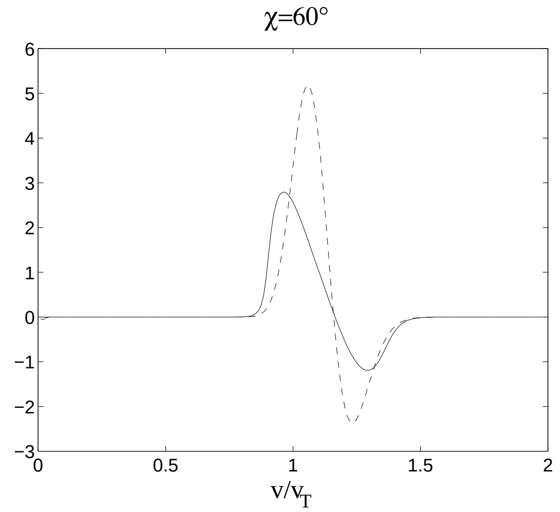

Figure 1. The phase averaged change in particle energy $\langle \Delta W \rangle$ after one pass through the beam as a function of its initial velocity module for a pitch angle of 60 degrees. Dashed line – quasilinear, solid line – nonlinear model.

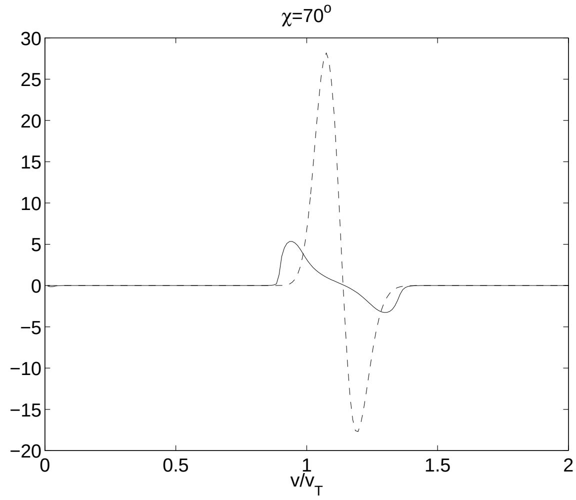

Figure 2. Same as Figure 1 but for a pitch angle of 70 degrees.

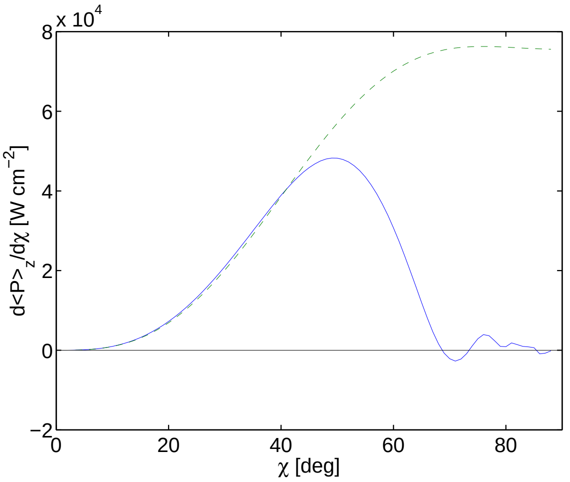

Figure 3. Distribution of the absorbed power over initial electron pitch angles. Dashed line - quasilinear, solid line - nonlinear model.

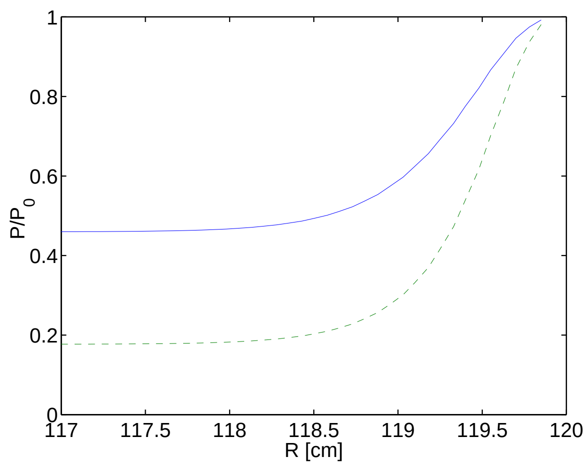

Figure 4. Fraction of transmitted power in the linear model (dashed) and in the nonlinear model (solid) with $P_0 = 400$ kW.

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