26th EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 1773 - 1776

Analytical and Computational Investigations of Improved-Confinement Stellarators

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1. Introduction

The stellarator configurations with poloidal direction of lines B=constant on magnetic surfaces have been previously shown to possess some attractive features. First of all, zero magnetic-axis curvature at the locations of the longitudinal magnetic field minima provides the possibility to improve the confinement of charged particles: by a corresponding choice of the near-axis magnetic surface cross-sections, it is possible to eliminate locally lost orbits and even to satisfy the condition of quasi-isodynamicity (QI) for reflected particles of deep to moderately-deep trapping condition [1]. Secondly, the choice of the system with poloidal direction of constant B contours on magnetic surfaces was shown to increase the plasma pressure limit with respect to local stability [2] by improving the local stability condition for ballooning modes which are very localised along the magnetic field lines. Despite the possible decrease of the average magnetic well, the Mercier modes could still remain stable under these circumstances.

In the present paper, such configurations are studied both analytically, up to second-order terms in the expression for B in an expansion with respect to the distance from the magnetic axis, and numerically.

2. The strength of the magnetic field in near-axis approximation

Using the method of expansion with respect to the distance from the magnetic axis with curvature k and torsion κ, and the corresponding magnetic surface parameterisation (for detail, see, e.g. [3]), one can obtain the expression for B up to second-order in a. For vacuum systems, when the diamagnetic effect is negligible, this expression has the form:

$$B^2 = B_0^2 b^2\{1 + 2a(A_c\cos\theta + A_s\sin\theta) + a^2[3(A_c\cos\theta + A_s\sin\theta)^2 + A_{20} + A_{2c}\cos 2\theta + f_0 + f_c\cos 2\theta + f_s\sin 2\theta]\} \ . \quad (1)$$

Here

$$f_0 = \frac{1}{bR^2}\left[-(\kappa_n^2R^2)\frac{sh^2\eta}{ch^2\eta} + \frac{1}{4}\left(\eta'^2 + \frac{b'^2}{b^2}\right)ch\eta + \frac{\eta''}{2}sh\eta - \frac{b''}{2b}ch\eta\right] ,$$

$$f_c = \frac{1}{bR^2}\left[(\kappa_n^2R^2)\left(1 + \frac{1}{ch^2\eta}\right)sh\eta - \frac{1}{4}\left(\eta'^2 + \frac{b'^2}{b^2}\right)sh\eta - \frac{\eta''}{2}ch\eta + \frac{b''}{2b}sh\eta\right] ,$$

$$f_s = -\frac{1}{bR^2}\left[(\kappa_nR)'th\eta + (\kappa_nR)\eta'\left(1 + \frac{1}{ch^2\eta}\right)\right] , \quad (2)$$

$$\kappa_n = \kappa - \delta'/R \ , \quad A_c = k\frac{\exp(-\eta/2)}{\sqrt{b}}\cos\delta \ , \quad A_s = k\frac{\exp(\eta/2)}{\sqrt{b}}\sin\delta \ ,$$

$$A_{2c} = -2(\alpha_1A_c + \alpha_2A_s) \ , \quad A_{20} = A_c(\xi_1 + \alpha_1) + A_s(\xi_2 - \alpha_2) \ . \quad (3)$$

The parameters η and δ describe the elliptical magnetic surface shape, the function b describes the magnetic field inhomogeneity on the magnetic axis, prime denotes the derivative with respect to the longitudinal coordinate ζ, and ξ_i and α_i are the plasma column shifts and triangularities, $2\pi R$ is the magnetic axis length.

For systems with poloidal direction of lines B=constant, the following conditions of particle confinement improvement can be formulated for the region near $\zeta = 0$.

1) The condition of local pseudosymmetry (PS) (there are no islands formed by the constant B lines near the region $\zeta = 0$). To first order in a, this condition requires $k(0) = 0$. If it is fulfilled, the expression for B can be written as

$$\begin{aligned} B^2 &= B_0^2 b^2 \left\{ 1 + a^2 \left[\left(A_{20} - b \left(\frac{A_c^2 + A_s^2}{2b'} \right)' + f_0 \right) \right. \right. \\ &+ \left. \left. \left(A_{2c} - b \left(\frac{A_c^2 - A_s^2}{2b'} \right)' + f_c \right) \cos 2\theta + \left(-b \left(\frac{A_c A_s}{2b'} \right)' + f_s \right) \sin 2\theta \right] \right\} , \quad (4) \end{aligned}$$

and to fulfill this condition in the second approximation, the coefficients of the $\cos 2\theta$ and the $\sin 2\theta$ terms in Eq. (4) should be zero for $\zeta = 0$.

2) To diminish the particle radial excursion, one can try in addition to fulfill the QI condition [1] which requires $J_{||} = \oint V_{||} dl = J_{||}(a)$. In the flux coordinate system with straight magnetic field lines a, θ^*, ζ^* with $B = B(a, \zeta^*)$, it can be formulated as the independence of the combination

$$\Delta\varphi = \varphi(a, \theta_0^* + \iota\zeta_0^*, \zeta_0^*) - \varphi(a, \theta_0^* - \iota\zeta_0^*, -\zeta_0^*) \quad (5)$$

on θ_0^*. Here $\varphi(a, \theta_0^*, \zeta_0^*)$ is the potential part of $\mathbf{B}$ in the covariant representation in a, θ_0^*, ζ_0^* flux coordinates. In a linear approximation with respect to a, the expression for φ acquires the form (for simplicity superscripts (*) are omitted for θ, ζ):

$$\begin{aligned} \varphi &= \varphi_0(\zeta) + \frac{akb^{3/2}}{b'} \{ [\exp(-\eta/2) \cos\delta \cos(\iota\zeta - \eta_0^*) + \exp(\eta/2) \sin\delta \sin(\iota\zeta - \eta_0^*)] \cos\theta_0 \\ &+ [-\exp(\eta/2) \cos\delta \sin(\iota\zeta - \eta_0^*) + \exp(\eta/2) \sin\delta \cos(\iota\zeta - \eta_0^*)] \sin\theta_0 \} . \quad (6) \end{aligned}$$

Here the parameter η_0^* describes the slope of the magnetic field lines. It can be extracted from the equation: $-\iota + \partial\eta_0^*/\partial\zeta = \kappa_n R / ch\eta$.

Different possibilities to fulfill the QI condition can be considered. For example, for $b' \sim \zeta$, $k \sim \zeta$, $\delta \sim \zeta$, $\eta = \eta(0) + \eta''(0)\zeta^2/2$, $\kappa = \kappa(0) + \kappa''(0)\zeta^2/2$, the term with $\cos\theta_0$ is even and drops out from the expression for $\Delta\varphi$. The term with $\sin\theta_0$ is odd. If its component that is linear with respect to ζ vanishes, that is if

$$\kappa_n R / ch\eta = -\delta' \exp(\eta) , \quad (7)$$

the longitudinal invariant acquires the form $J_{||} \sim \zeta_0^2(1 + \gamma a \zeta_0^2 \sin\theta_0)$. It is worth to emphasize that condition (7) is local, i.e. it depends on the magnetic surface structure near the region $\zeta = 0$ only. The region in which $J_{||}$ does not depend on θ_0 can be expanded if the terms of higher order in ζ can be eliminated too. In principle, it can be done up to any given order. Nevertheless, the requirement to fulfill this condition on the whole magnetic surface in general is incompatible with the periodicity conditions. If the curvature of the magnetic axis is proportional to ζ^2, then for the ordering used above, it is impossible to fulfill the QI condition even at lowest order. In this case we can consider another condition.

3) The condition of the existence of closed B=constant surfaces near the region $\zeta = 0$. This condition requires the coefficient of a^2 in the expression (4) for B to be positive for all θ near the region $\zeta = 0$. It can be considered in combination with the condition of

local PS also. If it is fulfilled, the deeply-trapped particles are confined inside the closed surface B=constant and cannot drift out of the plasma column.

3. Some examples of numerical investigations

The possibilities to fulfill some of the conditions of particle confinement improvement discussed above have been verified numerically. The results are presented in the Table and in Figs. 1-4. Two different cases were considered in the near-axis approximation for 8-periods systems.

Case (1) corresponds to a system with $k \sim \zeta^2$. Here the parameters were found for a system which possesses a vacuum magnetic well, an absolute minimum of B near the region $\zeta = 0$ and in which the constant B contours on a flux surface do not form islands in this region to first and second order in a (Fig.1). The QI condition is not satisfied here. Case (2) correspond to a system with $k \sim \zeta$ (Fig. 2). For the parameters shown in the Table, this system displays a vacuum magnetic well, the B=constant lines do not show islands and is QI to leading order (condition (7) is fulfilled). The surfaces B=constant in this system are open.

3D equilibrium numerical calculations for a 5-periods Helias system were produced using the VMEC and TERPSICHORE codes. It was shown that there exist equilibria without islands of lines B=constant in the middle of the system period. Magnetic flux surface cross-sections and contours of B=constant on a boundary magnetic surface are presented in Figs. 3 and 4.

Parameters of magnetic configurations with a vacuum magnetic well and no islands of lines B=constant near the region $\zeta = 0$. The functions $k, \kappa, \eta, ...$ have the form: $k(\zeta) = \sum_n (k_{cn} \cos nN\zeta + k_{sn} \sin nN\zeta)$.

	k_{c0}	k_{c1}	k_{c2}	k_{s1}	κ_{c0}	κ_{c1}	η_{c0}	η_{c1}	b_{c0}	b_{c1}	δ_{s1}	α_{1c0}	α_{1c1}
1	1.20	-1.2	0	0	0.07	6.57	0.33	-0.28	1.014	-0.014	0	0.02	0.07
2	0.95	0	-0.95	2.4	0	3.0	0.75	-0.25	1.05	-0.05	-0.45	0.05	0.08

4. Conclusions

The results of both analytical and numerical investigations of different possibilities to control the behaviour of the B=constant lines (on flux contours) and the B=constant surfaces are presented. The conditions of local PS and QI are obtained and investigated in a near-axis approximation as well as the condition for the existence of an absolute minimum of B. It is shown that the conditions of local PS, QI and magnetic well existence can be satisfied simultaneously in the near-axis approximation. Configurations were found also that possess an absolute minimum of B, a vacuum magnetic well and no island formation in the B=constant contours. The condition of QI is not fulfilled, however, in such type of configurations.

Acknowledgements

We thank Prof. M.Q. Tran for supporting this work. This work is partially sponsored by the Fonds National Suisse de la Recherche Scientifique and by EURATOM, and by Russian Foundation for Fundamental Research, project No 97-02-17695. One of the authors (M.I.M) made a part of this work during his visit to the Institute of Advanced Energy (Kyoto University). He would like to thank the Administrations of the Institute and Prof. T. Obiki for invitation and supporting of these investigations.

References

[1] S. Gori, W. Lotz, J. Nührenberg, Theory of Fusion Plasmas (Varenna 1996), Editrice Compositori, Bologna (1996), p.335.

[2] M.Yu. Isaev, W.A. Cooper, V.D. Shafranov, Proc. 25th EPS Conf. on Controlled Fusion and Plasma Phys., Prague, 1998, p.1738.

[3] M. Yu. Isaev, V.D. Shafranov, Sov. J. Plasma Phys. **16** (1990) 419.

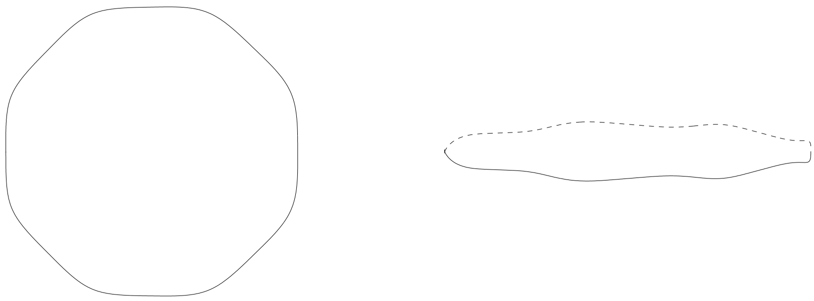

Fig. 1. Two projections of magnetic axis for the case that corresponds to a system with $k \sim \zeta^2$. For the parameters shown in the Table, this system possesses a vacuum magnetic well, an absolute minimum of B near the region $\zeta = 0$ and the lines B=constant display no islands in this region to first and second order in a. The condition of quasi-isodynamicity is not satisfied here.

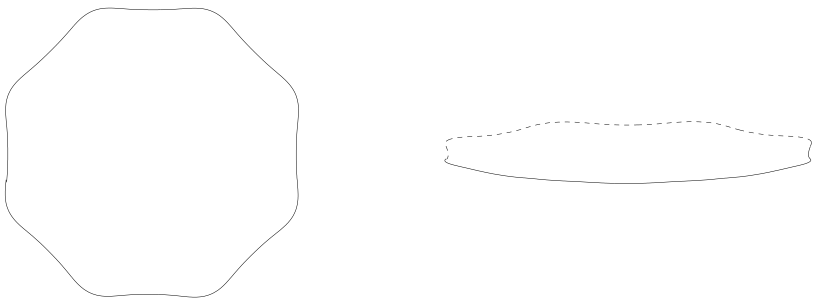

Fig. 2. Two projections of magnetic axis for the case that corresponds to a system with $k \sim \zeta$ near the region $\zeta = 0$. For the parameters shown in the Table, this system has a vacuum magnetic well, no islands formed by the lines B=constant to first and second order in a and is quasi-isodynamical to leading order.

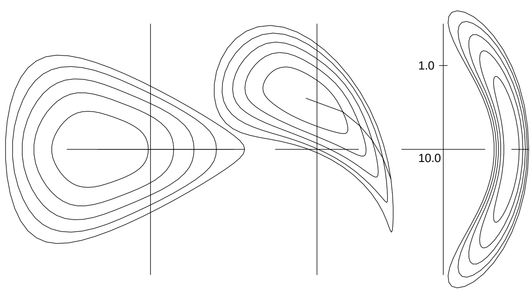

Fig. 3. Magnetic flux surface cross-sections at the beginning, at one quarter and at the middle of the period for a 5-period Helias with B=constant islands absent near midperiod of the system.

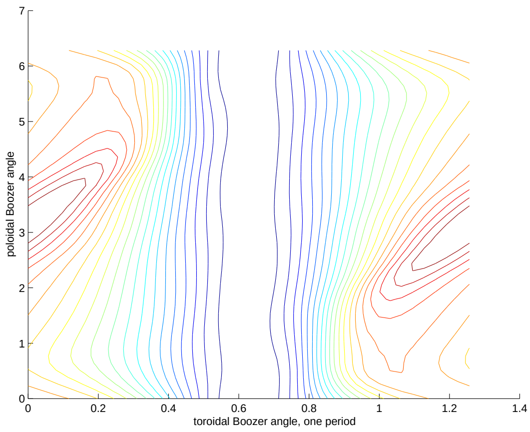

Fig. 4. Contours of B=constant on a boundary magnetic surface for a 5-period Helias without islands of lines B=constant at the middle of the system period.