ON GLOBAL H-MODE SCALING LAWS FOR JET

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Investigation of the scaling of the energy confinement time τ_E with various plasma parameters has since long been an interesting, albeit not uncontroversial topic in plasma physics. Various global scaling laws have been derived for Ohmic as well as (NBI and/or RF heated) L-mode discharges [1-5]. Due to the scarce availability of computerised, extensive and validated H-mode datasets, systematic statistical analysis of H-mode scaling behaviour has hitherto been limited. A common approach is to fit the available H-mode data by an L-mode scaling law (e.g., Kaye-Goldston, Rebut-Lallia) with one or two adjustable constant terms. In this contribution we will consider the alternative approach of fitting all free parameters of various simple scaling models to two recently compiled datasets consisting of about 140 ELM-free and 40 ELMy H-mode discharges, measured at JET in the period 1986-1988. From this period, approximately all known H-mode shots have been included that satisfy the following criteria: D-injected D^+ discharges with no RF heating, a sufficiently long ($\geq$ 300 ms) and regular P_{NBI} flat-top, and validated main diagnostics. Normally, 3 time points per discharge were selected. For future reference, we call these two datasets in this paper *ELMF* and *ELMY*, respectively.

We start our discussion with the empirical JET scaling law presented last year by Keilhacker [6] at the IAEA Conference in Nice:

$$\tau_{dia} = (0.63 \pm .02) I_p^{.75\pm.08} B_t^{.5\pm.08} \langle n_e \rangle^{.2\pm.1} (P_{tot} - \dot{W}_{dia})^{-.7\pm.05}, \qquad (1)$$

This law is based on a large subset of *ELMF*, consisting of those single null (SN) shots from 1986 and 1988, in which during some time interval $\dot{W}_{dia}/P_{tot} < 0.30$ (i.e. without relatively early disruptions), and containing some, but not all of the very latest 1988 5 MA shots. The regression was made taking only one timepoint per shot, where W_{dia} approximately reached its maximum value. The global energy confinement time is defined by $\tau_{dia} = W_{dia}/P_c$, where W_{dia} is the diamagnetically determined energy content, P_c is an abbreviation for $P_{tot} - \dot{W}_{dia}$, P_{tot} is the ohmic plus the NBI-injected power, B_t is the toroidal magnetic field, I_p the plasma current, and $\langle n_e \rangle$ the volume averaged electron density, all quantities being evaluated at the selected time point. The errors indicate one std (estimated standard deviation) of the estimated coefficients. The units used in formula (1) are: τ_{dia} (sec), I (3 MA), B_t (2.5 T), $\langle n_e \rangle$ ($4.10^{19}\mathrm{m}^{-3}$), and P_c (10 MW). These units correspond roughly the average values in the dataset. After a logarithmic transformation, a linear model was fitted. This approach presumes that a constant level of statistical errors exists on logarithmic scale (or, equivalently, constant relative errors on ordinary scale). The usual flaw of ordinary least squares, namely the assumption that only the response variable and none of the 'explanatory variables' are measured with statistical error, was avoided by presuming, not very differently from [7], 10% measurement error on τ_{dia} and W_{dia}, and 5% on $\langle n_e \rangle$ and P_c, and applying special regression techniques [8] based on theory of 'functional relationships' [9-12]. (The reader may have noted that, by definition of τ_{dia}, the above assumption on the errors implies that the errors in $\log\tau_{dia}$ and $\log W_{dia}$ are correlated with correlation coefficient $r = 0.875$.) Obviously, one gets an equivalent scaling law for W_{dia} by just adding +1 to the coefficient of $P_{tot} - \dot{W}_{dia}$ in (1). The assumed measurement errors in τ_{dia} and W_{dia} are more or less consistent with the residual rmse (root mean squared error) of 8% from ordinary regression.

To classify the various types of scaling laws, it seems useful to distinguish between 'scientific' scaling laws, in which the dependence of the energy confinement time τ_E on physically relevant, though possibly not directly controllable parameters (like the axis temperature $T_e(0)$ or a descriptor of the ELM activity, Δ_E) is analysed, and engineering ones, in which only directly controllable parameters (like heating power, refuelling rate, wall conditioning, etc.) are taken as independent variables. In practice, scaling laws are often of a hybrid type, in which the two objectives are mixed. In fact, looking at (1), one can see that the inclusion of P_{tot} instead of the electron temperature T_e implies an engineering aspect. The scaling law is, however, not a fully engineering one, since for instance $\langle n_e \rangle$ during H-mode is not an independently controllable parameter.

In table 1, the results are presented of an empirical analysis of the datasets from a more strictly engineering point of view. In this table, $\langle n_{ohm} \rangle$ denotes the volume-averaged electron density during the stationary ohmic discharge phase just before the onset of NBI. From the descriptive statistics one can see that the continuous variables were, generally, varied over quite a large range, B_t being changed least. However, from the correlation matrix it clear that the 'controllable' variables were not at all varied independently. The determinant $|R|$ of this matrix (i.e. the product of its eigenvalues) is some measure of the global dependency. In fact, under the hypothesis of independence, for large normal samples, $X = -N\log|R| \sim \chi^2_{p(p-1)/2}$. In our case, obviously, $X = 623 \gg \chi^2_{6;\alpha}$ for any reasonable level α. Clearly, less correlation between the independent variables at roughly the present ranges, which may be feasible to achieve, even if one takes into account operational machine limits, would have a beneficial effect on the precision and the robustness of the regression. The two binary variables Δ_{DN} and Δ_{87} indicate whether $(\Delta = 1)$ or not $(\Delta = 0)$ the shot was double null and/or run in 1987. In the scaling law section, coefficients and their std's are given that correspond directly to expressions like (1). Ordinary least squares [13] has been used for simplicity. (The presence of interaction terms destroys the linearity in the functional relationship model. The measurement errors tend to increase the absolute values of the estimates and certainly increase their std's. Experience with the simple model (1) suggests by 5 to 10% and 50 to 100%, respectively.) The quantity τ_{mhd} represents the equilibrium-based confinement time, in which the parallel as well as the perpendicular thermal and beam energy contributions are taken into account, whereas τ_{dia} is obtained from the perpendicular contributions. The subscript $_{rc}$ denotes a rough correction for radiation $(\tau_{rc,dia} = W_{dia}/(P_c - \alpha P_{rad}))$, where P_{rad} denotes the total radiated power (from the central plasma as well as the X-point) and α was taken to be 30%. From an analysis of a few shots, this approximation appeared not too bad, although of course P_{rad} and P_{NBI} depend both on density. Obviously, a standard implementation of a more accurate radiation correction would be desirable. From this simple approximation, one can see however that the engineering scaling laws are, except for their constant terms, not very sensitive to the radiation correction. In order to keep the scaling law an engineering one, no radiation correction was applied to the independent variable P_c. In a scientific approach, one may do so, in order to get a better fit, or at least to have the same scaling laws for τ_{rc} and W.

The reader might object that the laws in table 1 are not purely engineering ones, because of the presence of $\dot{W}$. The relationships presented can however be interpreted as applying to the quasi-stationary state where $\dot{W} = 0$, which is reached in many JET discharges. The more serious objection that knowledge on the presence of ELM's is not yet an engineering quantity, is encountered by making an appropriate discriminant analysis, the details of which will be presented on a poster. It has been known for some time that the shots from 1987 were somewhat worse than the other H-mode shots. This is quantified in column Δ_{87}, where one can see that the overall degradation ranges between 20% and 30%. After some searching for the cause, it turned out that during and prior to the period (december 1987) in which these 18 H-mode shots were run, the He wall conditioning was stopped for 6 weeks and the condition of the wall was directly influenced by a number of disruptions. Hence, from an engineering point of view, one might interpret the absolute values of these exponents as 'the gain in H-mode confinement time due to careful wall conditioning'. In table 1, $I_p \otimes I_p$ denotes an interaction term, which is simply a quadratic term on logarithmic scale. On ordinary scale we have, in our units, $\tau_{dia} \sim I_p^{.67-.46 \log I_p}$, i.e. for $I_p = 3$ MA the exponent is .67, which diminishes by .23 if the current is a factor $\sqrt{e} = 1.65$ larger, i.e. at $I_p \simeq 5$ MA. The presence of more interaction terms has been investigated, but only the significant ones have been retained in the table.

Compared to the ELM-free shots, the ELMy shots have a worse degradation with power and a smaller exponent for I_p. However, no significant saturation with current was found in *ELMY*. A remarkable fact is the difference in (I_p, B_t) dependence between τ_{dia} and τ_{mhd} in *ELMY*. As $I_p^a B_t^b \sim I_p^{a+b} q_{cyl}^b$, this can be interpreted as difference in q_{cyl} dependence at constant I_p. One might be concerned about the difference in power dependence between (1) and table 1. It should be remembered, however, that in (1) this exponent has to be interpreted at constant instantaneous $\langle n_e \rangle$, and in table 1 at constant $\langle n_{ohm} \rangle$. As P_c and $\langle n_e \rangle$ are positively correlated, a larger $\langle n_e \rangle$ from more power, at constant $\langle n_{ohm} \rangle$, counteracts the power degradation in (1). Of course, the quadratic term has been chosen as a simple expedient for describing the deviation from linearity, and the scaling law should not be extrapolated too far from the experimental range. The saturation with current is obvious for the ELM-free shots. It is not perfectly clear, however, whether this is due to intrinsic machine limitations, or to the fact that the 5 MA H-mode shots were not yet fully optimised. In the future, it is intended to compare the present scaling laws with other types of scaling laws at JET, and to make similar analyses for ASDEX.

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Table 1. Engineering scaling laws for the global energy confinement time of JET H-mode shots, from ordinary least squares regression.

descriptive statistics	units	av.	std	(min,max)	correlation matrix R (on logarithmic scale)			
Dataset ELMF (ELM-free, D into D^+, NBI only, 140 shots, N = 420 datapoints)								
τ_{dia}	sec	0.81	0.17	(0.4, 1.45)	I_p	B_t	P_c	$\langle n_{ohm} \rangle$
I_p	MA	3.4	0.7	(2.0, 5.2)	1			
B_t	T	2.4	0.4	(1.7, 3.5)	**.49**	1		
P_c	MW	7.2	2.5	(1.4,16.7)	.22	**.40**	1	
$\langle n_{ohm} \rangle$	$10^{19}/m^3$	1.6	0.5	(0.8, 3.1)	**.78**	**.56**	.24	1
Δ_{87}		13%		(0,1)	eigenvalues:			
Δ_{DN}		6%		(0,1)	2.4, 0.9, 0.5, 0.21			
Dataset ELMY (with ELM's, D into D^+, NBI only, SN, 40 shots, N = 120 datapoints)								
τ_{dia}	sec	0.75	0.18	(0.4, 1.14)	I_p	B_t	P_c	$\langle n_{ohm} \rangle$
I_p	MA	4.1	0.7	(3.0, 5.2)	1			
B_t	T	2.7	0.4	(1.7, 3.5)	.43	1		
P_c	MW	8.7	3.3	(1.4,16.7)	**.55**	.43	1	
$\langle n_{ohm} \rangle$	$10^{19}/m^3$	2.0	0.5	(1.0, 3.1)	**.75**	**.48**	**.47**	1
					eigenvalues:			
					2.5, 0.7, 0.6, 0.24			

scaling laws: ELMF

	C	I_p	$I_p \otimes I_p$	B_t	P_c	$\langle n_{ohm} \rangle$	Δ_{87}	Δ_{DN}	rmse
τ_{dia}	.705 (.008)	.67 (.03)	−.46 (.06)	.36 (.04)	−.50 (.02)	.16 (.03)	−.18 (.01)	−.04 (.015)	7.0%
$\tau_{rc,dia}$	.890 (.012)	.77 (.04)	−.55 (.07)	.28 (.04)	−.44 (.02)	.12 (.03)	−.20 (.01)	−.06 (.02)	8.8%
τ_{mhd}	.690 (.008)	.45 (.03)	−.55 (.06)	.57 (.04)	−.46 (.01)	.14 (.03)	−.32 (.01)	−.08 (.015)	7.7%
$\tau_{rc,mhd}$	.870 (.015)	.57 (.04)	−.64 (.07)	.50 (.05)	−.43 (.02)	.11 (.04)	−.36 (.02)	−.10 (.02)	10 %

scaling laws: ELMY

	C	I_p	B_t	P_c	$P_c \otimes P_c$	$\langle n_{ohm} \rangle$	rmse
τ_{dia}	.610 (.020)	.27 (.08)	.34 (.08)	−.87 (.04)	−.52 (.08)	.26 (.06)	10.5%
$\tau_{rc,dia}$	.795 (.030)	.23 (.09)	.35 (.09)	−.75 (.04)	−.50 (.08)	.28 (.06)	11.3%
τ_{mhd}	.585 (.030)	−.32 (.11)	.89 (.11)	−.85 (.05)	−.59 (.10)	.25 (.08)	14.5%
$\tau_{rc,mhd}$	.745 (.035)	−.30 (.12)	.92 (.12)	−.74 (.05)	−.57 (.11)	.25 (.08)	15.2%

In this table, av. stands for average, std for estimated standard deviation, P_c for $P_{tot} - \dot{W}_{dia}$ or $P_{tot} - \dot{W}_{mhd}$, and τ_{rc} for a roughly radiation corrected confinement time. The units for the scaling laws are: τ (sec), I_p (3 MA), B_t (2.5 T), P_c (10 MW), and $\langle n_{ohm} \rangle$ ($2.10^{19} m^{-3}$). The std's of the estimated coefficients are given in parentheses. The correlations in bold-face are at least 8 (for *ELMF*) or 5 (for *ELMY*) times as large as their std under the (unlikely) hypothesis that the corresponding true correlation is zero.