A 1+1 D MODEL OF ION-IMPURITY PFIRSCH-SCHLÜTER TRANSPORT IN A ROTATING TOKAMAK PLASMA

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Introduction

We address in this paper the question of particle and angular momentum transport in a highly collisional tokamak plasma which, 1) contains a single impurity species of sufficiently high concentration that the friction of the electrons on the ions and the impurities is negligible, and 2) is toroidally rotating with a velocity such that (at least) the impurities are supersonic. Under the latter conditions (typical of neutral beam injected plasmas) the transport analysis intrinsically requires a 2-D treatment: differently from the conventional case of small rotation, the densities and the elctric potential are no more flux surface functions, due to the action of inertial forces.

Compared to previous ones ([3,4,5] and references therein) our model [1,2] contains (all and) only the purely neoclassical contributions pertaining to the high collisionality regime (no *ad hoc* terms), and we can afford (in principle) arbitrarily large poloidal variations of the relevant plasma parameters.

Brief outline of the model

We build a hydrodynamic model by taking velocity moments of the Fokker-Planck equation, and the resulting hierarchy is truncated assuming the plasma is in the high collisionality regime (details are to be found in [1,2]). The set of moment equations is then expanded in the small parameter δ_{pi} (ratio between the ion Larmor radius in the poloidal field and a characteristic radial scale length). To each order in δ_{pi} corresponds a separate time scale, and we present here the essential features of the steady states obtained up to the second order, which describes the slowest evolution in our problem (the fastest, zero-th order time scale, being of the order of the inverse of the ion transit frequency).

Our model includes all effects related to toroidicity (in the Pfirsch- Schlüter limit), inertia and viscous stress tensor. We do not attempt here to discuss energy transport; therefore a constant uniform temperature is assumed, and its value is chosen such as to give, for densities in the experimental range, both ion species well in the Pfirsch-Schlüter regime (thermal friction should not qualitatively change our results). A low beta, large aspect ratio equilibrium with circular concentric magnetic surfaces is taken as the fixed background for this transport model.

Zero-th order time scale

To this order the plasma is described by the balance between parallel inertial, pressure, and electric forces (also, quasineutrality of the plasma is imposed). The flow is rigid toroidal separately on each magnetic surface, and species independent ($\mathbf{V}_j^{(0)} = V_\phi \mathbf{e}_\phi = \omega^{(0)} R \mathbf{e}_\phi$) [5, 6, 7]. The density distribution at steady state is given by

$$n_j^{(0)}(r,\theta) = \bar{n}_j(r) \frac{A_j^{(0)}}{< A_j^{(0)} >}$$

where we defined

$$A_j^{(0)}(r,\theta) \equiv \exp\left(\frac{m_j \omega^{(0)2} R^2}{2T}\right) \left[\frac{n_i^{(0)}(\theta=0) + Z n_Z^{(0)}(\theta=0)}{n_i^{(0)} + Z n_Z^{(0)}}\right]^{Z_j}.$$

To this order the average quantities $\bar{n}_j$ and $\omega^{(0)}$ are taken as given. Notice that the density poloidal distribution is up-down symmetric to this order, due to the absence of ion-impurity friction.

First order time scale

Ion-impurity friction drives the evolution to this order through the factor [1, 2]

$$\left(m_i - \frac{m_Z}{Z}\right) \frac{R^2}{T} \omega^{(0)} \frac{\partial \omega^{(0)}}{\partial r} + \left(\frac{\partial}{\partial r} \mathrm{Log} \frac{\bar{n}_i}{< A_i^{(0)} >} - \frac{1}{Z} \frac{\partial}{\partial r} \mathrm{Log} \frac{\bar{n}_Z}{< A_Z^{(0)} >}\right)$$

proportional to the difference in the first order diamagnetic flows. The first term is the new contribution to the transport fluxes coming from rotation, and for $\omega^{(0)} \to 0$ the driving force reduces to the well known result in the absence of rotation [8]. The mass and parallel momentum balances for ions and impurities are evolved to steady state to determine the first order poloidal distributions. The radial gradients in the driving force are taken as given quantities, because they vary on the slower, second order time scale. A typical result for an ion Mach number of 0.75 is shown in Fig.1 (ions=dashed, impurities=solid, $\theta = \pi$ outboards), where the impurity species is Oxygen VI, as for all the other numerical examples presented in this paper. Notice that the densities (given here normalized to their respective averages) are now up-down asymmetric, due to friction; also, nonzero steady state poloidal flows are obtained.

Second order time scale and 1+1 D algorithm

The evolution of the average ion and impurity density, and of the toroidal angular momentum of the plasma is described by

$$\frac{\partial < n_j^{(0)} >}{\partial t_2} = \frac{1}{eZ_j}\frac{1}{rR_0}\frac{\partial}{\partial r}(\frac{r}{B_{\theta 0}} < RF_{\| j}^{(1)} >) + < S_j >$$

$$\frac{\partial}{\partial t_2}(\sum_j m_j < Rn_j^{(0)} V_{\phi j}^{(0)} >) = \frac{1}{rR_0}\frac{\partial}{\partial r}(\frac{r}{B_{\theta 0}}\omega^{(0)} \sum_j \frac{m_j}{eZ_j} < R^2 F_{\| j}^{(1)} >) + \sum_j < RM_{\phi j} >$$

where $F_{\|}^{(1)}$ is the first order ion-impurity parallel friction (classical contributions have been presently neglected but will be included in future numerical experiments). As boundary conditions we impose zero particle fluxes (therefore we also set $S_j = 0$ in order to get a steady state), and a flux of angular momentum proportional to the toroidal rotation velocity.

One sees that the dependence of the radial fluxes on $\bar{n}$ and $\omega^{(0)}$ is not explicitely known, differently from the low rotation case, but only implicitely through flux surface moments of the friction. The latter results in turn from the steady state reached by the fast first order evolution, whose driving force contains radial gradients evolving only on the slow second order time scale. The 1+1 D algorithm can thus be summarized as follows: 1) start with given radial profiles, 2) compute the zero-th and first order steady states on several different magnetic surfaces, 3) update the radial profiles integrating one step forward in time the second order equations, 4) go to 2) and repeat until a steady state is reached on the slowest time scale.

In Fig.2 the initial (solid) and steady state (thick) radial profiles of $\bar{n}_i$, $\bar{n}_Z$ (in m^{-3}) and ion Mach number are shown, for the case of an isolated plasma. (The dashed line indicates the profile foreseen by the theory [8] for the case of low rotation.) One notices impurity peaking towards the center, and that the whole plasma rotates in this case as a rigid body. The results corresponding to a uniform momentum source applied on the plasma cross section are shown in Fig.3. The density profiles are now sensibly different from the case of low rotation, because non zero rotation gradients are sustained by the source at steady state.

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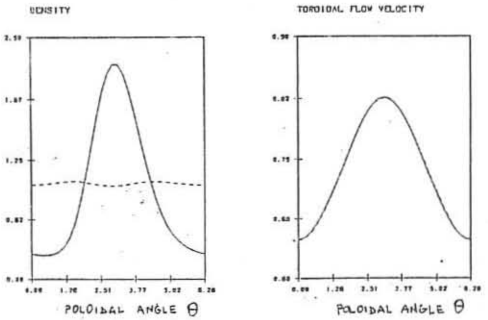

Fig. 1

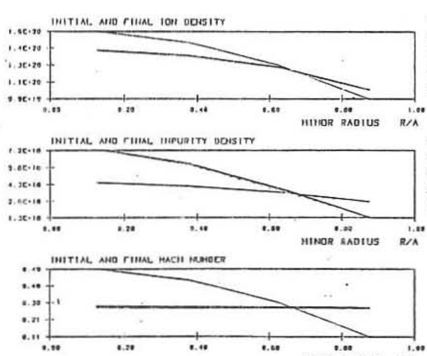

Fig. 2

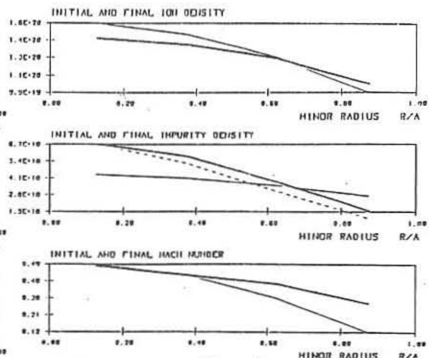

Fig. 3