APPROXIMATE CONSTRUCTION OF RATIONAL MAGNETIC SURFACES IN ANALYTIC VACUUM STELLARATOR FIELDS

W. Dommaschk, F. Herrnegger, A. Schlüter

Max-Planck-Institut für Plasmaphysik
IPP-EURATOM Association
D-8046 Garching bei München

1.Introduction. A systematic method is applied to reduce the size of magnetic islands in analytic vacuum field configurations where, because of technical reasons, the magnetic fields should be described by a set of Dommaschk potentials[1] of low order. In typical cases, fields with 5 or 6 toroidal periods of length L_P and poloidal mode numbers up to 4 (including axisymmetric fields) define a last closed magnetic surface with aspect ratio $A \approx 11$ to 13 and a value of the twist t (i.e. angle of rotational transform divided by 2π) of $t_b = 5/7$ near the boundary.

The method of measure-preserving tangential mapping [2,3] is used to analyse the vicinity of the fixed points of the mapping. In case of 5 field periods and $t = 5/7$, the field line is closed while making 7 toroidal and 5 poloidal revolutions. The mapping of the plane $\phi = 0$ onto itself takes seven field periods (R, ϕ, Z are ordinary polar coordinates). Because of the stellarator symmetry, the matrix of the mapping can be obtained by integrating the field lines over just the half of that number, namely 7/2 periods. The residue R^* of the fixed points of the mapping[2] and its internal twist t_{is} are computed from the trace of that matrix: the fixed point is of the O-type if $R^* > 0$, and is of the X-type if $R^* < 0$. Because of the stellarator symmetry, one fixed point (to be determined) of the mapping of the plane $\phi = 0$ lies inboard at $Z = 0$, and another distinct fixed point at $Z = 0$ inboard at $\phi = L_P/2R_T$ (the torus radius R_T is the reference length). The residues of these two fixed points are intended to get at small values by varying some values of the field potentials. In addition, the normalized magnetic flux $F = \oint \mathbf{A} \cdot d\mathbf{x}$ between these two closed field lines is computed and, in some cases, iterated to zero by varying another partial field ($\mathbf{A}$ is the vector potential of the magnetic field, $\mathbf{x}$ the radius vector, the line integral is performed over both closed field lines).

2.Results. For demonstration, essentially one nonresonant axisymmetric field[5] is varied to make the absolute values of the two residues sufficiently small for both closed field lines of the connectivity fitting $t = 5/7$. The aspect ratio is nearly kept fixed. Fig. 1 shows an example of a configuration with 7 intersections of one closed field line of the O-point type (left column: aspect ratio $A \approx 13$, $t_{ax} = 0.73$) and a configuration where these islands have been removed as observed from the figure (right column: $t_{ax} = 0.77$). Figure 2 shows a configuration with 6 field periods, small Pfirsch-Schlüter currents (measured by $\langle j_{\parallel}/j_{\perp} \rangle = 0.91$), $t_{ax} = 0.65$, $t_b = 0.96$, and without visible islands.

In a case where the islands are close to the boundary, the island structure was investigated in some detail as shown in the Figs. 3 to 6; Figure 3 shows the island structure

of three different field configurations with decreasing size of the islands. The islands of the configuration WAD321 $(R^* = 0.37 \cdot 10^{-5})$ are surrounded by an ergodic region whereas the islands of the configuration WAD042 $(R^* = 33.95 \cdot 10^{-5})$ are very thin and are surrounded by smooth magnetic surfaces; the configuration WAD847 shows two systems of islands not conneted to each other $(R^* = -1.44 \cdot 10^{-5})$. Fig. 4 shows the internal twist ι_{is} (over seven field periods) of the configuration WAD321 as function of the major half-axis of the magnetic surfaces within the island. The internal twist number is almost zero at the fixed point corresponding to the fact that the residue R^* is very small. Nevertheless the islands are still of finite size (left graph of Fig. 3).
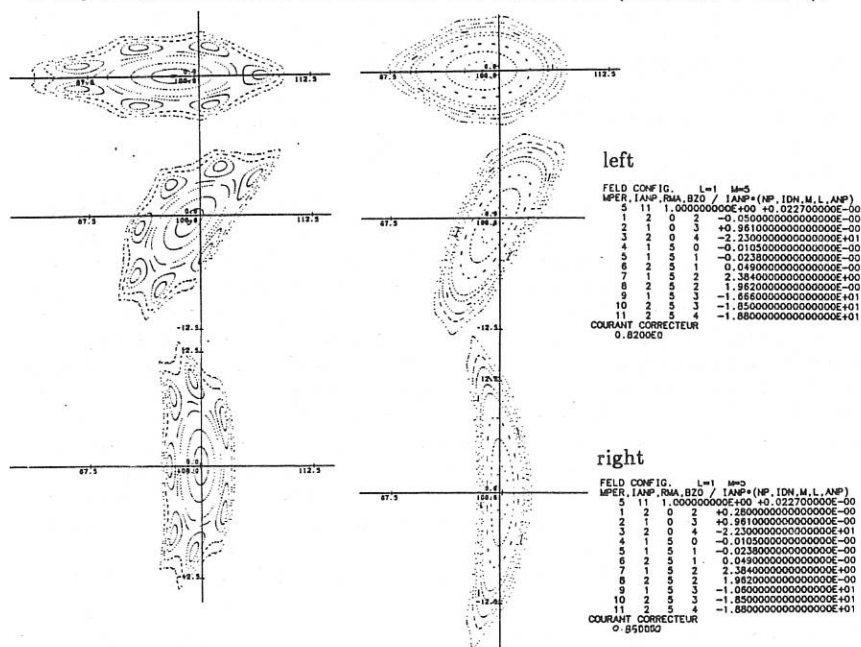

Fig.1. Reduction of island size by additional axisymmetric field components.

If, in addition, the constraint of vanishing flux between the two closed field lines is used to determine a further field and keeping both residues sufficiently small, the resulting field configuration WAD042 (right graph of Fig. 3) shows the desired property that the island size is negligibly small. One observes that the flux F decreases by three orders of magnitude simultaneously as the island size decreases.
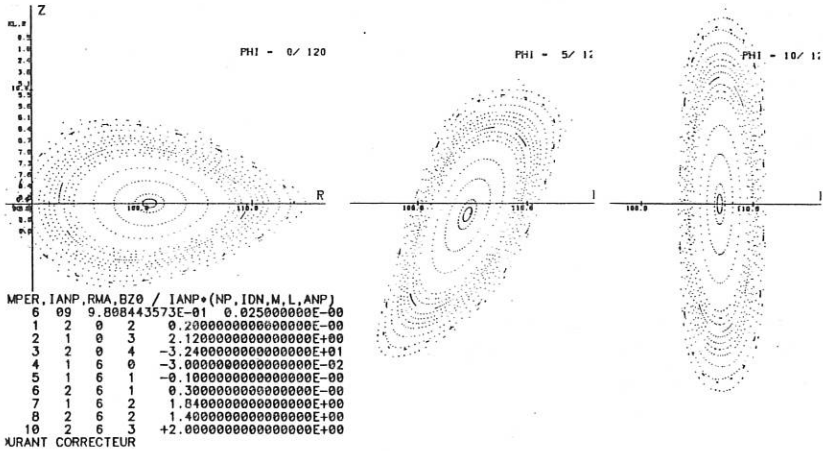
Fig.2. Field configuration with 6 field periods. The Dommaschk potentials are given in the Table where the notation of Ref.5 is used.
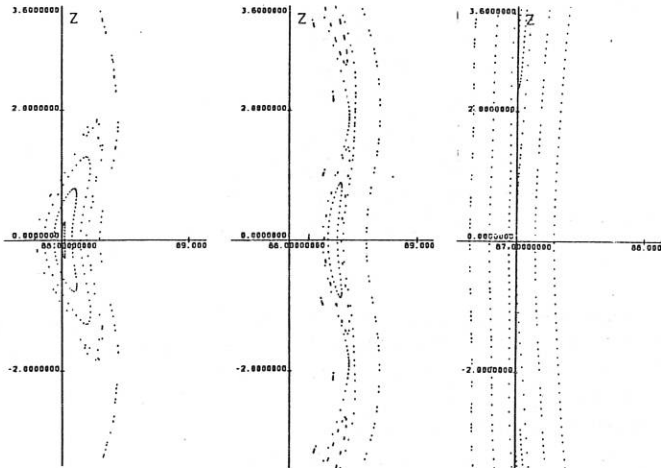

WAD321: $F = 7.4 \cdot 10^{-6}$ WAD847: $F = 6.3 \cdot 10^{-8}$ WAD042: $F = 1.0 \cdot 10^{-9}$

Fig.3. Island structure in the boundary region and residual flux in various configurations with 5 field periods.The boundary region of Helias configurations is discussed in Ref.6.
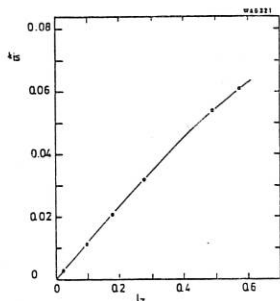
Fig.4. Internal twist t_{is} of the island as function of half-axis l_y (configuration WAD321).
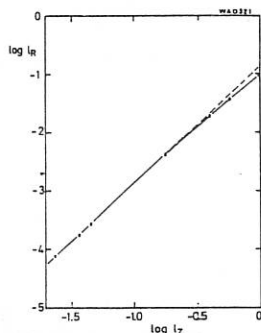

Fig.5. Dependence of the half-axis l_R of the island surfaces on l_z giving the asymptotic relation $l_R \sim l_Z^2$ as l_Z goes to zero.
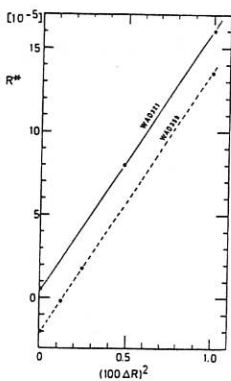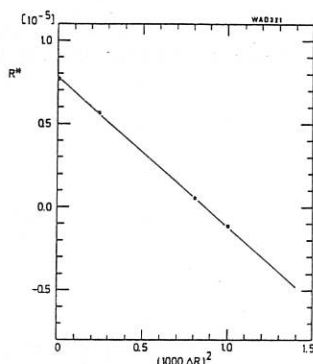

Fig.6. Residue R^* as funtion of the grid size ΔR used to compute the matrix of the mapping; left graph for islands located at inboard side and right graph for islands located at outboard side. A typical grid size used here is $\Delta R = \Delta Z = 10^{-4}$ for $R_T = 100$.

This algorithm is being used in an optimization procedure to approach analytically vacuum magnetic fields with the desired field qualities of, e.g., magnetic well, small secondary currents at t_b around 0.8, and reduced particle drift.

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