STUDIES OF ISLANDS IN STELLARATOR VACUUM FIELDS BY SOLVING A NEUMANN PROBLEM

P. MERKEL
Max-Planck-Institut für Plasmaphysik
IPP-EURATOM Association
D-8046 Garching bei München
Federal Republic of Germany

1. Introduction

Stellarator vacuum fields can be obtained by solving the Neumann problem for Laplace's equation inside the boundary of a toroidal domain. By this method a vacuum field with vanishing normal component at the boundary is computed, so that the prescribed boundary of the domain becomes a magnetic surface. For appropriately chosen parameters one finds solutions where also the magnetic field in the interior consists of a set of "good" nested surfaces [1], as shown in Fig. 1. Of course, a finer resolution of the solutions shows that islands are still present and that the field structure is in general complicated. However, one can conclude from these studies that in general the analyticity of the boundary benignly influences the regularity of the surfaces; for example, finding low aspect ratio solutions with good surfaces is easy. In the present paper the question of the regularising effect of the solution method on the structure of islands is considered. A numerical study of a class of toroidal Heliac vacuum fields was performed. A configuration with $n_p = 4$ periods, small shear and a rotational transform per period of $\iota \approx 1/3$ has been chosen, so that sizeable islands are found, although a boundary value problem is solved for a smooth boundary.

The Heliac fields are computed with the NESCOIL code. In [1], [2] one finds a detailed description: on a closed surface surrounding the Heliac boundary a surface current distribution is determined in such a way that the field $\vec{B}$ approximates the true solution of the Neumann problem. This is achieved by requiring that the normal component of $\vec{B}$ be minimized on the Heliac surface

$$F = \int_{\partial R} (\vec{B} \cdot \vec{n})^2 \mathrm{d}f = \mathrm{min!}$$

This method solves the boundary value problem approximately, but with arbitrarily high accuracy. One period of the outer current carrying surface and the Heliac surface are given by a parameter representation $r(u,v)$, $z(u,v)$, $v = \frac{n_p}{2\pi}\varphi$, where v is proportional to the toroidal angle, u is a poloidal angle-like variable, (r,φ,z) are cylindrical coordinates, and n_p is the number of periods.

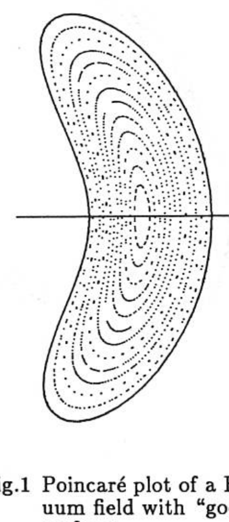

Fig.1 Poincaré plot of a Helias [3] vacuum field with "good" magnetic surfaces

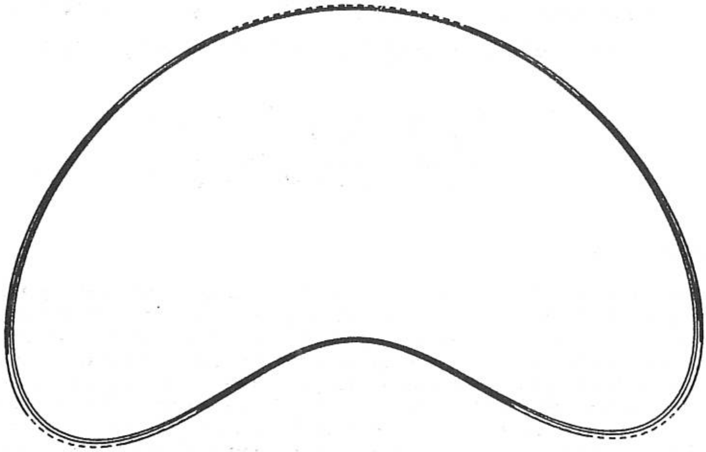

Fig.2a

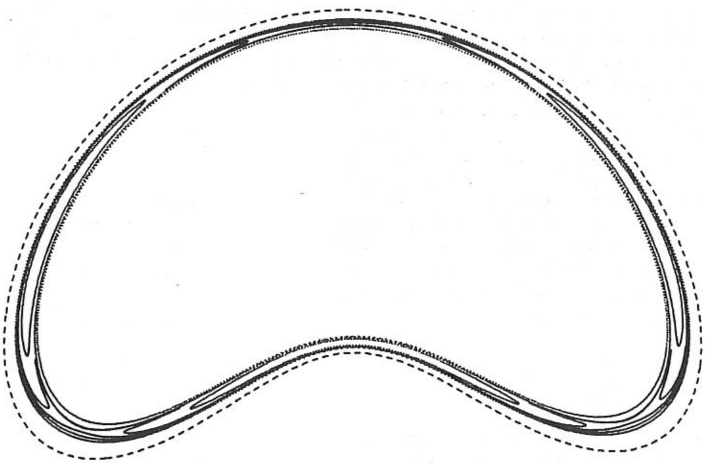

Fig.2b

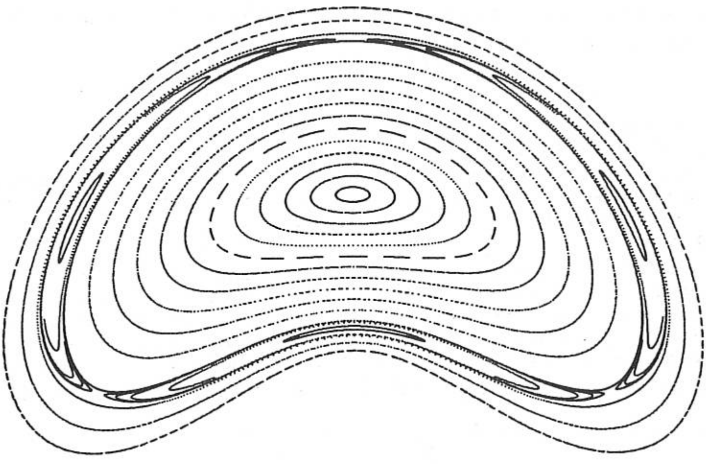

Fig.2c

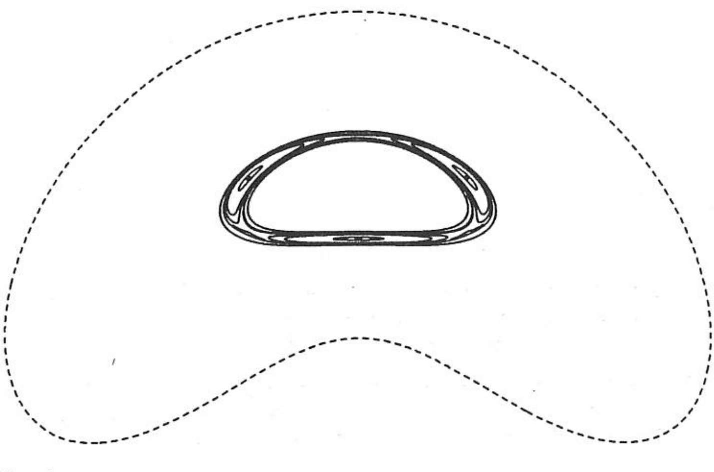

Fig.2d

Fig.2a-2d Poincaré plots of the islands at $\iota = 1/3$ are shown for different radial positions. Values of the parameter p are $p = 0.3113$, 0.313, 0.32 and 0.33. The dashed line is the Heliac boundary.

The surface current density with the same periodicity is expressed by a potential $\Psi(u,v)$ defined on the surface: $\vec{j} = \vec{n} \times \text{Grad } \Psi(u,v)$.

2. Results

The Heliac boundary is defined by the Fourier harmonics of $r(u,v)$ and $z(u,v)$ in (u,v)-space: $r_{0,0} = 10.$, $r_{01} = z_{01} = 0.96$, $r_{11} = z_{11} = 1.$, $r_{21} = z_{21} = 0.38$, $r_{2-1} = -z_{2-1} = 0.11$ and $-r_{1-1} = z_{1-1} = p$ (for notation see [2]). The class of Heliac configurations presented in Fig. 2 is obtained by varying the value of p between $0.3113 < p < 0.334$. In this way the position of the $\iota = 1/3$ resonance is shifted from the boundary to the magnetic axis.

Figures 2a-2d show Poincaré plots of the $\iota = 1/3$ islands for different positions. The Poincaré plot in Fig. 2c shows that apart from the $\iota = 1/3$ islands solutions with "good" nested surfaces are found. The boundary value problem is solved with high accuracy: For a number $N_u = 128$ and $N_v = 256$ mesh points in real space and a number of $m = 32$ poloidal and $|n| \leq 6$ toroidal Fourier modes for the potential $\Psi(u,v)$ the maximal local error on the boundary $\epsilon = (\vec{B}\cdot\vec{n})/|\vec{B}|$ is 10^{-6} ($\vec{n}$ = normal unit vector on the boundary).

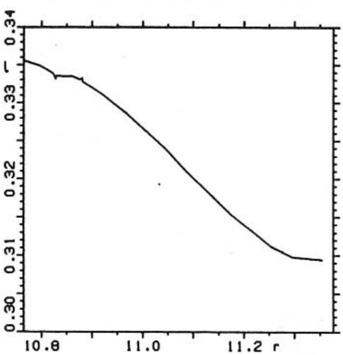

Fig. 3

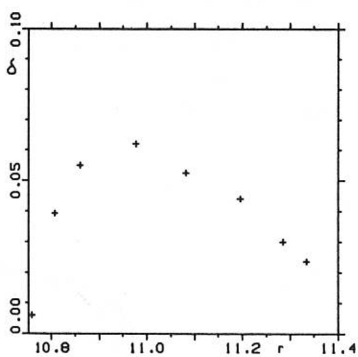

Fig. 4

Fig.3 Profile of the rotational transform for the case shown in Fig.2c.

Fig.4 The relative island thickness δ versus its radial position with the Heliac boundary at $r = 10.76$ and the magnetic axis at $r = 11.4$.

The island thickness δ versus its radial position is plotted in Fig. 4. At the Heliac boundary the condition $\vec{B} \cdot \vec{n} = 0$ forces the island thickness to $\delta = 0$. Close to the magnetic axis and close to the boundary the island thickness is in accordance with analytical arguments.

References

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[3] J. Nührenberg, R. Zille, Phys. Lett. A **114** (1986) 129.