RESISTIVE BALLOONING STABILITY OF ADVANCED STELLARATORS

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Abstract

Recently it has been shown that the problem of resistive ballooning stability of plasmas in axisymmetric toroidal magnetic field configurations can be treated and solved applying a variational approach [1]. The method was suggested by the observation that resistive ballooning modes, in the framework of single-fluid MHD theory, can be conceived as Euler-Lagrange equations of an appropriately constructed Lagrangian. In the present paper we extend this method to three-dimensional configurations. A unique treatment for both stellarators and tokamaks becomes possible by specifying the MHD equilibria assigned for ballooning stability analysis in terms of suitable flux coordinates.

This approach and a stability investigation for the advanced stellarator Helias will be presented.

Using particular field line coordinates in the covering space the equations describing resistive ballooning modes [2] in a three-dimensional equilibrium configuration can be written in the form

$$\frac{1}{\mu_0}\mathbf{B}\cdot\nabla\left\{\frac{k^2}{B^2}\mathbf{B}\cdot\nabla u\right\} + \left\{K\frac{dp}{dV} - \frac{\rho\gamma^2 k^2}{B^2}\right\}u + K\frac{dp}{dV}v = 0 \qquad (1)$$

$$\mathbf{B}\cdot\nabla\left\{\frac{\mathbf{B}\cdot\nabla v}{B^2}\right\} - K\frac{dp}{dV}\left\{\frac{\eta}{\gamma} + \frac{\rho\gamma^2}{\mu_0((\nabla p\cdot(\mathbf{k}\times\nabla\sigma))^2}\right\}u$$
$$-\left\{\frac{\eta}{\gamma}\left(K\frac{dp}{dV} + \frac{\rho\gamma^2 k^2}{B^2}\right) + \frac{\mu_0\rho\gamma^2}{B^2}\frac{1+\beta}{\beta}\right\}v = 0 \qquad (2)$$

Here

$$K \equiv 2\boldsymbol{\kappa}\cdot(\mathbf{k}\times\nabla\sigma)(\mathbf{k}\times\nabla\sigma)\cdot\nabla V \qquad D \equiv 1 + \frac{\eta k^2}{\mu_0\gamma} \qquad (3)$$

$$\mathbf{k} \equiv -\frac{2\pi n}{\Psi'(V)}\nabla\tau \qquad (4)$$

with $\mathbf{k}$ being the wave vector, $\boldsymbol{\kappa}$ the curvature vector, V the volume enclosed by a magnetic surface and Ψ being the poloidal magnetic flux; all other quantities have their usual meaning. (V, τ, σ) are right-handed flux coordinates in the covering space [3] of a magnetic surfaces resulting from the Clebsch representation $\mathbf{B} = \nabla V \times \nabla \tau$. σ is related to the arc length s along a field line by $d\sigma = ds/B$.

After an appropriate transformation of the equations (1) and (2) the problem to be solved can be seen to be equivalent to the stationarity conditions with respect to $\mathbf{u}$ of the quadratic functional

$$L(\gamma, V, \sigma_0) = \int_{-\infty}^{+\infty} \mathcal{L}(\gamma, V, \sigma, \sigma_0, u(\sigma), \dot{u}(\sigma)) d\sigma \tag{5}$$

with the Lagrange density

$$\mathcal{L} = \frac{1}{2}(\dot{u}^T \cdot \mathbf{P} \cdot \dot{u} - u^T \cdot \mathbf{Q} \cdot u) \tag{6}$$

$\mathbf{u} = (u^1, u^2, u^3, u^4)$ comprises real and imaginary parts of u and v in equations (1) and (2) and $\dot{\mathbf{u}} = du/d\sigma$ the components $du^k/d\sigma$, $k = 1, .., 4$. $\mathbf{Q}$ and $\mathbf{P}$ are equilibrium determined symmetric matrices with nonlinear dependence on the the complex growth rate γ. Thus unstable resistive ballooning modes $\mathbf{u}$ are those stationary points of L with $\text{Real}\{\gamma\} > 0$. $\mathbf{P}$ and $\mathbf{Q}$ are equilibrium determined real symmetric matrices with nonlinear dependence on the the complex growth rate γ. They have the structure

$$\mathbf{P} = \begin{pmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & -P_{11} & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} \\ 0 & 0 & P_{34} & -P_{33} \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} & 0 \\ Q_{12} & -Q_{11} & 0 & -Q_{13} \\ Q_{13} & 0 & Q_{33} & Q_{34} \\ 0 & -Q_{13} & Q_{34} & -Q_{33} \end{pmatrix} \tag{7}$$

where the matrix elements are determined by equilibrium quantities.

The field line coordinates (V, τ, σ) can be easily related to Boozer's coordinates [4] which were used to investigate ideal ballooning modes for stellarator configurations [5].

In the present paper, for the case of resistive ballooning modes, we apply a variational approach which already turned out to be successful in the case of axially symmetric equilibrium configurations [1].

The computational procedure will be applied to a Helias configuration which originally is described in Boozer's coordinates (V, u, v). Fig. 1. shows a poloidal cut of magnetic surfaces reconstructed from data in the straight-field line flux coordinates (V, u, v).

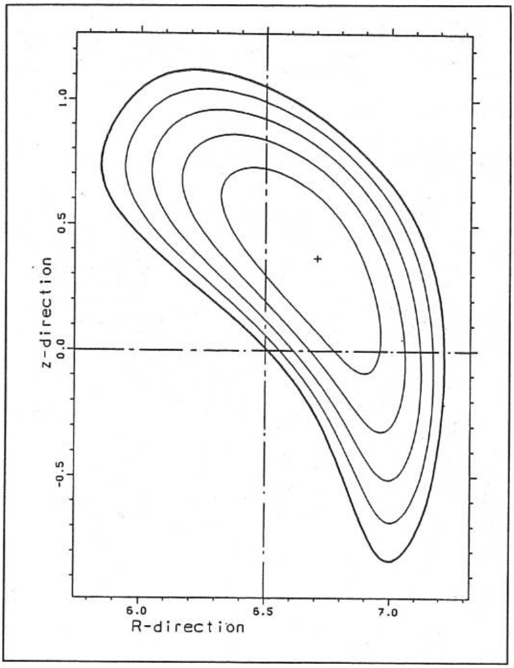

Fig. 1: Poloidal cut of magnetic surfaces of the Helias configuration reconstructed from data in straight-field line flux coordinates.

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