# MODULAR-RIPPLE TRANSPORT IN STELLARATORS

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The future stellarator experiment, Wendelstein VII-X, will be the first to employ the Helical-Axis Advanced Stellarator (Helias) concept. A distinct advantage of Helias configurations over classical stellarator/torsatron devices is their significantly reduced neoclassical transport rates, both in collisional regimes and in the long-mean-free-path regime where particles trapped in the local *helical* ripple wells of the stellarator's magnetic field make the dominant contribution to transport. Unlike the classical, continuously wound stellarator/torsatron, however, the magnetic field of Wendelstein VII-X will be produced by a large number of *discrete* non-planar coils. These *modular* coils give rise to a further ripple in the magnetic field strength and introduce a neoclassical loss mechanism not otherwise present. The magnitude of this modular ripple may be made small by using a large number of coils — added expense and restricted plasma access are the principal drawbacks to this approach.

In the present work, the additional transport introduced by the modular ripple is calculated by analytically solving the bounce-averaged kinetic equation. The solution assumes a general magnetic field model so that complicated helical ripple profiles composed of several harmonics may be treated. The deformation of modular ripple wells due to the presence of the other magnetic field harmonics is fully accounted for. This solution is used to determine the level of modular ripple transport to be expected in Wendelstein VII-X candidates for configurations with both 10 and 12 coils per field period.

The set of magnetic coordinates $(r, \theta_0, \phi)$ will be chosen to represent physical space [1]; $r$ is the minor radial variable defining a toroidal flux surface, $\theta_0$ labels a field line and $\phi$ is the usual toroidal angle. The poloidal angle, $\theta$, is related to the angular variables through the rotational transform, $\iota$, by the expression $\theta = \theta_0 + \iota\phi$. In this system, the guiding-center drift-kinetic equation may be expressed as

$$\frac{v_\parallel}{R}\frac{\partial F}{\partial \phi} + \dot{\theta}_0 \frac{\partial F}{\partial \theta_0} + \dot{r}\frac{\partial F}{\partial r} = \nu B^{-1} v_\parallel \frac{\partial}{\partial \mu}\left(m v_\parallel \mu \frac{\partial F}{\partial \mu}\right) , \tag{1}$$

where $v_\parallel$ is the particle velocity along a field line, $dl = Rd\phi$ is the differential distance along a field line, $\nu$ is the 90 degree deflection frequency and $\mu = mv_\perp^2/2B$ is the magnetic moment. The magnitude of the magnetic field, $B$, is given by

$$\frac{B}{B_0} = -\delta(r)\cos Np\phi + \sum_{m,\ell} C_{m,\ell}(r)\cos \ell\theta \cos mp\phi + \sum_{m,\ell} S_{m,\ell}(r) \sin \ell\theta \sin mp\phi \ , \tag{2}$$

with $p$ the number of field periods of the stellarator and $N$ the number of discrete modular coils which make up each field period. The first term in this expression represents the modular ripple with magnitude $\delta$, while the Fourier harmonic expansions describe the toroidal and helical nature of the stellarator field. The only restriction on such harmonics is that they be of rather low order, i.e. $m, \ell \ll N$.

Given the density and temperature ranges at which Wendelstein VII-X is expected to operate, the bulk plasma ions will find themselves in the long-mean-free-path or *collisionless* regime. In other words, the bounce frequency of ions trapped in modular ripple wells, $\omega_\delta \approx \sqrt{\delta} v_{th} Np/(2\pi R)$, is much greater than the frequency with which particles are collisionally removed from such wells, $\nu_{eff} = \nu/\delta$. As this is the case, the longitudinal adiabatic invariant, $J = \oint m v_\parallel \, dl$, is a constant of a trapped particle's motion and equation (1) may be simplified by application of the bounce-averaging operator, $\oint dl/v_\parallel$.

The resulting bounce-averaged kinetic equation is then solved in lowest non-trivial order by expanding the distribution function in terms of a local Maxwellian, $F_M$, and a perturbation term, $f$. Finally, the ratio of $\dot{\theta}_0/\nu_{eff}$ is assumed to be small, an assumption valid for all but the most energetic ions given the expected plasma parameters of Wendelstein VII-X. With the above assumptions, the kinetic equation to be solved is

$$\frac{1}{qB_0 r} \frac{\partial B}{\partial \theta_0} \frac{\partial F_M}{\partial r} \frac{\partial}{\partial \mu}(\mu J) = \nu \frac{\partial}{\partial \mu}\left(\mu J \frac{\partial f}{\partial \mu}\right) . \tag{3}$$

The form of equation (3) is made possible since particles trapped in modular ripple wells have very low parallel velocity and hence $\dot{r} \approx -(\mu/qB_0 r)(\partial B/\partial \theta_0)$ and $(\partial/\partial\mu)(\mu J) \approx -\oint(\mu B/v_\parallel)dl$. One may then solve equation (3) in the regions of phase space where local modular ripple wells exist (i.e. the range of $\mu$ values which satisfy $\kappa/B_{max} < \mu < \kappa/B_{min}$, with $\kappa = mv^2/2$ and $B_{min}, B_{max}$ the local minimum and maximum values of $B$ which define the ripple well) with the boundary conditions that $\partial f/\partial \mu$ be finite in the region of solution and that $f = 0$ for non-ripple-trapped particles. The solution for $f$ is

$$f = \frac{1}{qB_0 r} \frac{\partial B}{\partial \theta_0} \frac{\partial F_M}{\partial r} \frac{(\mu - \kappa/B_{max})}{\nu} , \tag{4}$$

and the associated particle flux is given by

$$\Gamma_\delta = \frac{1}{\pi (2m^3)^{1/2}} \int d\theta_0 \int d\phi \int d\kappa \int d\mu \, \frac{B}{\sqrt{\kappa - \mu B}} \dot{r} f . \tag{5}$$

Carrying out the $\mu$ integration, the particle flux may be written in the form $\Gamma_\delta = A_\delta(\theta_0, \phi) W(\kappa)$, where

$$A_\delta(\theta_0, \phi) = \frac{2\sqrt{2}}{15\pi} \int_0^{2\pi} d\theta_0 \int_0^{2\pi} d\phi \left(\frac{1}{B\epsilon_t} \frac{\partial B}{\partial \theta_0}\right)^2 (1 - B/B_{max})^{3/2} (4 + B/B_{max}) , \tag{6}$$

where $\epsilon_t = r/R_0$ is the inverse aspect ratio of the given flux surface. The quantity

$A_\delta(\theta_0, \phi)$ contains all information relating to the magnetic field structure and is thus a convenient quantity to use in describing the neoclassical transport characteristics of a given device.

A general analytic solution of equation (6) is impossible given the very complicated magnetic field geometry introduced by equation (2). Difficulties appear not only in the expression for $B$ itself, but also in the accompanying deformation of individual modular ripples, greatly complicating any analytic expression for $B_{max}$. An expression very similar to equation (6) has been analytically evaluated by several authors for rippled tokamaks in limiting cases for simple magnetic field profiles [2-4]; in the general case, these authors resorted to numerical integration. The latter approach will also be adopted here, except that no analytic form for $B_{max}$ will be derived or assumed. Instead, for each point on a flux surface, the condition $\partial B/\partial \phi = 0$ along a field line is *numerically* solved to determine whether a local modular ripple well exists at that point, and if so, what the local value of $B_{max}$ is.

The approach described above is used to compare the level of neoclassical transport due to particles trapped in the modular ripple with that due to particles trapped in the stellarator's *helical* ripple. The comparison will be made for the Wendelstein VII-X candidate designated HS5-7, a five field period Helias with rotational transform varying from $\iota\!\!\!- = .76$ on axis to $\iota\!\!\!- = .96$ at the plasma edge. The principal Fourier harmonics of the HS5-7 magnetic field, shown in Figure 1, place this configuration in the class of *transport-optimized* stellarators [5]. As such, the geometrical factor $A_h$ which characterizes *helical* ripple transport is greatly reduced

$$A_h \approx \frac{64}{9}\epsilon_h^{3/2}\left(\frac{C_{0,1}}{\epsilon_t}\right)^2\left\{1 - \frac{6}{5}\frac{\sigma\epsilon_h}{C_{0,1}} + 0.385\left(\frac{\sigma\epsilon_h}{C_{0,1}}\right)^2\right\}, \tag{7}$$

where $\epsilon_h$ and $\sigma$ are averages over the magnitudes of the various magnetic-field harmonics. (For the simplest model stellarator field, $\epsilon_h$ is identified with the magnitude of the helical ripple, $\sigma = 0$, $C_{0,1} = \epsilon_t$ and thus $A_h = 64\epsilon_h^{3/2}/9$.)

It is possible to realize the magnetic field of HS5-7 (or any other Helias) through a number of different modular coil configurations, differing in the number of discrete coils per field period, $N$. The principal magnetic-field harmonics are essentially identical in these various cases, only the magnitude of the modular ripple, $\delta$, and its periodicity are effected. Large values of $N$ reduce $\delta$ significantly but also greatly limit experimental access to the plasma. Ideally, the value of $N$ should be as small as possible to maximize access, but also large enough to insure that modular-ripple transport does not spoil the optimization represented by equation (7).

The HS5-7 configuration is currently being considered in $N = 10$ and $N = 12$ versions. The relative magnitude of the modular ripple in each case is indicated by the dashed lines in Figure 1. The ratio of $A_\delta/A_h$ is also calculated in the two cases as a function of plasma radius and presented in Figure 2. The $N = 10$ version shows values of this ratio considerably in excess of one near the plasma edge. On the other hand, the ratio of $A_\delta/A_h < 1$ at all radii for the $N = 12$ version of HS5-7. As this latter version also allows sufficient plasma access, 12 coils per field period is deemed to be the optimum value for HS5-7.

## REFERENCES

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## FIGURE CAPTIONS

FIGURE 1. Fourier components of the HS5-7 magnetic field are plotted vs. normalized radius. The magnitude of the *modular* ripple appears as the dashed line for configurations with both 10 and 12 modular coils per field period.

FIGURE 2. Ratio of $A_\delta/A_h$ vs. normalized radius for the Wendelstein VII-X candidate HS5-7. Results are presented for configurations with both 10 and 12 modular coils per field period.