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On curvature squared terms in $\mathcal{N} = 2$ supergravity

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Abstract

We present the $\mathcal{N} = 2$ supersymmetric completion of a scalar curvature squared term in a completely gauge independent form. We also elaborate on its component structure.

1 Introduction

Recently, there has been renewed interest in R^2 gravity [1, 2, 3] and $\mathcal{N} = 1$ supergravity [4]. In particular, it has been confirmed that the pure R^2 gravity theory is ghost free [3]. This provides a rationale to look more closely at the structure of curvature squared terms in four-dimensional (4D) $\mathcal{N} = 2$ supergravity. On dimensional grounds, all such terms should be given by chiral integrals.

The $\mathcal{N} = 2$ locally supersymmetric invariant $I_{C^2_{abcd}}$ containing the Weyl tensor squared (which coincides with the action for $\mathcal{N} = 2$ conformal supergravity) was constructed by Bergshoeff, de Roo and de Wit almost thirty five years ago [5]. However, the $\mathcal{N} = 2$ supersymmetric extension $I_{R^2_{ab}-\frac{1}{3}R^2}$ of the term $R^{ab}R_{ab} - \frac{1}{3}R^2$ was obtained only two years ago [6]. A special combination of the super-Weyl invariants $I_{C^2_{abcd}}$ and $I_{R^2_{ab}-\frac{1}{3}R^2}$ constitutes the $\mathcal{N} = 2$ Gauss-Bonnet term [6]. In this note we describe a third curvature squared invariant – a locally supersymmetric extension of the R^2 term that is of special interest in the context of the ideas advocated in [1, 2, 3, 4]. Although the invariant has been discussed in [7, 8], there has not appeared a complete description of the invariant due to some missing elements. In particular, the invariant was given in [7] in a special gauge and the explicit component action has never been worked out. In this note we provide a more complete exposition of the invariant and construct it in a gauge independent form.

This note is organised as follows. In section 2 we present the superspace description of curvature squared invariants within $\mathcal{N} = 2$ superspace. In section 3 we elaborate on the component structure of a $\mathcal{N} = 2$ supersymmetric invariant containing a curvature squared term. Section 4 is devoted to a discussion of our results.

We have included a couple of technical appendices. Appendix A provides the essential details of the formulation for $\mathcal{N} = 2$ conformal supergravity [9] in SU(2) superspace [10], while Appendix B summarises the important details of conformal superspace [11].

2 The curvature squared invariants in superspace

In this section we use the formulation for $\mathcal{N} = 2$ conformal supergravity [9] in SU(2) superspace [10]. Some technical details concerning this supergravity formulation are collected in Appendix A. We proceed by recalling the explicit structure of the invariants $I_{C^2_{abcd}}$ and $I_{R^2_{ab}-\frac{1}{3}R^2}$.

The invariant containing the Weyl tensor squared is

$$I_{C^2_{abcd}} = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}\,W^{\alpha\beta}W_{\alpha\beta} \;+\; \text{c.c.}\;, \tag{2.1}$$

where $W_{\alpha\beta}$ is the super-Weyl tensor, see Appendix A, and $\mathcal{E}$ is the chiral density, see, e.g., [12] for the definition of $\mathcal{E}$.

The invariant containing $R^{ab}R_{ab} - \frac{1}{3}R^2$ is

$$I_{R^2_{ab}-\frac{1}{3}R^2} = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}\,\Xi \;+\; \text{c.c.}\;, \tag{2.2}$$

where Ξ denotes the following composite scalar [6]:

$$\Xi := \frac{1}{6}\bar{\mathcal{D}}^{ij}\bar{S}_{ij} + \bar{S}^{ij}\bar{S}_{ij} + \bar{Y}_{\dot\alpha\dot\beta}\bar{Y}^{\dot\alpha\dot\beta}\;, \qquad \bar{\mathcal{D}}_{ij} := \bar{\mathcal{D}}_{\dot\alpha(i}\bar{\mathcal{D}}^{\dot\alpha}_{j)}\;. \tag{2.3}$$

The torsion superfields S_{ij}, $W_{\alpha\beta}$ and $Y_{\alpha\beta}$ and their conjugates $\bar{S}^{ij}$, $\bar{W}_{\dot\alpha\dot\beta}$ and $\bar{Y}_{\dot\alpha\dot\beta}$ are defined in Appendix A. The fundamental properties of Ξ are as follows [6]:
(i) it is covariantly chiral,

$$\bar{\mathcal{D}}^{\dot\alpha}_i\Xi = 0\;; \tag{2.4}$$

(ii) its super-Weyl transformation is

$$\delta_\sigma\Xi = 2\sigma\Xi - 2\bar\Delta\bar\sigma\;. \tag{2.5}$$

Here $\bar\Delta$ denotes the chiral projection operator [12, 13]

$$\begin{aligned}\bar\Delta &= \frac{1}{96}\Big((\bar{\mathcal{D}}^{ij}+16\bar{S}^{ij})\bar{\mathcal{D}}_{ij} - (\bar{\mathcal{D}}^{\dot\alpha\dot\beta} - 16\bar{Y}^{\dot\alpha\dot\beta})\bar{\mathcal{D}}_{\dot\alpha\dot\beta}\Big)\\ &= \frac{1}{96}\Big(\bar{\mathcal{D}}_{ij}(\bar{\mathcal{D}}^{ij}+16\bar{S}^{ij}) - \bar{\mathcal{D}}_{\dot\alpha\dot\beta}(\bar{\mathcal{D}}^{\dot\alpha\dot\beta} - 16\bar{Y}^{\dot\alpha\dot\beta})\Big)\;,\end{aligned} \tag{2.6}$$

with $\bar{\mathcal{D}}^{\dot\alpha\dot\beta} := \bar{\mathcal{D}}^{(\dot\alpha}_k\bar{\mathcal{D}}^{\dot\beta)k}$. The main properties of $\bar\Delta$ can be formulated using a super-Weyl inert scalar U as follows:

$$\bar{\mathcal{D}}^{\dot\alpha}_i\bar\Delta U = 0\;, \tag{2.7a}$$

$$\delta_\sigma U = 0 \quad\Longrightarrow\quad \delta_\sigma\bar\Delta U = 2\sigma\bar\Delta U\;, \tag{2.7b}$$

$$\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathrm{d}^4\bar\theta\,E\,U = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}\,\bar\Delta U\;. \tag{2.7c}$$

Here E denotes the full superspace density.

The super-Weyl invariance of (2.2) follows from the relations (2.5) and (2.7c) in conjunction with the identity

$$\bar{\mathcal{D}}^{\dot\alpha}_i\sigma = 0 \quad\Longrightarrow\quad \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathrm{d}^4\bar\theta\,E\,\sigma = 0\;, \tag{2.8}$$

for any covariantly chiral scalar σ.

As shown in [6], the functional

$$\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}\,\Big\{W^{\alpha\beta}W_{\alpha\beta} - \Xi\Big\} \tag{2.9}$$

is a topological invariant being related to the difference of the Gauss-Bonnet and Pontryagin invariants.

The specific feature of the invariants (2.1) and (2.2) is that they do not involve any conformal compensator.[1] However, such a compensator is required in order to construct a supersymmetric extension of the R^2 term, and it should be the improved tensor multiplet [15].

The tensor (or *linear*) multiplet can be described in curved superspace by its gauge invariant field strength $\mathcal{G}^{ij}$ which is defined to be a real SU(2) triplet subject to the covariant constraints [16, 17]

$$\mathcal{D}_\alpha^{(i}\mathcal{G}^{jk)} = \bar{\mathcal{D}}_{\dot\alpha}^{(i}\mathcal{G}^{jk)} = 0 ~. \tag{2.10}$$

These constraints are solved in terms of a chiral prepotential Ψ [18, 19, 20, 21] via

$$\mathcal{G}^{ij} = \frac{1}{4}\Big(\mathcal{D}^{ij} + 4S^{ij}\Big)\Psi + \frac{1}{4}\Big(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}\Big)\bar{\Psi} ~, \qquad \bar{\mathcal{D}}^i_{\dot\alpha}\Psi = 0 ~, \tag{2.11}$$

which is invariant under shifts $\Psi \to \Psi + \mathrm{i}\Lambda$, with Λ a *reduced chiral* superfield,[2]

$$\bar{\mathcal{D}}^{\dot\alpha}_i\Lambda = 0 ~, \qquad \Big(\mathcal{D}^{ij} + 4S^{ij}\Big)\Lambda = \Big(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}\Big)\bar{\Lambda} ~. \tag{2.12}$$

The super-Weyl transformation laws of Ψ and $\mathcal{G}^{ij}$ are

$$\delta_\sigma\Psi = \sigma\Psi \quad \Longrightarrow \quad \delta_\sigma\mathcal{G}^{ij} = (\sigma + \bar\sigma)\mathcal{G}^{ij} ~. \tag{2.13}$$

The improved tensor multiplet is characterised by the condition $\mathcal{G}^2 := \frac{1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij} \neq 0$.

Using the improved tensor multiplet one can construct the following reduced chiral superfield $\mathbb{W}$:

$$\begin{aligned}\mathbb{W} &= -\frac{1}{24\mathcal{G}}(\bar{\mathcal{D}}_{ij} + 12\bar{S}_{ij})\mathcal{G}^{ij} + \frac{1}{36\mathcal{G}^3}\bar{\mathcal{D}}_{\dot\alpha k}\mathcal{G}^{ki}\bar{\mathcal{D}}^{\dot\alpha}_l\mathcal{G}^{lj}\mathcal{G}_{ij} \\ &= -\frac{\mathcal{G}}{8}(\bar{\mathcal{D}}_{ij} + 4\bar{S}_{ij})\frac{\mathcal{G}^{ij}}{\mathcal{G}^2} ~.\end{aligned} \tag{2.14}$$

[1]Super-Weyl anomalies in $\mathcal{N}=2$ superconformal theories coupled to supergravity [14] are given by linear combinations of the integrands in (2.1) and (2.2).

[2]The field strength $\mathcal{W}$ of an Abelian vector multiplet is a reduced chiral superfield.

The regular procedure to derive $\mathbb{W}$ is described in [22]. This multiplet (up to normalisations) was discovered originally in [15] using superconformal tensor calculus. It was later reconstructed in curved superspace [21] with the aid of the results in [15] and [23].

The supersymmetric completion of the R^2 term will prove to be described by the invariant

$$I_{R^2} = \int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathcal{E} \, \mathbb{W}^2 + \text{c.c.} \tag{2.15}$$

In the next section we will explicitly show that the above invariant does indeed contain a R^2 term at the component level.

The scalar curvature squared invariant (2.15) is analogous to the one constructed in five dimensions in [24]. There a supersymmetric completion of a R^2 term was obtained by considering the Chern-Simons coupling between a vector multiplet and two identical composite vector multiplets constructed out of the tensor multiplet. A similar procedure was performed in superspace in [25]. In contrast to five dimensions the component action corresponding to (2.15) contains a R^2 term without the need to impose any gauge condition.

3 Supersymmetric invariants in components

The curvature squared invariants (2.1) and (2.2) are independent of any compensator and were reduced to components in [6]. In order to perform component reduction of the R^2 invariant (2.15) it is advantageous to lift the superspace actions to conformal superspace.

Besides the R^2 invariant (2.15) is it also worth elaborating on the component structure of the tensor multiplet action [15], which in the formulation of [9] is given by

$$S_{\text{tensor}} = -\int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathcal{E} \, \Psi \mathbb{W} + \text{c.c.} \; , \tag{3.1}$$

where $\mathbb{W}$ is given in eq. (2.14). The above action can be shown to contain an Einstein-Hilbert term upon imposing a certain gauge.

In this section we first lift the descriptions of the tensor multiplet action and the R^2 invariant to conformal superspace and then reduce them to components. The salient details of conformal superspace are summarised in Appendix B.

3.1 Invariants in conformal superspace

The tensor multiplet is described in conformal superspace by a real primary superfield $\mathcal{G}^{ij} = \mathcal{G}^{ji}$ of dimension 2,

$$\mathbb{D}\mathcal{G}^{ij} = 2\mathcal{G}^{ij} \ , \quad K_A \mathcal{G}^{ij} = 0 \ , \tag{3.2}$$

satisfying the constraint

$$\nabla_\alpha^{(i}\mathcal{G}^{jk)} = 0 \ . \tag{3.3}$$

The tensor multiplet can be described by a two-form gauge potential. Its superform formulation in conformal superspace can be found in [26]. The tensor multiplet can be solved in terms of an unconstrained chiral prepotential Ψ,

$$\mathcal{G}^{ij} = \frac{1}{4}\nabla^{ij}\Psi + \frac{1}{4}\bar{\nabla}^{ij}\bar{\Psi} \ , \tag{3.4}$$

where we have defined $\nabla^{ij} := \nabla^{\alpha(i}\nabla_\alpha^{j)}$.

We can lift the superspace expressions for the tensor multiplet and the scalar curvature squared actions to conformal superspace. In conformal superspace, the tensor multiplet action is defined by (3.1) but with the composite $\mathbb{W}$ now constructed with the covariant derivative of conformal superspace:

$$\mathbb{W} = -\frac{1}{24\mathcal{G}}\bar{\nabla}_{ij}\mathcal{G}^{ij} + \frac{1}{36\mathcal{G}^3}\bar{\nabla}_{\dot{\alpha}k}\mathcal{G}^{ki}\bar{\nabla}^{\dot{\alpha}}_l\mathcal{G}^{lj}\mathcal{G}_{ij} = -\frac{\mathcal{G}}{8}\bar{\nabla}_{ij}\frac{\mathcal{G}^{ij}}{\mathcal{G}^2} \ . \tag{3.5}$$

One can check that $\mathbb{W}$ is indeed a vector multiplet since it is a primary superfield of dimension 1 satisfying the reduced chiral constraints

$$\bar{\nabla}^i_{\dot{\alpha}}\mathbb{W} = 0 \ , \quad \nabla^{ij}\mathbb{W} = \bar{\nabla}^{ij}\bar{\mathbb{W}} \ . \tag{3.6}$$

The above expression for $\mathbb{W}$ degauges to the one given by eq. (2.14) upon using the degauging procedure given in [11].

It is important to note that the action (3.1) only involves $\mathcal{G}^{ij}$ without a compensating vector multiplet. This is in contrast to, for example, the $\mathcal{N} = 2$ supersymmetric BF action[3] which is described by

$$S_{BF} = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \Psi\mathcal{W} + \text{c.c.} \ , \tag{3.7}$$

[3] In general, a BF theory on a d-dimensional orientable manifold is a Schwarz-type topological gauge theory with action $S_{(d,n)} = \int B_n \wedge \mathrm{d}A_{d-n-1} = \int B_n \wedge F_{d-n}$, where B_n and A_{d-n-1} are differential forms and F_{d-n} is the gauge invariant field strength associated with A_{d-n-1}. For a review of BF theories, see [27]. The action (3.7) is a supersymmetric generalisation of $S_{(4,2)}$.

where $\mathcal{W}$ is the chiral field strength of a vector multiplet.

In conformal superspace, the scalar curvature squared action is described by eq. (2.15) but with the composite $\mathbb{W}$ replaced by the one given in eq. (3.5). It is important to emphasise that the invariant (2.15) has the same type as the vector multiplet action,

$$S_{\text{vector}} = \int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathcal{E} \, \mathcal{W}^2 + \text{c.c.} \tag{3.8}$$

with the vector superfield strength replaced by the composite $\mathbb{W}$. The component vector multiplet and tensor multiplet actions were given in [26] in our notation and conventions.[4] In the next subsection we apply the results of [26] to elaborate on the component structure of the invariants S_{tensor} and I_{R^2}.

3.2 The tensor multiplet action and scalar curvature squared invariant in components

Here we identify the component fields of the Weyl multiplet of conformal supergravity in accordance with [11]. The vierbein $e_m{}^a$, the gravitino $\psi_m{}_i^\alpha$, the U(1) and SU(2) gauge fields A_m and $\phi_m{}^{ij}$, and the dilatation gauge field b_m are defined as follows:

$$\begin{aligned} e_m{}^a &:= E_m{}^a| \ , \quad \psi_m{}_i^\alpha := 2E_m{}_i^\alpha| \ , \quad \bar{\psi}_m{}^i_{\dot\alpha} := 2E_m{}^i_{\dot\alpha}| \ , \\ A_m &:= \Phi_m| \ , \quad \phi_m{}^{ij} := \Phi_m{}^{ij}| \ , \quad b_m := B_m| \ . \end{aligned} \tag{3.9}$$

The component (or bar) projection of a superfield $V(z)$ is defined in the usual way $V| := V(z)|_{\theta=\bar\theta=0}$. There are several *composite* gauge connections. The spin connection $\omega_m{}^{ab}$, the special conformal $\mathfrak{f}_m{}^a$ and S-supersymmetry connections $\phi_m{}^i_\alpha$,

$$\omega_m{}^{ab} := \Omega_m{}^{ab}| \ , \quad \mathfrak{f}_m{}^a := \mathfrak{F}_m{}^a| \ , \quad \phi_m{}^i_\alpha := 2\mathfrak{F}_m{}^i_\alpha| \ , \tag{3.10}$$

are all composed of the previously defined component fields. Their expressions can be found in [11, 26].

The Weyl multiplet also contains some non-gauge fields. These are encoded in the components of $W_{\alpha\beta}$ as follows:

$$W_{ab} = W^+_{ab} + W^-_{ab} \ , \quad W^+_{ab} := (\sigma_{ab})^{\alpha\beta} W_{\alpha\beta}| \ , \quad W^-_{ab} := -(\tilde\sigma_{ab})_{\dot\alpha\dot\beta} \bar{W}^{\dot\alpha\dot\beta}| \ , \tag{3.11a}$$

$$\Sigma^{\alpha i} := \frac{1}{3}\nabla^i_\beta W^{\alpha\beta}| \ , \quad D := \frac{1}{12}\nabla^{\alpha\beta}W_{\alpha\beta}| = \frac{1}{12}\bar\nabla_{\dot\alpha\dot\beta}\bar{W}^{\dot\alpha\dot\beta}| \ . \tag{3.11b}$$

[4]However, here we denote the Lorentz curvature constructed from the spin connection by $R_{ab}{}^{cd}$ instead of $\hat{\mathcal{R}}_{ab}{}^{cd}$.

The component field $W^{\pm}_{ab}$ satisfies the self-duality relation $\frac{\mathrm{i}}{2}\varepsilon_{ab}{}^{cd}W^{\pm}_{cd} = \pm W^{\pm}_{ab}$. As in [26], to avoid cluttered notation, we will often use $W_{\alpha\beta}$ also for the corresponding component field. It should be clear from context to which we are referring. In what follows, we will also make use of the following bosonic covariant derivative:

$$\nabla'_a = e_a{}^m\Big(\partial_m + \frac{1}{2}\omega_m{}^{bc}M_{bc} + \phi_m{}^{ij}J_{ij} + \mathrm{i}A_mY + b_m\mathbb{D}\Big) \ . \tag{3.12}$$

The matter components of the tensor multiplet are defined as follows:

$$G^{ij} := \mathcal{G}^{ij}| \ , \quad \chi_{\alpha i} := \frac{1}{3}\nabla^j_\alpha \mathcal{G}_{ij}| \ , \quad F := \frac{1}{12}\nabla^{ij}\mathcal{G}_{ij}| \ . \tag{3.13}$$

There is also an additional component field, the two-form b_{mn}. Its supercovariant field strength is given by

$$\begin{aligned}\tilde{h}^a &= \frac{\mathrm{i}}{24}(\sigma^d)^\alpha{}_{\dot\beta}[\nabla^i_\alpha, \bar\nabla^{\dot\beta}_j]\mathcal{G}^j{}_i| \\ &= \frac{1}{2}\varepsilon^{abcd}(\frac{1}{3}h_{bcd} - \mathrm{i}(\sigma_{cd})_\alpha{}^\beta\psi_b{}^\alpha_k\chi^k_\beta - \mathrm{i}(\tilde\sigma_{cd})^{\dot\alpha}{}_{\dot\beta}\bar\psi_b{}^k_{\dot\alpha}\bar\chi^{\dot\beta}_k + (\sigma_b)_\alpha{}^{\dot\beta}\psi_c{}^\alpha_k\bar\psi_d{}^l_{\dot\beta}G^k{}_l) \ ,\end{aligned} \tag{3.14}$$

where

$$h_{abc} = 3e_a{}^me_b{}^ne_c{}^p\partial_{[m}b_{np]} \ . \tag{3.15}$$

The tensor multiplet action was reduced from conformal superspace to components in [26]. The action up to fermion contributions is

$$\begin{aligned}S_{\text{tensor}} = \int \mathrm{d}^4x\, e\,\Big(&\frac{1}{2G}|F|^2 - G\Big(\frac{1}{3}R + D\Big) - \frac{1}{2G}\Big(\tilde{h}^a\tilde{h}_a - G_{ij}\nabla'_a\nabla'^aG^{ij}\Big) \\ &-\frac{1}{4G^3}G_{ij}\nabla'^aG^{ik}\nabla'_aG^{jl}G_{kl} + \frac{1}{2}\varepsilon^{mnpq}b_{mn}f_{pq}\Big) + \text{fermion terms} \ ,\end{aligned} \tag{3.16}$$

where

$$f_{mn} = 2\partial_{[m}\Gamma_{n]} + \frac{1}{4G^3}\partial_mG^{ik}\partial_nG_k{}^jG_{ij} \ , \quad G = \mathcal{G}| \tag{3.17}$$

and

$$\Gamma_m := \frac{1}{2G}\phi_m{}^{ij}G_{ij} + \frac{1}{2G}e_m{}^a\tilde{h}_a + \text{fermion terms} \ . \tag{3.18}$$

We see that in the gauge where $G = 1$, S_{tensor} contains the Einstein-Hilbert term with a wrong sign.

The component form for the invariant I_{R^2} may be obtained by replacing the component fields of the vector multiplet in the vector multiplet action in [26] with the component fields of the composite $\mathbb{W}$. The invariant up to fermionic contributions is

$$I_{R^2} = \int \mathrm{d}^4x\, e\,\Big(- \nabla'^a(G^{-1}F)\nabla'_a(G^{-1}\bar{F}) + \frac{1}{G^2}|F|^2D - \frac{1}{6G^2}R|F|^2$$

$$
+\frac{1}{8}X^{ij}X_{ij} - 2f^{ab}f_{ab} - \frac{2}{G}(\sigma^{ab})_{\dot\alpha\dot\beta}\bar{F}\bar{W}^{\dot\alpha\dot\beta}f_{ab} + \frac{2}{G}(\sigma^{ab})_{\alpha\beta}FW^{\alpha\beta}f_{ab}
$$

$$
-\frac{1}{2G^2}\bar{W}_{\dot\alpha\dot\beta}\bar{W}^{\dot\alpha\dot\beta}\bar{F}^2 - \frac{1}{2G^2}W^{\alpha\beta}W_{\alpha\beta}F^2\Big) + \text{fermion terms} \ , \qquad (3.19)
$$

where

$$
X^{ij} = \frac{1}{G}\Big(-2\nabla'_a\nabla'^a G^{ij} + 2DG^{ij} + \frac{2}{3}RG^{ij}\Big) + \frac{1}{G^3}\Big(\nabla'^a G^{ik}\nabla'_a G^{jl}G_{kl}
$$

$$
+\tilde{h}^a\tilde{h}_a G^{ij} + G^{ij}F\bar{F} - 2\tilde{h}^a\nabla'_a G^{k(i}G^{j)}{}_k\Big) + \text{fermion terms} \qquad (3.20)
$$

and

$$
f_{ab} = e_a{}^m e_b{}^n f_{mn} \ . \qquad (3.21)
$$

One can see that a R^2 term arises in the $X^{ij}X_{ij}$ contribution to the invariant (3.19). It should be emphasised that the invariant (3.19) is independent of any gauge choice.

4 Discussion

In this paper we have completed the description of all off-shell curvature squared invariants in $\mathcal{N}=2$ supergravity. Such invariants are described by the linear combination

$$
I = AI_{C^2_{abcd}} + BI_{R^2_{ab}-\frac{1}{3}R^2} + CI_{R^2} \ , \qquad (4.1)
$$

where A, B and C are real parameters, and the invariants $I_{C^2_{abcd}}$, $I_{R^2_{ab}-\frac{1}{3}R^2}$ and I_{R^2} are given by the equations (2.1), (2.2) and (2.15), respectively. The bosonic sector of $I_{R^2} = \int \mathrm{d}^4x\, e\, L_{R^2}$ requires some discussion.

First of all, let us consider the part of L_{R^2} containing the auxiliary scalar D:

$$
L_{R^2} = -\frac{D}{2G^2}\Big(-4\nabla'^a G\nabla'_a G + \nabla'^a G^{ij}\nabla'_a G_{ij} - 2\tilde{h}^a\tilde{h}_a
$$

$$
+2\nabla'^a\nabla'_a G^{ij}G_{ij} - \frac{4}{3}RG^2 - 4|F|^2\Big) + D^2 + \cdots \ , \qquad (4.2)
$$

where the ellipsis represents terms not directly involving D. We see that the equation of motion for D is consistent and allows one to integrate D out. To understand the importance of this result, it is worth recalling why two compensators are required in ordinary $\mathcal{N}=2$ supergravity [15]. The D-terms in the vector and tensor multiplet Lagrangians are

$$
L_{\text{vector}} = 4|\phi|^2 D + \cdots \ , \qquad L_{\text{tensor}} = -DG + \cdots \ , \qquad (4.3)
$$

where $\phi = \mathcal{W}|$. It is seen that the equation of motion for D is contradictory if one chooses $L_{\text{SG}} = -L_{\text{vector}}$ or $L_{\text{SG}} = -L_{\text{tensor}}$. However, the supergravity Lagrangian

$L_{\rm SG} = -L_{\rm vector} - L_{\rm tensor}$ leads to a sensible equation of motion for D, which is $G = 4|\phi|^2$. Now, looking at the Lagrangian (4.2), we see that we can circumvent the need of having two compensators. We can have the invariant I_{R^2} on its own or we can have a linear combination of I_{R^2} and $S_{\rm tensor}$, and still have consistent dynamics.

The second important feature of I_{R^2} concerns the component field $W^{\alpha\beta}$ and its conjugate, which are auxiliary in ordinary $\mathcal{N} = 2$ supergravity. The equation of motion for $W^{\alpha\beta}$, which is derived from I_{R^2}, allows one to integrate $W_{\alpha\beta}$ out (if we assume $F \neq 0$) giving

$$I_{R^2} = \int {\rm d}^4x\, e \Big(- \nabla'^a(G^{-1}F)\nabla'_a(G^{-1}\bar{F}) + \frac{1}{G^2}|F|^2 D \\ -\frac{1}{6G^2}R|F|^2 + \frac{1}{8}X^{ij}X_{ij} + 2f^{ab}f_{ab} + \text{fermion terms}\Big) . \tag{4.4}$$

Comparing with eq. (3.19), one notices that there is a sign flip in the $f^{ab}f_{ab}$ term. In the above action the auxiliary field has not yet been eliminated. In ordinary supergravity F was auxiliary, however, now it becomes dynamical. It should also be noted that upon eliminating the auxiliary field D in the combined invariant of the form $S_{\rm tensor}+I$, we generate not only an R^2 term but also an R and a potential $|F/G|^4$ term. This will be discussed below.

Let us further elaborate on the elimination of the auxiliary field D. To begin with we consider the invariant

$$\alpha S_{\rm vector} + \beta S_{\rm tensor} + \gamma I_{R^2} \ , \tag{4.5}$$

where α, β and γ are arbitrary constants. The bosonic component action can be constructed by using (3.16) and (3.19), and the vector multiplet action in [26]. One can check that upon eliminating the auxiliary field D, the R^2 term cancels.[5] It is worth mentioning that in five dimensions there also exists a R^2 invariant in the standard Weyl multiplet [24, 25], which involves an auxiliary scalar field.[6] Upon eliminating the auxiliary field, the R^2 term can be similarly shown to vanish.

The component action of (4.5) will contain the potential contribution

$$-\frac{1}{\gamma}\Big(\frac{\alpha}{2}G - 2\beta|\phi|^2\Big)^2 \ , \tag{4.6}$$

as well as the term

$$-\frac{4}{3}\beta R|\phi|^2 \ . \tag{4.7}$$

[5] We are grateful to Sergey Ketov for pointing out this observation.

[6] A ρR^2 invariant in the standard Weyl multiplet was given in [24] since a different gauge condition was used.

Upon imposing an appropriate gauge, the first term contains a cosmological term, while the second gives rise to an Einstein-Hilbert term in the gauge $\phi = \text{const}$.

Although the invariant I_{R^2} does not give rise to a pure R^2 term alone upon integrating out the auxiliary field D, it can still lead to a non-trivial R^2 contribution if one adds to it another invariant. For instance, one can make use of the invariant (2.2) and consider the linear combination

$$\mathcal{A} S_{\text{tensor}} + \mathcal{B} I_{R^2} + \mathcal{C} I_{R^2_{ab} - \frac{1}{3} R^2} \ . \tag{4.8}$$

The invariant $I_{R^2_{ab} - \frac{1}{3} R^2}$ contains a D^2 term and no RD term at the component level, see [6]. This allows one to keep a R^2 contribution from I_{R^2} upon eliminating the auxiliary field D. The invariant will also obtain an Einstein-Hilbert term and a cosmological constant in a gauge where $G = 1$.

It should be mentioned that one can fix the special conformal transformations, dilatations and break the SU(2) R-symmetry down to U(1) by imposing the following gauge conditions on the improved tensor multiplet

$$B_A = 0 \ , \quad \mathcal{G}^{ij} = \delta^{ij} \mathcal{G} \ , \quad \mathcal{G} = 1 \ . \tag{4.9}$$

These conditions correspond to the following choice for the component fields:

$$b_m = 0 \ , \quad G = 1 \ , \quad \chi_\alpha^i = \bar{\chi}^{\dot{\alpha}}_i = 0 \ , \quad G_{ij} = \delta_{ij} G \ . \tag{4.10}$$

The first gauge choice fixes the special conformal summetry, the second fixes dilatations, the third fixes the S-supersymmetry transformations and the last breaks the SU(2) R-symmetry to U(1). Upon imposing the above gauge choice, the R^2 invariant (2.15) coincides with the R^2 invariant in [7], which was only specified in the above gauge in superspace. Our invariant, however, is described in both conformal superspace and the conventional superspace formulation of [9] without specifying any gauge condition on the compensator. As demonstrated, it is also readily reduced to components using the results of [26].

It may be shown that no R^2 invariant can be constructed with a compensating vector multiplet only. However, one can generalise the invariant I_{R^2} by coupling it to some vector multiplets. One can consider the following simple generalisation of I_{R^2}:

$$\int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathcal{E} \, \mathcal{F} + \text{c.c.} \ , \tag{4.11}$$

where $\mathcal{F}$ is a homogeneous function of degree two in $\mathcal{W}^{\hat{I}} = (\mathbb{W}, \mathcal{W}^I)$ with $\mathcal{W}^I$ denoting a number of vector multiplets. Such invariants were considered only in the gauge (4.9) in [7]. Using the results of [26] one can reduce the action to components.

So far we have considered constructing a R^2 invariant using a single compensator. Is it possible to use both a compensating tensor and vector multiplet together to generate a R^2 term? In principle, other invariants may be constructed with the use of the results in [22]. For instance, one can consider a projective-superspace Lagrangian $\mathcal{L}^{++}(v)$ of the form

$$\mathcal{L}^{++} = \frac{(H^{++})^2}{G^{++}} \ , \qquad H^{++}(v) = H^{ij} v_i v_j \ , \qquad G^{++}(v) = G^{ij} v_i v_j \ , \tag{4.12}$$

where $H^{ij} = \nabla^{ij} \mathcal{W}$ and $v^i \in \mathbb{C}^2 \setminus \{0\}$ denotes homogeneous coordinates for $\mathbb{C}P^1$. Using the results of [22], the corresponding invariant may be cast in the form of a BF action, $\int \mathrm{d}^4 x \, \mathrm{d}^4 \theta \, \mathcal{E} \, \Psi \mathbb{W}_2 + \text{c.c.}$, where

$$\mathbb{W}_2 = -\frac{\mathcal{G}}{16} \bar{\nabla}_{ij} \Big\{ \frac{1}{\mathcal{G}^4} \Big(\delta^i_{(k} \delta^j_{l)} - \frac{1}{2\mathcal{G}^2} \mathcal{G}^{ij} \mathcal{G}_{kl} \Big) H^{(kl} H^{mn)} \mathcal{G}_{mn} \Big\} \ . \tag{4.13}$$

It can be shown that the invariant contains a $\frac{|\phi|^2}{G} R^2$ term at the component level. In pure supergravity, $|\phi|^2 = G$ on the mass shell, and then the above invariant gives a R^2 term. However, this condition does not hold in general. We conclude that our R^2 invariant (2.15) has no obvious alternative.

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A SU(2) superspace

This appendix contains a brief summary of the formulation for $\mathcal{N} = 2$ conformal supergravity [9] in SU(2) superspace [10]. Our notation and conventions follow those of [28].

To describe SU(2) superspace one begins with a curved $\mathcal{N} = 2$ superspace $\mathcal{M}^{4|8}$ parametrised by local coordinates $z^M = (x^m, \ \theta^\mu_\imath, \ \bar{\theta}^\imath_{\dot{\mu}} = (\theta_{\mu\imath})^*)$, where $m = 0, 1, \cdots, 3$, $\mu = 1, 2$, $\dot{\mu} = 1, 2$ and $\imath = \underline{1}, \underline{2}$. The structure group is chosen to be $\mathrm{SL}(2, \mathbb{C}) \times \mathrm{SU}(2)$, and the covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}_i)$ have the form

$$\begin{aligned} \mathcal{D}_A &= E_A + \Phi_A{}^{kl} J_{kl} + \frac{1}{2} \Omega_A{}^{bc} M_{bc} \\ &= E_A + \Phi_A{}^{kl} J_{kl} + \Omega_A{}^{\beta\gamma} M_{\beta\gamma} + \bar{\Omega}_A{}^{\dot{\beta}\dot{\gamma}} \bar{M}_{\dot{\beta}\dot{\gamma}} \ . \end{aligned} \tag{A.1}$$

Here $E_A = E_A{}^M(z)\partial_M$ is the supervielbein, with $\partial_M = \partial/\partial z^M$, $J_{kl} = J_{lk}$ are generators of the group SU(2) and M_{ab} are the Lorentz generators. The one-forms $\Omega_A{}^{bc}$ and $\Phi_A{}^{kl}$ are the Lorentz and SU(2) connections.

The generators act on the covariant derivatives as follows:

$$[M_{\alpha\beta}, \mathcal{D}^i_\gamma] = \varepsilon_{\gamma(\alpha}\mathcal{D}^i_{\beta)} \ , \qquad \big[J_{kl}, \mathcal{D}^i_\alpha\big] = -\delta^i_{(k}\mathcal{D}_{\alpha l)} \ . \tag{A.2}$$

The algebra of covariant derivatives is [9]

$$\begin{aligned} \{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} &= \quad 4S^{ij}M_{\alpha\beta} + 2\varepsilon^{ij}\varepsilon_{\alpha\beta}Y^{\gamma\delta}M_{\gamma\delta} + 2\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar{W}^{\dot\gamma\dot\delta}\bar{M}_{\dot\gamma\dot\delta} \\ &\quad +2\varepsilon_{\alpha\beta}\varepsilon^{ij}S^{kl}J_{kl} + 4Y_{\alpha\beta}J^{ij} \ , \qquad\qquad (\text{A.3a}) \\ \{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot\beta}_j\} &= -2\mathrm{i}\delta^i_j(\sigma^c)_\alpha{}^{\dot\beta}\mathcal{D}_c + 4\delta^i_jG^{\delta\dot\beta}M_{\alpha\delta} + 4\delta^i_jG_{\alpha\dot\gamma}\bar{M}^{\dot\gamma\dot\beta} + 8G_\alpha{}^{\dot\beta}J^i{}_j \ . \quad (\text{A.3b}) \end{aligned}$$

The explicit expressions for the commutator $[\mathcal{D}_a, \mathcal{D}^j_\beta]$ can be found in [9]. Here the real four-vector $G_{\alpha\dot\alpha}$, the complex symmetric tensors $S^{ij} = S^{ji}$, $W_{\alpha\beta} = W_{\beta\alpha}$, $Y_{\alpha\beta} = Y_{\beta\alpha}$ and their complex conjugates $\bar{S}_{ij} := \overline{S^{ij}}$, $\bar{W}_{\dot\alpha\dot\beta} := \overline{W_{\alpha\beta}}$, $\bar{Y}_{\dot\alpha\dot\beta} := \overline{Y_{\alpha\beta}}$ are constrained by certain Bianchi identities [10, 9]. The latter comprise the dimension-3/2 identities

$$\mathcal{D}^{(i}_\alpha S^{jk)} = \bar{\mathcal{D}}^{(i}_{\dot\alpha} S^{jk)} = 0 \ , \tag{A.4a}$$

$$\bar{\mathcal{D}}^{\dot\alpha}_i W_{\beta\gamma} = 0 \ , \tag{A.4b}$$

$$\mathcal{D}^i_{(\alpha} Y_{\beta\gamma)} = 0 \ , \tag{A.4c}$$

$$\mathcal{D}^i_\alpha S_{ij} + \mathcal{D}^\beta_j Y_{\beta\alpha} = 0 \ , \tag{A.4d}$$

$$\mathcal{D}^i_\alpha G_{\beta\dot\beta} = -\frac{1}{4}\bar{\mathcal{D}}^i_{\dot\beta} Y_{\alpha\beta} + \frac{1}{12}\varepsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot\beta j}S^{ij} - \frac{1}{4}\varepsilon_{\alpha\beta}\bar{\mathcal{D}}^{\dot\gamma i}\bar{W}_{\dot\beta\dot\gamma} \ , \tag{A.4e}$$

as well as the dimension-2 relation

$$\big(\mathcal{D}^i_{(\alpha}\mathcal{D}_{\beta)i} - 4Y_{\alpha\beta}\big)W^{\alpha\beta} = \big(\bar{\mathcal{D}}^{(\dot\alpha}_i\bar{\mathcal{D}}^{\dot\beta)i} - 4\bar{Y}^{\dot\alpha\dot\beta}\big)\bar{W}_{\dot\alpha\dot\beta} \ . \tag{A.5}$$

The algebra of covariant derivatives (A.3) is invariant under the super-Weyl transformations [9]

$$\delta_\sigma\mathcal{D}^i_\alpha = \frac{1}{2}\bar\sigma\mathcal{D}^i_\alpha + (\mathcal{D}^{\gamma i}\sigma)M_{\gamma\alpha} - (\mathcal{D}_{\alpha k}\sigma)J^{ki} \ , \tag{A.6a}$$

$$\begin{aligned} \delta_\sigma\mathcal{D}_a &= \frac{1}{2}(\sigma+\bar\sigma)\mathcal{D}_a + \frac{\mathrm{i}}{4}(\sigma_a)^\alpha{}_{\dot\beta}(\mathcal{D}^k_\alpha\sigma)\bar{\mathcal{D}}^{\dot\beta}_k + \frac{\mathrm{i}}{4}(\sigma_a)^\alpha{}_{\dot\beta}(\bar{\mathcal{D}}^{\dot\beta}_k\bar\sigma)\mathcal{D}^k_\alpha \\ &\quad -\frac{1}{2}\big(\mathcal{D}^b(\sigma+\bar\sigma)\big)M_{ab} \ , \qquad\qquad (\text{A.6b}) \end{aligned}$$

with the parameter σ being an arbitrary covariantly chiral superfield,

$$\bar{\mathcal{D}}_{\dot\alpha i}\sigma = 0 \ , \tag{A.7}$$

provided the dimension 1 components of the torsion transform as follows:

$$\delta_\sigma S^{ij} = \bar\sigma S^{ij} - \frac{1}{4}\mathcal{D}^{\gamma(i}\mathcal{D}^{j)}_\gamma \sigma \ , \tag{A.8a}$$

$$\delta_\sigma Y_{\alpha\beta} = \bar\sigma Y_{\alpha\beta} - \frac{1}{4}\mathcal{D}^k_{(\alpha}\mathcal{D}_{\beta)k}\sigma \ , \tag{A.8b}$$

$$\delta_\sigma W_{\alpha\beta} = \sigma W_{\alpha\beta} \ , \tag{A.8c}$$

$$\delta_\sigma G_{\alpha\dot\beta} = \frac{1}{2}(\sigma+\bar\sigma)G_{\alpha\dot\beta} - \frac{\mathrm{i}}{4}\mathcal{D}_{\alpha\dot\beta}(\sigma-\bar\sigma) \ . \tag{A.8d}$$

As is seen from (A.8c), the super-Weyl tensor $W_{\alpha\beta}$ transforms homogeneously.

B Conformal superspace

In this appendix we present the salient details of the superspace formulation of $\mathcal{N}=2$ conformal supergravity [11], known as conformal superspace. The SU(2) superspace of the previous section may be viewed as a gauged fixed version of conformal superspace [11], which gauges the entire superconformal group $\mathrm{SU}(2,2|2)$. Our conventions and presentation follows that of [26].

The covariant derivatives of conformal superspace $\nabla_A = (\nabla_a, \nabla_\alpha^i, \bar\nabla^{\dot\alpha}_i)$ have the form

$$\nabla_A = E_A + \frac{1}{2}\Omega_A{}^{ab}M_{ab} + \Phi_A{}^{ij}J_{ij} + \mathrm{i}\Phi_A Y + B_A\mathbb{D} + \mathfrak{F}_A{}^B K_B \ . \tag{B.1}$$

Here Y is the generator of the U(1) subgroup of the $\mathcal{N}=2$ R-symmetry group $\mathrm{SU}(2)\times\mathrm{U}(1)$, and $K^A = (K^a, S^\alpha_i, \bar S^i_{\dot\alpha})$ are the special superconformal generators, while the one-forms Φ_A, B_A and $\mathfrak{F}_A{}^B$ are the corresponding connections.

The Lorentz, SU(2), U(1) and dilatation generators act on the spinor covariant derivatives as

$$[M_{ab},\nabla^i_\alpha] = (\sigma_{ab})_\alpha{}^\beta\nabla^i_\beta \ , \quad [J_{ij},\nabla^k_\alpha] = -\delta^k_{(i}\nabla_{\alpha j)} \ , \tag{B.2a}$$

$$[Y,\nabla^i_\alpha] = \nabla^i_\alpha \ , \quad [\mathbb{D},\nabla^i_\alpha] = \frac{1}{2}\nabla^i_\alpha \ . \tag{B.2b}$$

The S-supersymmetry generators S^α_i transform under Lorentz, SU(2), $\mathrm{U(1)_R}$ and dilatations as

$$[M_{ab},S^\gamma_i] = -(\sigma_{ab})_\beta{}^\gamma S^\beta_i \ , \quad [J_{ij},S^\gamma_k] = -\varepsilon_{k(i}S^\gamma_{j)} \ , \tag{B.3a}$$

$$[Y,S^\alpha_i] = -S^\alpha_i \ , \quad [\mathbb{D},S^\alpha_i] = -\frac{1}{2}S^\alpha_i \ . \tag{B.3b}$$

Among themselves, the generators K^A obey the algebra

$$\{S^\alpha_i,\bar S^j_{\dot\alpha}\} = 2\mathrm{i}\delta^j_i(\sigma^a)^\alpha{}_{\dot\alpha}K_a \ , \tag{B.4}$$

while the special conformal generators K^A act on ∇_B as

$$\{S^\alpha_i, \nabla^j_\beta\} = 2\delta^j_i \delta^\alpha_\beta \mathbb{D} - 4\delta^j_i M^\alpha{}_\beta - \delta^j_i \delta^\alpha_\beta Y + 4\delta^\alpha_\beta J_i{}^j \ , \quad [S^\alpha_i, \nabla_b] = \mathrm{i}(\sigma_b)^\alpha{}_{\dot\beta} \bar\nabla^{\dot\beta}_i \ , \tag{B.5a}$$

$$\left[K^a, \nabla^j_\beta\right] = -\mathrm{i}(\sigma^a)_\beta{}^{\dot\beta} \bar S^j_{\dot\beta} \ , \quad [K^a, \nabla_b] = 2\delta^a_b \mathbb{D} + 2M^a{}_b \ . \tag{B.5b}$$

The algebra of covariant derivatives is

$$\{\nabla^i_\alpha, \nabla^j_\beta\} = 2\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar W_{\dot\gamma\dot\delta}\bar M^{\dot\gamma\dot\delta} + \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar\nabla_{\dot\gamma k}\bar W^{\dot\gamma\dot\delta}\bar S^k_{\dot\delta} - \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta}\nabla_{\gamma\dot\delta}\bar W^{\dot\delta}{}_{\dot\gamma}K^{\gamma\dot\gamma} \ , \tag{B.6a}$$

$$\{\nabla^i_\alpha, \bar\nabla^{\dot\beta}_j\} = -2\mathrm{i}\delta^i_j \nabla_\alpha{}^{\dot\beta} \ , \tag{B.6b}$$

$$\begin{aligned}[][\nabla_{\alpha\dot\alpha}, \nabla^i_\beta] = &-\mathrm{i}\varepsilon_{\alpha\beta}\bar W_{\dot\alpha\dot\beta}\bar\nabla^{\dot\beta i} - \frac{\mathrm{i}}{2}\varepsilon_{\alpha\beta}\bar\nabla^{\dot\beta i}\bar W_{\dot\alpha\dot\beta}\mathbb{D} - \frac{\mathrm{i}}{4}\varepsilon_{\alpha\beta}\bar\nabla^{\dot\beta i}\bar W_{\dot\alpha\dot\beta}Y + \mathrm{i}\varepsilon_{\alpha\beta}\bar\nabla^{\dot\beta}_j\bar W_{\dot\alpha\dot\beta}J^{ij} \\ &- \mathrm{i}\varepsilon_{\alpha\beta}\bar\nabla^i_{\dot\beta}\bar W_{\dot\gamma\dot\alpha}\bar M^{\dot\beta\dot\gamma} - \frac{\mathrm{i}}{4}\varepsilon_{\alpha\beta}\bar\nabla^i_{\dot\alpha}\bar\nabla^{\dot\beta}_k\bar W_{\dot\beta\dot\gamma}\bar S^{\dot\gamma k} + \frac{1}{2}\varepsilon_{\alpha\beta}\nabla^{\gamma\dot\beta}\bar W_{\dot\alpha\dot\beta}S^i_\gamma \\ &+ \frac{\mathrm{i}}{4}\varepsilon_{\alpha\beta}\bar\nabla^i_{\dot\alpha}\nabla^\gamma{}_{\dot\gamma}\bar W^{\dot\gamma\dot\beta}K_{\gamma\dot\beta} \ . \end{aligned} \tag{B.6c}$$

The super-Weyl tensor $W_{\alpha\beta} = W_{\beta\alpha}$ and its complex conjugate $\bar W_{\dot\alpha\dot\beta} := \overline{W_{\alpha\beta}}$ are superconformally primary, $K_A W_{\alpha\beta} = 0$, and obey the additional constraints

$$\bar\nabla^{\dot\alpha}_i W_{\beta\gamma} = 0 \ , \qquad \nabla^k_\alpha \nabla_{\beta k} W^{\alpha\beta} = \bar\nabla^{\dot\alpha}_k \bar\nabla^{\dot\beta k}\bar W_{\dot\alpha\dot\beta} \ . \tag{B.7}$$

In contrast to SU(2) superspace the entire algebra of covariant derivatives is constructed in terms of the super-Weyl tensor $W_{\alpha\beta}$.

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