MHD STABLE $\beta-$ REGIONS IN $\ell = 2,3$ STELLARATORS

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1. Introduction

An average magnetic well ($V'' < 0$) of the vacuum fields of small shear is one mechanism to stabilize[1,2] MHD-unstable free-boundary modes. In the present study, a class of $\ell = 2,3$ stellarator configurations, which are characterized by an average magnetic hill or well and small shear, are analysed with respect to free-boundary MHD modes. The analysis is performed by using the stellarator expansion procedure STEP[3].

The various vacuum field configurations are given in terms of Dommaschk potentials[4] and the notation of Refs.2,5 is used. Table 1 gives two examples of $l = 2,3$ vacuum fields. These configurations consist of $M = 5$ field periods of period length L_P and aspect ratio $A \approx 10$ ($A = R_T/a = ML_P/2\pi a$, a is the average minor plasma radius, R_T is the major torus radius). The value of the twist (angle of rotational transform divided by 2π) is in the range of $0.56 \leq \iota \leq 0.67$ (see Fig.2). The specific volume V' can be varied by changing the axisymmetric dipole and quadrupole fields[2,5] (resulting in a displacement of the magnetic axis with respect to the outermost magnetic surface) or by adjusting the $\ell = 3$ fields having 5 or 10 field periods around the torus (see Fig.1).

By these means the dependence of the MHD stability of global modes with low values of the toroidal mode number n on the magnetic well of the associated vacuum fields can be studied. In all cases the the pressure profile is approximately a parabolic function of the minor radius r, $p \simeq p_o(1-(r/a)^2)$, and the pressure p_o (measured by β_o on the magnetic axis) is varied (stability parameter). The finite-β equilibria are net-current free. As examples, the Figs. 1 and 2 show the magnetic surfaces, the twist ι and the specific volume V' as functions of r/R_T of two configurations with magnetic well and hill. The configuration FZH207B has a very small aspect ratio of $A \simeq 5$.

2. Stability Results

For the stability computations the equilibrium mass density ρ is assumed to be $\rho \sim \sqrt{p}$ and the plasma region is surrounded by a vacuum region with an electrically conducting wall at infinity. In the STEP procedure the mode number n is a free input parameter ($1 \leq n \leq 6$ for the present study) whereas the various poloidal Fourier modes associated with each n-value result from the computations. The m-numbers given in the Figures correspond to the resonant Fourier mode with the dominant amplitude. In all cases, about ten Fourier modes in the neighborhood of the resonant mode are sufficient.

In Fig.3 the normalized eigenvalues are plotted as functions of β_0 for several unstable free-boundary modes where the corresponding magnetic vacuum field has a magnetic hill of $(\Delta V'/V'_o)_{vac} = 2.81\%$. The modes are characterized by the mode numbers n, m and the node number in radial direction. All modes shown here have no radial nodes except those of the curve with bold squares ($n = 3, m = 5$) which have smaller absolute eigenvalues and are more localized than the corresponding modes without radial nodes.

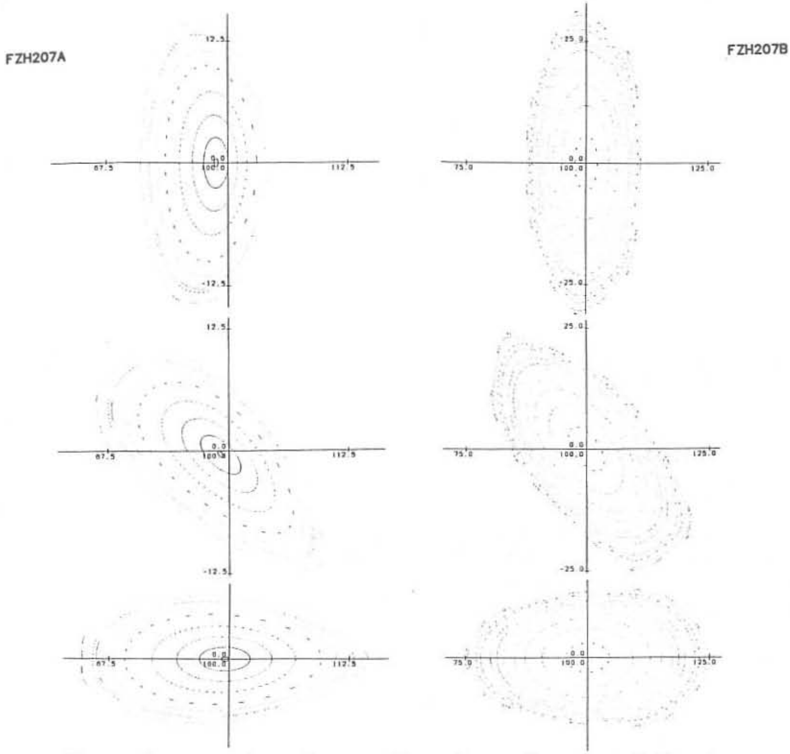

Fig.1. Contour plots of magnetic surfaces of vacuum fields at 0, $L_P/4$, $L_P/2$ of a field period with period lenth L_P;
left: $\iota_o = 0.57$, $\iota_b = 0.65$, $A = 10.3$, $\Delta V'/V_o' = -2.3\%$, $\langle j_\parallel/j_\perp \rangle_{ax} = 2.75$, $J^*_{ax} = 8.54$;
right: $\iota_o = 0.41$, $\iota_b = 0.67$, $A = 5$, $\Delta V'/V_o' = 14.3\%$, $\langle j_\parallel/j_\perp \rangle_{ax} = 2.95$, $J^*_{ax} = 9.73$.

Reducing the magnetic hill of the vacuum field to $(\Delta V'/V_o')_{vac} = 1.38\%$, the absolute eigenvalues are becoming smaller (see Fig.4). One observes that the $n = 3$ and $n = 6$ modes show a resonance feature, are unstable for small β_o-values (i.e. $-(\gamma R_T/v_A)^2 > 0$) and are stabilized at higher β_o-values ($\beta_o \geq 2.6\%$, second stability region for those modes). However modes with different n ($n = 4$, $n = 5$) appear which are localized very close to the magnetic axis or the plasma boundary. The stable β_o-window is very narrow in this case. For equilibrium configurations with $(\Delta V'/V_o')_{vac} = 0.96\%$ (and smaller, see Fig.5), the absolute eigenvalues of the unstable modes decrease further and a small stable region around $\beta_o = 2.3\%$ can be observed where no unstable free-boundary modes with $n = 1, 2, ..., 6$ investigated so far have been found. Another stable β-region is found for $3.3\% \leq \beta_o \leq 5\%$, the width of which depends on the magnetic well depth. In case of a magnetic well $(\Delta V'/V_o')_{vac} = -0.2\%$, no unstable global modes are observed for $\beta_o \leq 3.5\%$.

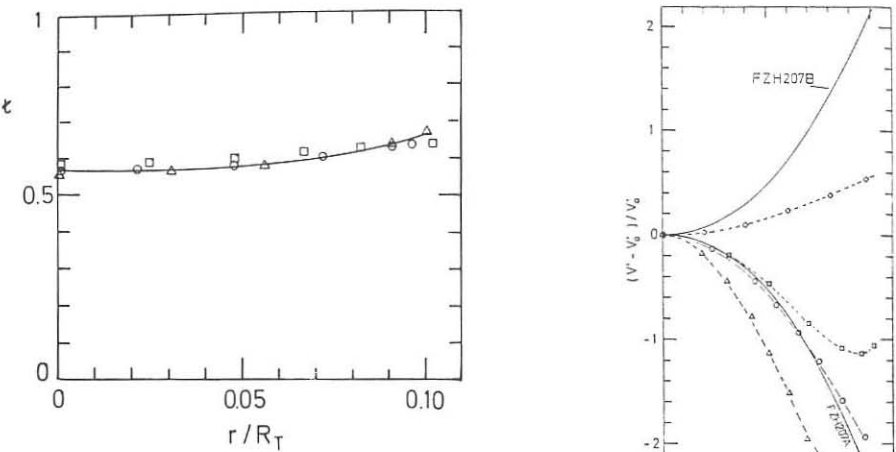

Fig.2. Twist ι and normalized specific volume $(V'-V_o')/V_o'$ as functions of r/R_T for various vacuum fields. The solid curves are for the configuration FZH207A from a field line tracing code and the corresponding values from STEP are indicated by open circles. The additional curves (squares and triangles) obtained by STEP show the variety of vacuum fields (V'' positive and negative) used for the stability computations.

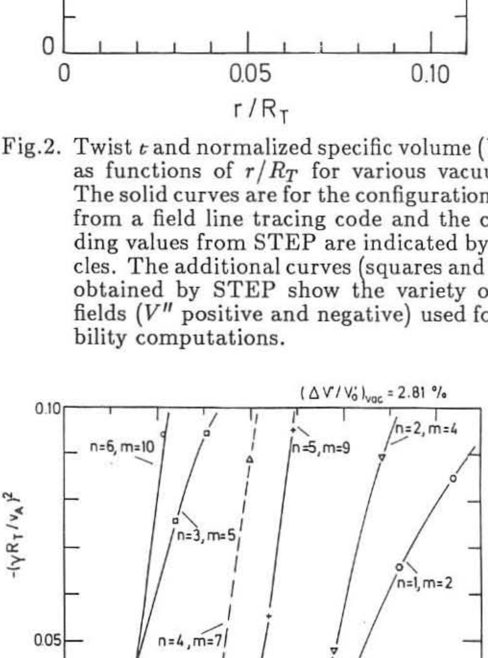

Fig.3. Normalized eigenvalues as functions of β_o for $n = 1, 2, ...6$ free-boundary modes with no or one radial node ($\iota_{o,vac} = 0.59$, $\iota_{b,vac} = 0.61$). The mode structure is changed if β_o increases. There are many other modes with smaller absolute eigenvalues.

Table 1 a
FIELD CONFIG. FZH207A CC=1.0, 3/88

MPER,IANP,RA,BZO/	IDN,M,L,ANP		
5	21	1.00E+00	-6.160E-03
1	0	3	-2.800E-01
1	5	1	-3.240E-02
2	5	1	3.240E-02
1	5	2	-1.550E+00
2	5	2	-1.550E+00
1	5	3	-1.500E-01
2	5	3	5.000E+00
1	5	4	-4.450E+01
2	5	4	-2.440E+01
1	5	5	-1.000E+02
2	5	5	1.100E+02
1	5	6	-4.810E+02
2	5	6	1.120E+02
1	10	3	-4.500E-00
2	10	3	4.500E-00
1	10	4	4.630E+01
2	10	4	4.580E+01
1	10	5	2.640E+02
2	10	5	-2.350E+02
1	10	6	3.000E+03
2	10	6	1.500E+03

Table 1 b
FIELD CONFIG. FZH207B CC=0.9, 3/88

MPER,IANP,RA,BZO/	IDN,M,L,ANP		
5	25	1.00E+00	6.161E-04
2	0	2	1.080E-01
1	0	3	-4.876E-04
2	0	4	-6.638E-04
1	0	5	7.005E+01
2	0	6	6.599E-01
1	5	1	2.238E-03
2	5	1	-2.239E-03
1	5	2	-1.544E+00
2	5	2	-1.546E+00
1	5	3	-2.152E+00
2	5	3	2.950E+00
1	5	4	-3.452E+01
2	5	4	-1.437E+01
1	5	5	-1.008E+02
2	5	5	1.106E+02
1	5	6	-4.813E+02
2	5	6	1.119E+02
1	10	3	8.502E-01
2	10	3	-8.507E-01
1	10	4	2.631E+01
2	10	4	2.584E+01
1	10	5	2.638E+02
2	10	5	-2.346E+02
1	10	6	2.991E+03
2	10	6	1.505E+03

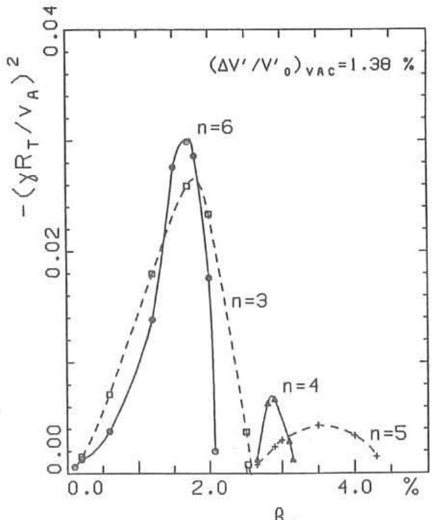

Fig.4. Eigenvalues vs β_o for $n/m = 6/10,\ 3/5,\ 5/9,\ 4/7$ free-boundary modes.

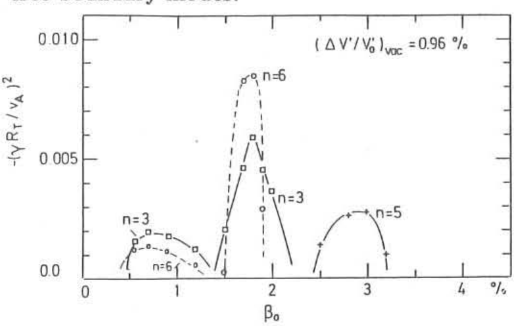

Fig.5. Eigenvalues vs β_o for $n/m = 6/10,\ 3/5,\ 5/8$ free-boundary modes.

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