OPTIMIZATION OF COILS FOR STELLARATORS

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1. Introduction

In stellarator configuration studies the optimization of plasma equilibrium properties and the realization of the magnetic field geometry by external currents can be treated separately. The Helias class of MHD stable stellarators has been obtained by studying fixed boundary equilibria [1]. The equilibria are characterized by strongly reduced parallel current density and a magnetic well. In order to realize these equilibria experimentally a method has been developed to determine external coils which will produce the appropriate magnetic field (NESCOIL code). In [2], [3] one finds a detailed description: On a closed surface surrounding the plasma equilibrium a surface current distribution is determined such that the field, $\vec{B}$, produced approximates the vacuum field of the plasma configuration. This is achieved by requiring that the normal component of $\vec{B}$ is minimized on the plasma surface

$$F \equiv \int_{\partial R} (\vec{B} \cdot \vec{n})^2 \mathrm{d}f = \min! \tag{1.1}$$

The outer surface is given by mapping the unit square $0 \leq u < 1$, $0 \leq v < 1$ of the two angle-like variables, u and v, onto one period of the surface:

$$\begin{aligned} r &= \sum_{m=0,n=-n_b}^{m_b,n_b} \hat{r}_{mn} \cos 2\pi(mu + nv) \ , \\ z &= \sum_{m=0,n=-n_b}^{m_b,n_b} \hat{z}_{mn} \sin 2\pi(mu + nv) \ , \\ \varphi &= \frac{2\pi}{n_p} v, \end{aligned} \tag{1.2}$$

where (r, φ, z) are cylindrical coordinates and the usual stellarator symmetry is assumed. The surface current density with the same periodicity can be expressed by a potential $\Psi(u, v)$ defined on the surface:

$$\vec{j} = \vec{n} \times \text{Grad } \Psi(u, v) \ , \tag{1.3}$$

where Grad is the gradient operator on the surface ($\vec{x}_u \cdot \text{Grad}\, \Psi = \Psi_u$, $\vec{x}_v \cdot \text{Grad}\, \Psi = \Psi_v$), $\vec{n}$ is the exterior normal to ∂D, and $\vec{x}_u, \vec{x}_v$ are the derivatives with respect to u and v respectively (Fig. 1). The general ansatz for $\Psi(u,v)$ can be written as

$$\Psi(u,v) = \sum_{m=0,n=-N}^{M,N} \hat{\Psi}_{mn} \sin 2\pi(mu+nv) - \frac{I_p}{n_p} v - I_t u \ , \tag{1.4}$$

where I_p and I_t are the net poloidal and toroidal surface currents, respectively.

If the closed contours of the surface current are sufficiently simple then the current may be discretized into a finite number of infinitely thin filaments, which will approximate the external coils of the configuration. However, the coils for a realistic device have to satisfy a number of additional conditions: The minimal (maximal) plasma – coil distance must be prescribed. The curvature of the coils and the current density in the coils must not exceed certain limits.

For this purpose a further optimization step has been added: The outer current carrying surface is varied systematically by varying the Fourier coefficients $\hat{r}_{mn}$, $\hat{z}_{mn}$. Again, F is minimized, but this time the above mentioned side-conditions are expressed as constraints (penalty functions) in the minimization.

2. Applications

The Helias configuration is considered to be the most promising candidate for the W VII-X device [4]. Preliminary data for the W VII-X are: major radius 5 m, aspect ratio $A \approx 10$, magnetic field $B = 4$ Tesla, minimal coil – plasma distance $d_{\min} \geq$ 0.2 m, maximal coil curvature $\kappa_{\max} = 0.2$ m and maximal coil current density $j_{\max} =$ 40 MA/m^2.

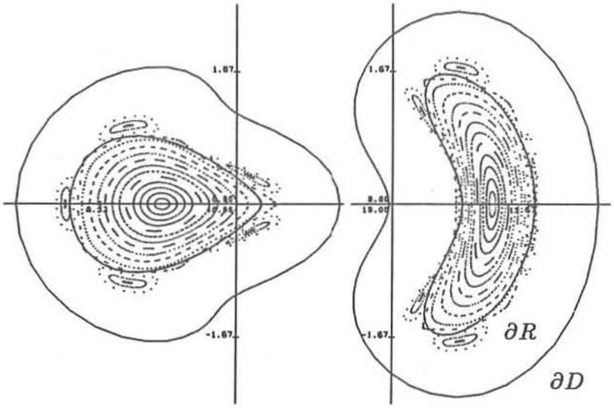

Fig.1 Poincaré plot of the 5081 Helias field. Fourier harmonics $\hat{\Psi}_{mn}$ of the potential taken into account have $m \leq 4$, $|n| \leq 4$. Fourier coefficients $\hat{r}_{mn}$, $\hat{z}_{mn}$ of optimized current carrying boundary have $m \leq 4$, $|n| \leq 4$.

Two optimized coil configurations which have been computed for the Helias cases 5081 and 5099 are presented here [5]. Characteristic properties of the 5081 Helias are: number of periods $n_p = 5$, aspect ratio $A = 10$, rotational transform $0.8 \lesssim \iota \lesssim 1$, ballooning stable up to $\langle\beta\rangle = 0.05$. In Fig. 2 one period of a coil system is shown. Fig. 1 shows the Poincaré plot of the magnetic field produced by 10 coils per period. A magnetic well of 1.9 % is obtained. The configuration may also be of interest as a candidate for an island-divertor concept with divertor regions at the tips of the bean-shaped and the triangular cross-sections.

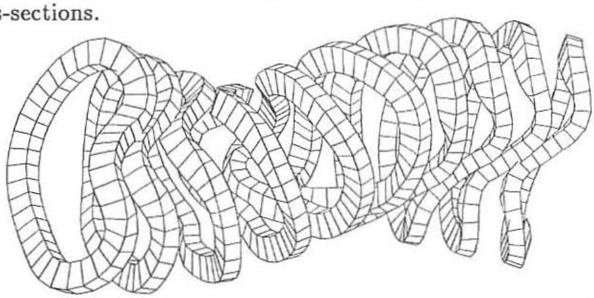

Fig.2 One period of modular coils for the 5081 Helias stellarator. Coils per period $N_c = 10$, major radius $R = 5$ m, minimal and maximal distance of coils to plasma $d_{\min} = 0.19$ m, and $d_{\max} = 0.32$ m, maximal curvature at coil boundary $\kappa = 5\ \mathrm{m}^{-1}$, coil cross-section $q = 0.18 \times 0.28\ \mathrm{m}^2$.

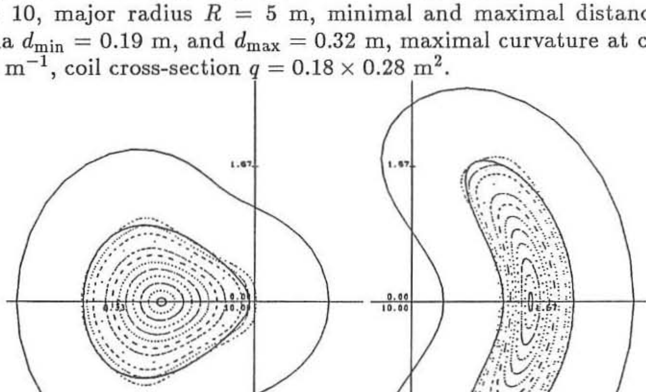

Fig.3 Poincaré plot of the 5099 Helias field. Fourier harmonics $\hat{\Psi}_{mn}$, of the potential taken into account have $m \leq 4$, $|n| \leq 3$. Fourier coefficients $\hat{r}_{mn}$, $\hat{z}_{mn}$ of optimized current carrying boundary have $m \leq 4$, $|n| \leq 4$.

The 5099 Helias case differs from the 5081 case only with respect to the rotational transform, which is almost constant with $\iota = 0.9$. The magnetic well is 1.8 %. Figure 3 shows the Poincaré plot for this case which is produced by the coils shown in Fig. 4. There are 10 coils per period each carrying the same current.

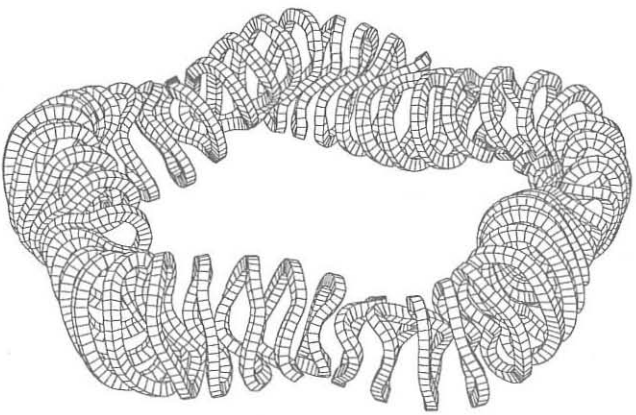

Fig.4 Modular coils for the 5099 Helias stellarator. Coils per period $N_c = 10$, major radius $R = 5$ m, minimal and maximal distance of coils to plasma $d_{\min} = 0.22$ m, and $d_{\max} = 0.34$ m, maximal curvature at coil boundary $\kappa = 4\ \mathrm{m}^{-1}$, coil cross-section $q = 0.2 \times 0.25\ \mathrm{m}^2$. A magnetic field $B = 4$ Tesla is achievable with current density $j = 40\ \mathrm{MA/m}^2$.

3. Summary

The NESCOIL code has been extended to take into account important practical constraints on the properties of stellarator coils. The code has been used to compute coil configurations for some possible W VII-X designs. Having found the coils producing the external field, one can study free boundary plasma equilibrium using the NEMEC code [6], [7]. In particular, the change in plasma properties with $\langle\beta\rangle$ and with the superposition of a vertical B_z-field can be studied.

4. References

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