Effect of measured toroidal flows on tokamak equilibria

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The magnetic confinement of a tokamak plasma is routinely calculated by solving the Grad-Shafranov equation describing the ideal magneto-hydrodynamic (MHD) equilibrium for the poloidal flux function for axisymmetric geometry. A basic assumption usually is that small effects of the centrifugal force due to toroidal and poloidal plasma rotation can be neglected. But in present-days unbalanced high-power beam-heated plasmas the toroidal plasma rotation is typically rather large. Toroidal plasma rotation velocities up to 300 km/s have been measured at the ASDEX Upgrade tokamak which corresponds to Mach numbers up to 0.5 in the plasma center. For other tokamaks (MAST, DIII-D, NSTX, JET) sonic Mach numbers approaching unity have also been reported [1] (and references therein). For such large Mach numbers the centrifugal force results in a redistribution of the particles on the flux surface. The surfaces of constant pressure do no longer coincide with the flux surfaces and significantly modified flux surface topology can arise. Reasonably accurate equilibrium reconstructions for moderate and high Mach numbers are necessary to study plasma stability since it is known that strong flows may considerably damp plasma instability [2]. Additionally, for an improvement of the flux surface geometry by multiple temperature measurements on the same flux surface (iso-flux constraint) [3] flows might have to be considered. If the underlying equilibrium equation is incomplete, a constraint from the measurement of the geometry of flux surfaces can lead to erroneous current distributions trying to compensate for the missing terms in the equilibrium equation.

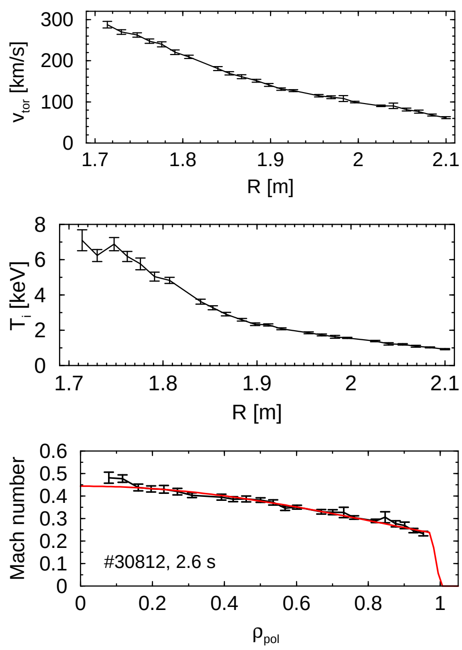

Figure 1: *Profiles of the toroidal rotation velocity and ion temperature as a function of the major radius for #30812, 2.6 s, and the corresponding Mach number profile as a function of* ρ_{pol} *and a parabolic fit.*

Various theoretical studies of toroidal and poloidal mass flow modifications to axisymmetric ideal MHD equilibria exist [4, 1] (and references therein). The purpose of the present paper is to study the effects of the toroidal flow on various equilibrium parameters quantitatively based on flow velocity profiles determined from measurements of the ion temperature and toroidal rotation velocity profiles at ASDEX Upgrade. The poloidal flow is not considered in the present work since it is typically much smaller and, therefore, only of minor importance [5].

The Grad-Shafranov equation for the poloidal flux function ψ describing ideal magneto-hydrodynamic equilibrium in two-dimensional tokamak geometry with purely toroidal flow is:

$$\left(R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}\right)\psi(R,z)=-4\pi^2\mu_0 R^2\left(\frac{\partial p_s}{\partial\psi}+\frac{p_s R^2}{2R_a^2}\frac{\partial M_t^2}{\partial\psi}\right)\exp\left(\frac{M_t^2R^2}{2R_a^2}\right)-\mu_0 F\frac{\partial F}{\partial\psi} \quad (1)$$

$F(\psi)$ is the poloidal diamagnetic current. Assuming rigid rotation on a flux surface, the angular velocity Ω can be evaluated from the measured toroidal rotation velocity v_t: $\Omega(\psi)=v_t/R$. The centrifugal force re-distributes particles on a flux surface according to a Boltzmann distribution resulting in a modified plasma pressure $p(R,\psi)=p_s\exp\left(\frac{m\Omega^2R^2}{2k_BT}\right)=p_s\exp\left(\frac{M_t^2R^2}{2R_a^2}\right)$ where $M_t^2:=\frac{m\Omega^2R_a^2}{k_BT}$ and R_a is the major radius of the magnetic axis. With temperature T, ion mass m, and $c_s^2=\frac{k_BT}{m}$ defining the velocity of sound, the toroidal Mach number is $M_t=\frac{R_a}{R}\frac{v_t}{c_s}$. Please note that p_s, T, Ω and, therefore, M_t are assumed to be constant on a poloidal flux surface whereas the pressure p is not a flux quantity due to the particle re-distribution on a flux surface.

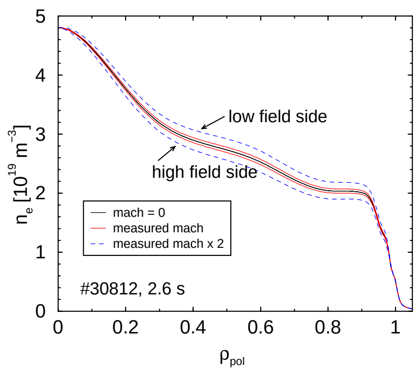

Figure 2: *Variation of the particle density for the static case (black line), the measured Mach number profile (red line), and multiplied by a factor of 2 (blue dashed line).*

The extension of the Grad-Shafranov equation with purely toroidal flow is included in the Grad-Shafranov solver IDE (Integrated Data analysis Equilibrium) [3] which is a free-boundary equilibrium code. The code can be run in an interpretative mode of reconstructing an equilibrium from measured data, or, to avoid effects of the plasma control, it can be run in a predictive mode with prescribed source terms of pressure and diamagnetic current. For the predictive mode to stabilize the inherent vertical instability of (1) an additional magnetic field from currents in external coils (V2o and V2u at ASDEX Upgrade) is applied which program-internally adjusts itself so that the magnetic axis is at a prescribed position (R_a, z_a). To study the effect of the toroidal rotation on the equilibrium, the predictive run can be performed in two different ways: 1. Keep the axis position constant for various Mach number profiles. The centrifugal force has to be balanced with the additional magnetic field, resulting in a shift of the last-closed flux surface (LCFS) towards the inner vessel wall [8]. 2. Since an experimental control of the magnetic axis position is not realistic, the second approach is to get rid of the appearing inward shift: In an additional iteration the stabilizing axis position is adjusted such that the currents in the external coils vanish. This gives the *natural* equilibrium not corrupted by the stabilizing condition.

T and v_t are measured at ASDEX Upgrade with a high-resolution charge exchange system [6]. Fig. 1 shows the measured T and v_t profiles for discharge #30812, 2.6 s, chosen for its large toroidal rotation velocity (1 MA, 2.6 T, $q_{95} = 4.5$, $n_e = 5.8 \times 10^{19}$ m^{-3}, 2.5 MW NBI and 0.6 MW ECRH heating), and the corresponding Mach number profile including a parabolic fit. To study the effect of the various Mach number profiles, we start from the source profiles of a reconstructed equilibrium in interpretative mode employing the measured Mach number profile. The Mach number profile is then scaled from 0 (static, no flow) to 2. A factor of 2 results in Mach numbers close to 1 at the magnetic axis and close to 0.8 at mid radius. Fig. 2 shows the variation of the particle density as a function of ρ_{pol} for the static case ($M_t = 0$), and due to the redistribution on the flux surface for the measured Mach number profile (red line), and its scaling by a factor of 2. In contrast to the density profile without flow being constant on a flux surface, the density decreases on the high-field-side (hfs) and increases on the low-field-side (lfs) of the flux surface with increasing Mach number. As the deviation is rather small for the measured Mach number of about 0.4 at mid-radius, the deviation increases approximately quadratically with the Mach number.

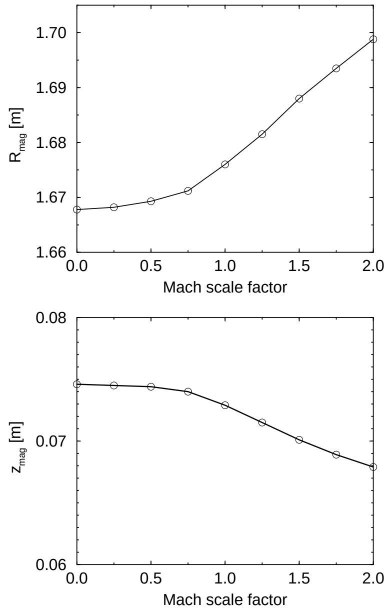

Figure 3: *Magnetic axis position as a function of the Mach number scaling.*

Fig. 3 shows the variation of the magnetic axis position (R_{mag}, z_{mag}) as a function of the scaling factor. The magnetic axis shows outward shifts of 0.8 cm for the measured Mach number profile and of 3.1 cm for a two-times larger Mach number profile. The radial shift starts with an approximately quadratic dependence on the Mach number with a transition into linearity for

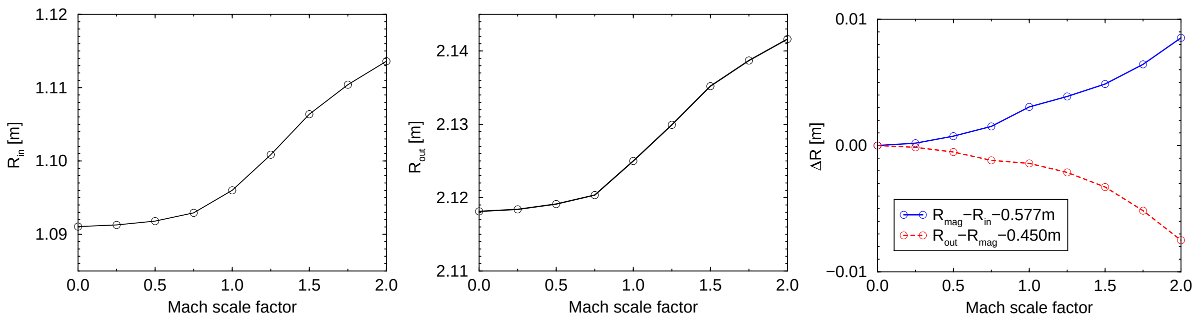

Figure 4: Variation of the innermost R_{in} and outermost R_{aus} position of the LCFS as a function of the Mach number scaling factor, and change of difference $R_{mag} - R_{in}$ and $R_{out} - R_{mag}$.

larger values resembling the observation of a theoretical study [7]. The shift for the vertical position of the magnetic axis is much smaller ($\sim$mm) because the centrifugal force acts in radial direction. Fig. 4 shows the variation of the innermost R_{in} and outermost R_{out} position of the LCFS as a function of the Mach number scaling factor. Both quantities increase with Mach number resulting in an overall outward shift of the LCFS.

This effect can neither be seen in the interpretative mode of the equilibrium reconstruction because the magnetic measurements determine the LCFS position robustly, nor in the predictive mode with magnetic axis position held constant. An artificial scaling of the Mach number profile in the interpretative mode results in modifications of the source profiles in the plasma center only. In the predictive mode with magnetic axis position held constant the LCFS shifts to the inner vessel wall [8]. This is consistent with the increase of the distance $R_{\text{mag}} - R_{\text{in}}$ shown in the right panel of Fig. 4. In contrast $R_{\text{out}} - R_{\text{mag}}$ decreases with Mach number showing an increased flux gradient on the lfs. Fig. 5 shows the relative change of the gradient of the flux in the midplane with respect to the gradient in the static case. The largest change of the flux gradient is at the lfs (outer) midplane at mid radius. The change of the gradient increases approximately quadratically up to about 5% for a Mach number of 0.8 at mid radius (scale 2.0). Although the plasma shows a shift of the LCFS to the lfs, a compression on the lfs and a stretching on the hfs, there is nearly no effect on the plasma volume, plasma surface, horizontal or vertical extension, or the ellipticity.

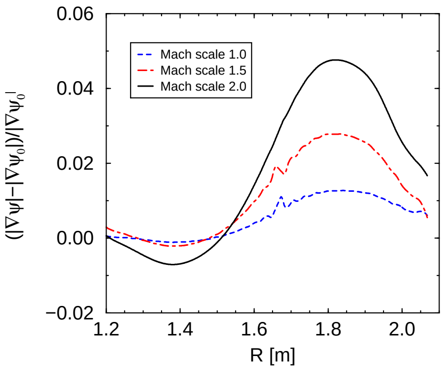

Figure 5: *Relative change of the midplane flux gradient with flow with respect to the static case.*

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