BRIEF COMMUNICATION

Approximate Quasi-isodynamicity at Finite Aspect Ratio in a Stellarator Vacuum Magnetic Field

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Abstract

A stellarator vacuum field is found in which, at finite aspect ratio ($\mathrm{A} \approx 40$), the contours of the second adiabatic invariant of nearly all particles reflected inside that surface are poloidally closed.

Introduction

Stellarators as toroidal magnetic confinement devices without internal net toroidal current were invented by L. Spitzer. Only later it became clear that the collisionless confinement of reflected particles poses a severe problem for the viability of stellarators as fusion devices while in tokamaks sufficiently close to axisymmetry reflected particles are confined due to the two-dimensional symmetry. For stellarators being genuinely three-dimensional devices without symmetry this problem was solved by the invention of three classes of toroidal configurations with quasi-helical symmetry [1], quasi-axial symmetry [2] and quasi-isodynamicity [3]. V.D. Shafranov was the first in Russia who understood the significance of these discoveries. He organized the corresponding investigations in Russia as well as international collaborations. Results of this work in connection to the subject of the present paper, quasi-isodynamic configurations, are found, e.g. in [4-6]).

Recently, it has been shown that a stellarator vacuum magnetic field not close to any of the above three concepts can exhibit excellent collisionless particle confinement [7]. In that field the topographies of the contours of the second adiabatic invariant are complex; in particular there are contours which do not encircle the magnetic axis. Also, the maximum of the field strength B on a magnetic surface is local, i.e. does not occur as a poloidally closed line as is necessary for a strictly quasi-isodynamic configuration [5]. Here it is shown that stellarator vacuum field configurations exist that are closer to quasi-isodynamicity.

Procedure

The starting point was the configuration of [8] for which the condition of quasi-isodynamicity is almost satisfied on the magnetic axis, which means that the contours of the second adiabatic invariant $\mathcal{J}$ exhibit a stagnation point on the magnetic axis. A diagnostic for quasi-isodynamicity (qi) was developed which avoids the computation of $\mathcal{J}$ and, instead, evaluates the condition that the lengths along the fieldlines between equal values of the field strength on both sides of the minimum of B be independent of the field line label [5]. This diagnostic was applied to measure the extent of violation of qi on a magnetic surface at about a third of the minor radius of the configuration. An optimization then minimized this extent with the constraint that the strength of B form poloidally closed lines since this a necessary condition for qi. Computational details are sketched in the appendix.

Results

Figure 1 compares the topographies of B for the configuration of [8]and the result obtained here. Figure 1a can be compared to Fig. 3 (B at about a fifth of the minor radius) of [8] and shows that at about a third of the minor radius the necessary condition that the contours of B be poloidally closed is somewhat violated near the minimum of B (about 1 percent poloidal variation) while Fig. 1b shows that this property is nearly perfectly satisfied in the present result; the variation of the maximum of B is about 1.5 permil, the one of the minimum of B 0.7 permil. These contours are shown in magnetic coordinates, θ (poloidal) and ϕ (toroidal); while B_{max} occurs at $\phi = const$ (for vanishing toroidal current) [5], there is no analogous property for qi configurations at B_{min}. Accordingly, the B_{min} line reflects the shapes of the contour lines needed for qi between B_{min} and B_{max}.

Figure 2 compares the contours of the qi diagnostic for the two configurations. Here the abscissa is the variation of B between B_{min} and B_{max} while the ordinate again is the poloidal magnetic coordinate θ; contour lines independent of θ indicate perfect qi. So, the present configuration is significantly closer to qi. Since the optimization is carried out at only a third of the small radius only geometrical boundary coefficients up to poloidal index $m = 2$ have been used and, accordingly, Fourier coefficients of the field strength beyond m=2 are insignificant.

Figure 3 shows $\mathcal{J}$ contours. While in the configuration [8] a major fraction of the $\mathcal{J}$-contours was not poloidally closed, here, for reflection values between $1.01 \cdot B_{min}$ and $0.99 \cdot B_{max}$, poloidally closed $\mathcal{J}$-contours encompass the plasma core up to a third (or larger) of the plasma radius. So, as has to be expected, only very deeply or very shallowly trapped particles can get lost collisionlessly.

A computation of the actual collisionless loss of particles (α-particles followed for 1 sec in a fusion-size (5 T, 10^3 m^3) device) shows it to be very small: of a thousand mono-energetic particles started at normalized minor radius 0.3 with random starting points and pitch angles only seven are lost and are very deeply reflected particles.

Discussion

This case study shows that quasi-isodynamicity can be approached in a vacuum magnetic field. The quality of collisionless particle confinement is similar to a finite-β qi configuration [6]. The residual loss here appears to be related to decreasing quality of the $\mathcal{J}$-contours very close to the minimum of the field strength (already indicated at 1.01 B_{min}) so that an additional investigation is needed to see whether this type of loss can be eliminated, too.

Acknowledgments

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Appendix

The computational procedure relies on evaluation of the strength of the magnetic field along field lines. The optimization towards poloidally closed contours of B is done with three conditions: the minimum and the maximum of B should occur as poloidally closed lines and B should be monotonously increasing between B_{min} and B_{max}. Then (as in [5]) B can be used on both sides of B_{min} as independent variable; more precisely the normalized increment of B is the independent variable for the normalized increments of ϕ_+ and ϕ_- on both sides of B_{min}; finally, the variation of $\phi_+ + \phi_-$ with θ is used to measure the deviation from qi.

Figure captions

Fig. 1 a, b.

Contours of the strength of the magnetic field in Boozer magnetic cooerdinates at about a third of the minor radius of the configurations are shown; in a) for the configuration of [8], in b) for the present configuration.

Fig. 2 a, b.

Contours of the qi condition, $\phi_+ + \phi_-$, are shown; in a) for the configuration of [8], in b) for the present configuration.

Fig. 3

Contours in magnetic coordinates ($\sqrt{s}$ with s the normalized toroidal flux and θ) of the second adiabatic invariant. The reflection values of B are approximately 1.01 B_{min} for the first and approximately 0.99 B_{max} for the last contour plot. In between these values are $\approx 1.11 B_{min}$, $\approx 1.17 B_{min}$ and $\approx 1.22 B_{min}$ while $B_{max} \approx 1.33 B_{min}$. The inner circles approximately indicate the flux surface on which the optimization was performed.

References

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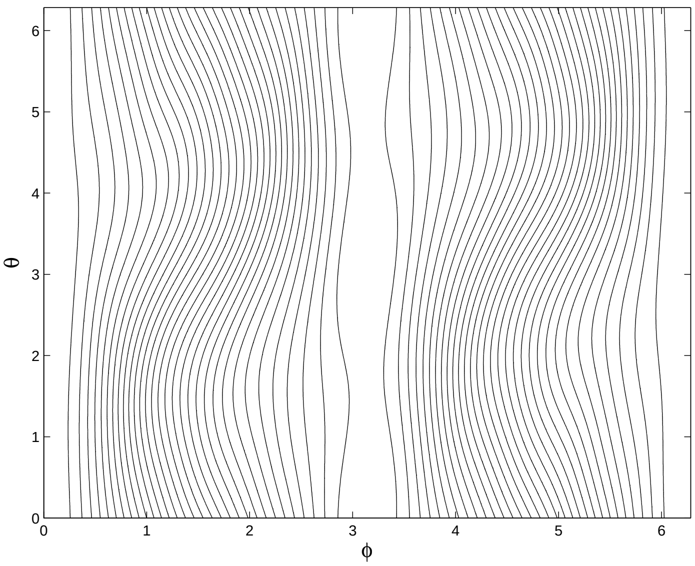

Figure 1a

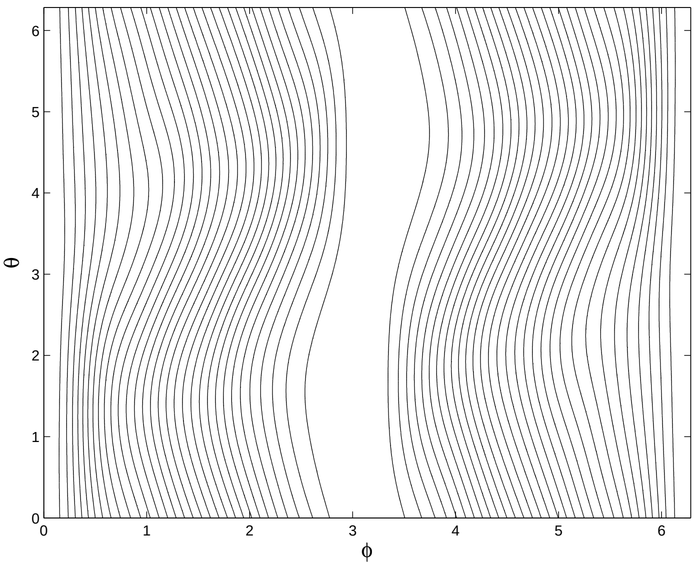

Figure 1b

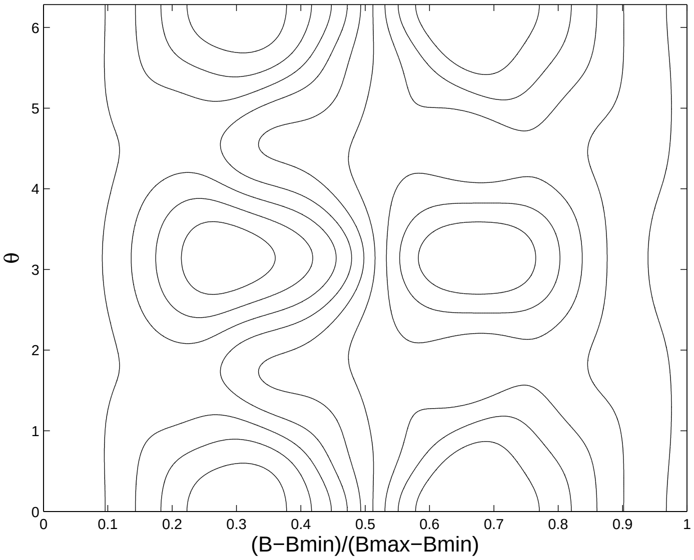

Figure 2a

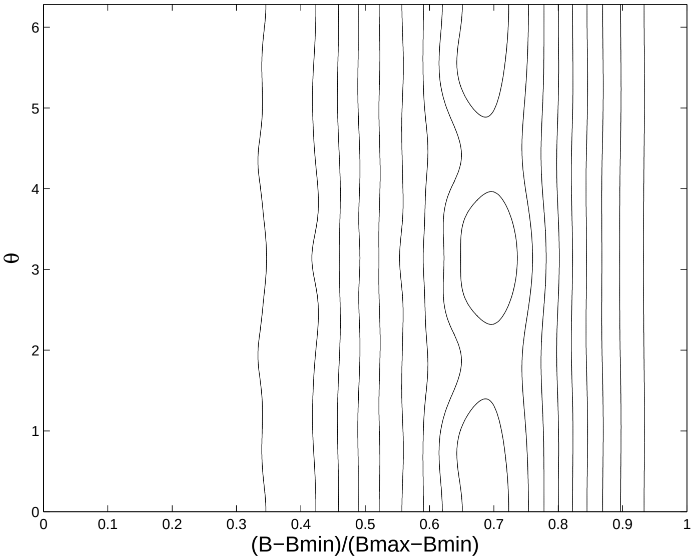

Figure 2b

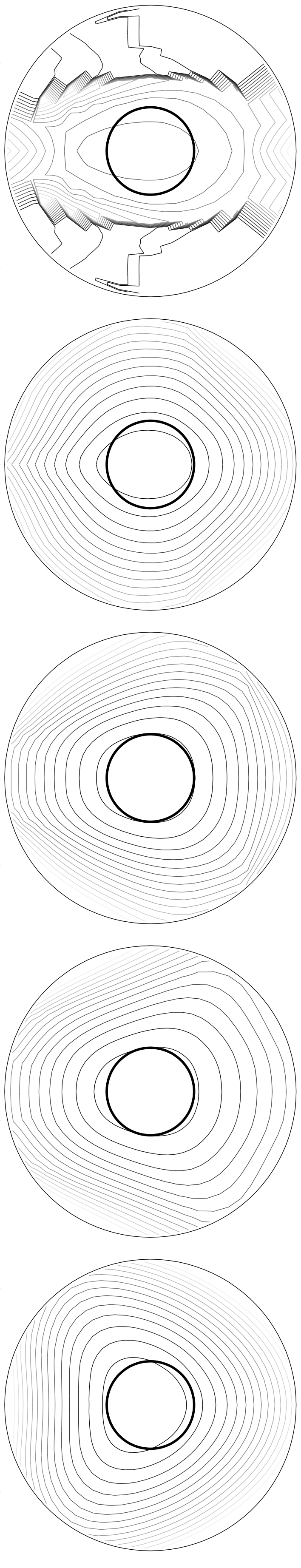

Figure 3