Hawking radiation from a five-dimensional Lovelock black hole

**Mahamat Saleh · Bouetou Bouetou Thomas ·
Timoleon Crepin Kofane**

Abstract We investigate Hawking radiation from a five-dimensional Lovelock black hole using the Hamilton-Jacobi method. The behavior of the rate of radiation is plotted for various values of the ultraviolet correction parameter and the cosmological constant. The results show that, owing to the ultraviolet correction and the presence of dark energy represented by the cosmological constant, the black hole radiates at a slower rate in comparison to the case without ultraviolet correction or cosmological constant. Moreover, the presence of the cosmological constant makes the effect of the ultraviolet correction on the black hole radiation negligible.

Keywords Hawking radiation, Lovelock black hole, Hamilton-Jacobi method

1 Introduction

Hawking radiation, which is closely related to the existence of a black hole's event horizon, is an important quantum phenomenon. Hawking radiation from black holes [1–11] is one of the most striking effects known or at least widely agreed to arise from the combination of quantum mechanics and general relativity. As one of the most important

Mahamat Saleh · Bouetou Bouetou Thomas · Timoleon Crepin Kofane
Department of Physics, Faculty of Science, University of Yaoundé I, P.O. Box 812, Yaoundé, Cameroon

Mahamat Saleh
Department of Physics, Higher Teachers' Training College, University of Maroua, P.O. Box 55, Maroua, Cameroon
E-mail: mahsaleh2000@yahoo.fr

Bouetou Bouetou Thomas
National Advanced School of Engineering, University of Yaoundé I, P.O. Box 8390, Yaoundé, Cameroon
E-mail: tbouetou@yahoo.fr

Kofane Timoleon Crepin
The Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, 01187 Dresden, Germany
E-mail: tckofane@yahoo.com

achievements of quantum field theory in curved spacetimes, the discovery of Hawking radiation lent support to the idea that a classical black hole could radiate a thermal spectrum of particles. Since Hawking's original work, several derivations of Hawking radiation have been proposed in the literature. Kraus and Wilczek [9,10] considered the modification of the formulas for black hole radiation resulting from the self-gravitation of the radiation and found that the particles no longer move along geodesics and that the action along the rays is no longer zero for a massless particle. They concluded that the radiation is no longer thermal but can be corrected in a definite way that they calculated. In 2000, Parikh and Wilczek, elaborating upon Kraus and Wilczek's work, presented another new derivation of Hawking radiation, in which Hawking radiation is treated as a quantum tunneling process [11]. By using this method, many studies were conducted to evaluate the black hole radiation from massless particles [12], massive particles [13], charged massive particles [1,14,15], and Dirac particles [16–18]. In 2012, Jiang and Han investigated black hole spectroscopy via adiabatic invariance by combining the black hole property of adiabaticity with the oscillating velocity of the black hole horizon obtained from the tunneling framework [19]. Recently, some related works have developed Jiang-Hans method to investigate the entropy spectra of different black holes [20–26]. Many other works involving the Hamilton-Jacobi method to investigate Hawking radiation can be found in the literature [13,27–32]. These works highlight the fact that black holes are not exclusively absorbing; they are also emitting radiation.

Recently, Cai *et al.* [33] investigated Hawking radiation of an apparent horizon in a Friedmann-Robertson-Walker universe using the Hamilton-Jacobi method. Using the same method, Gohar and Saifullah [34] investigated scalar field radiation from dilatonic black holes. In this paper, a five-dimensional Lovelock black hole is considered to investigate Hawking radiation by including the influence of the ultraviolet correction to the black hole.

2 Hawking radiation of the black hole

The five-dimensional Lovelock black hole metric is given by [35,36]

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega_3^2, \tag{1}$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\varphi^2 + \sin^2\theta\sin^2\varphi d\psi^2$,

$$f(r) = \frac{4\alpha - 4M + 2r^2 - \Lambda r^4/3}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}}, \tag{2}$$

M is the black hole mass, Λ is the cosmological constant, and α is the coupling constant of an additional term that represents the ultraviolet correction to Einstein theory.

For $\Lambda = 0$, the radial function (2) reduces to

$$f(r) = \frac{4\alpha - 4M + 2r^2}{4\alpha + r^2 + \sqrt{r^4 + 16M\alpha}}, \tag{3}$$

and the horizon radius is

$$r_H = \sqrt{2(M - \alpha)}. \tag{4}$$

When $\Lambda \neq 0$, the horizon radii, r_+ and r_-, for this background are given by

$$r_+ = \sqrt{\frac{3}{\Lambda}(1 + \sqrt{1 - 4\Lambda(M - \alpha)/3})} \tag{5}$$

and

$$r_- = \sqrt{\frac{3}{\Lambda}(1 - \sqrt{1 - 4\Lambda(M - \alpha)/3})}. \tag{6}$$

We can then convert the metric into the following form:

$$f(r) = \frac{-\frac{\Lambda}{3}(r^2 - r_+^2)(r^2 - r_-^2)}{4\alpha + r^2 + \sqrt{r^4 + \frac{4}{3}\alpha\Lambda r^4 + 16M\alpha}} \tag{7}$$

The Hamilton-Jacobi method is an alternate method for calculating black hole tunneling that makes use of the Hamilton-Jacobi equation as an ansatz [37]. This method is based on the work of Padmanabhan and his collaborators [38–40]. In general, the method involves using the WKB approximation to solve a wave equation. The simplest case to model is that of scalar particles, which therefore involves applying the WKB approximation to the Klein-Gordon equation. The result, to the lowest order of the WKB approximation, is a differential equation that can be solved by substituting a suitable ansatz. The ansatz is chosen by using the symmetries of the spacetime to assume separability. After substituting a suitable ansatz, the resulting equation can be solved by integrating along the classically forbidden trajectory, which starts inside the horizon and finishes at the outside observer (usually at infinity). Because this trajectory is classically forbidden, the equation will have a simple pole located at the horizon. Consequently, the method of complex path analysis must be applied to deflect the path around the pole.

Scalar particles under a gravitational background obey the Klein-Gordon equation. For a scalar particle moving in spacetime, the radiated particle obeys the Klein-Gordon equation for a scalar field ϕ:

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi - \frac{m^2}{\hbar^2}\phi = 0. \tag{8}$$

Applying the WKB approximation by assuming an ansatz of the form

$$\phi(t, r, \theta, \varphi) = \exp[\frac{i}{\hbar}I(t, r, \theta, \varphi) + I_1(t, r, \theta, \varphi) + O(\hbar)], \tag{9}$$

where I and I_1 are the components of the action approximated at the zeroth and first order, respectively, and then inserting this back into the Klein-Gordon equation will result in the Hamilton-Jacobi equation to the lowest order in $\hbar$:

$$g^{\mu\nu}\partial_\mu I\partial_\nu I + m^2 = 0. \tag{10}$$

For the Hamilton-Jacobi ansatz, the classically forbidden trajectory from inside to outside the horizon is given by [37]

$$\Gamma \propto \exp(-2\mathrm{Im}I). \tag{11}$$

For the five-dimensional Lovelock black hole metric, the Hamilton-Jacobi equation is explicitly

$$-f^{-1}(r)(\partial_t I)^2 + f(r)(\partial_r I)^2 + \frac{1}{r^2}(\partial_\theta I)^2$$
$$+\frac{1}{r^2 \sin^2 \theta}(\partial_\varphi I)^2 + \frac{1}{r^2 \sin^2 \theta \sin^2 \varphi}(\partial_\psi I)^2 + m^2 = 0. \tag{12}$$

Considering the symmetry of the black hole metric, we perform the following separation of variables for the action I:

$$I(t, r, \theta, \varphi) = -\omega t + W(r) + J(\theta, \varphi). \tag{13}$$

As a consequence, we have

$$\partial_t I = -\omega; \quad \partial_r I = W'(r); \quad \partial_\theta I = J_\theta; \quad \partial_\varphi I = J_\varphi, \tag{14}$$

where J_θ and J_φ are constants. Since ∂_t is the time-like killing vector for this coordinate system, ω is the energy of the particle as detected by an observer at infinity.

Having these expressions, we can transform Eq. (12) to

$$-\omega^2 f^{-1}(r) + f(r)(W'(r))^2 + r^{-2}(J_\theta)^2 + r^{-2} \sin^{-2} \theta (J_\varphi)^2 + m^2 = 0. \tag{15}$$

Solving for $W(r)$ yields

$$W_\pm(r) = \pm \int \frac{dr}{f(r)} \sqrt{\omega^2 - f(r)(m^2 + r^{-2}J_\theta^2 + r^{-2}\sin^{-2}\theta J_\varphi^2)} \tag{16}$$

since the equation was quadratic in terms of $W(r)$.

One solution corresponds to scalar particles moving away from the black hole and the other solution corresponds to particles moving toward the black hole. Imaginary parts of the action can only result from the pole at the horizon. The probability of crossing the horizon for outgoing particle is

$$\text{Prob(out)} \propto \exp(-2\text{Im}I) = \exp(-2\text{Im}W_+). \tag{17}$$

By using the residue theorem, the expression for the quantity W_+ is

$$W_+ = \frac{2i\pi\omega}{f'(r_{AH})}, \tag{18}$$

where r_{AH} represents the apparent horizon.

When $\Lambda = 0$, substituting Eqs. (3) and (4) into Eq. (18) yields

$$W_+ = \frac{2i\pi(M + \alpha)\omega}{\sqrt{2(M - \alpha)}}, \tag{19}$$

and the rate of radiation,

$$\Gamma \propto \exp\left(-\frac{4\pi(M + \alpha)}{\sqrt{2(M - \alpha)}}\omega\right), \tag{20}$$

is plotted in Figure 1.

For $\Lambda \neq 0$, the quantity W_+ transforms to

$$W_+ = -\frac{3i\pi(4\alpha + r_+^2 + \sqrt{r_+^4 + \frac{4}{3}\alpha\Lambda r_+^4 + 16M\alpha})\omega}{\Lambda r_+(r_+^2 - r_-^2)}. \tag{21}$$
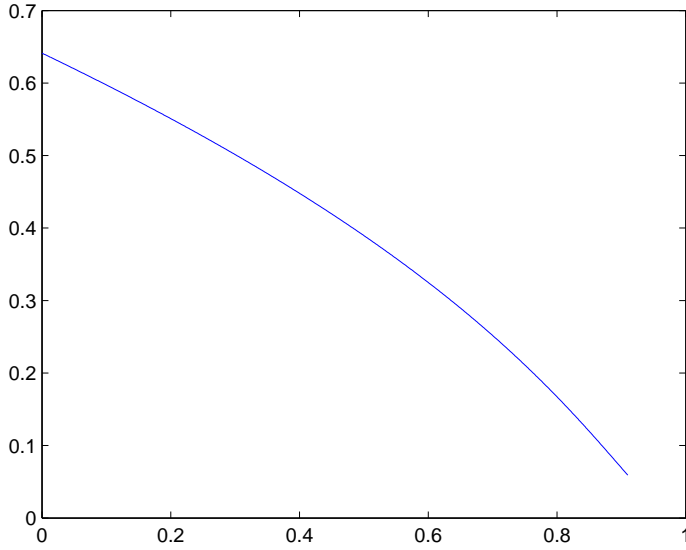
Fig. 1 Variation of W_+ with respect to the ultraviolet correction parameter α, for $\Lambda = 0$.

The relation between the outgoing wave and the incoming wave is given by

$$\Psi_{out} = \exp\left(-\frac{\pi\omega}{\kappa}\right)\Psi_{in},\tag{22}$$

where $\kappa = \left|\frac{f'(r_{AH})}{2}\right|$ is the surface gravity of the black hole and Ψ_{out} and Ψ_{in} are the outgoing wave and the incoming wave, respectively.

The scattering rate of the black hole horizon with a wave function is

$$\left|\frac{\Psi_{out}}{\Psi_{in}}\right|^2 = \exp\left(-\frac{2\pi\omega}{\kappa}\right) = \exp(-2\mathrm{Im}W_+),\tag{23}$$

since the classical theory of black holes tells us that an incoming particle is absorbed with a probability of one.

By substituting into Eq. (17), the probability Γ of the outgoing particle can be expressed as

$$\Gamma \propto \exp(-2\mathrm{Im}W_+) = \exp\left(\frac{6\pi(4\alpha + r_+^2 + \sqrt{r_+^4 + \frac{4}{3}\alpha\Lambda r_+^4 + 16M\alpha})\omega}{\Lambda r_+(r_+^2 - r_-^2)}\right).\tag{24}$$

Its behavior is plotted in Figure 2.

We can see that the probability Γ of the outgoing particle decreases for an increasing cosmological constant. This confirms the fact that dark energy reduces the rate of radiation, as demonstrated for the Reissner-Nordström black hole [32].

The actual value of the cosmological constant is slightly less than these values $(\sim 10^{-120})$ [41]. Considering that assertion, we plot the behavior of the quantity W_+
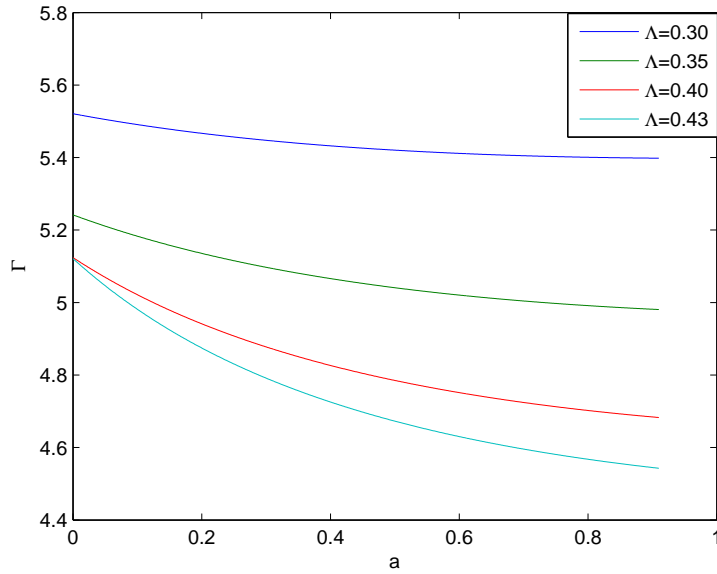
Fig. 2 Variation of W_+ with respect to the ultraviolet correction parameter α, for different values of the cosmological constant Λ.

with respect to the ultraviolet correction parameter α in Figure 3. From this figure, we can remark that, for the given value of Γ, this quantity seems to be independent of the ultraviolet correction parameter α, indicating that, when dark energy is considered, the effect of the ultraviolet correction becomes less perceptible.

3 Conclusion

In summary, we have used the Hamilton-Jacobi method to investigate Hawking radiation of a five-dimensional Lovelock black hole. Explicitly, we have plotted the behavior of the rate of radiation from the black hole. Figure 1 represents the variation of the rate of radiation with respect to the ultraviolet correction parameter a, when $\Lambda = 0$, while Figure 2 represents the variation of the rate of radiation with respect to the ultraviolet correction parameter a, for different values of the cosmological constant $\Lambda \neq 0$. We can conclude through these figures that the black hole radiates at a slower rate when the ultraviolet correction or the cosmological constant are increased. The actual value of the cosmological constant is $\Lambda \sim 10^{-120}$ and so the presence of the cosmological constant makes the effect of the ultraviolet correction on the black hole radiation negligible.
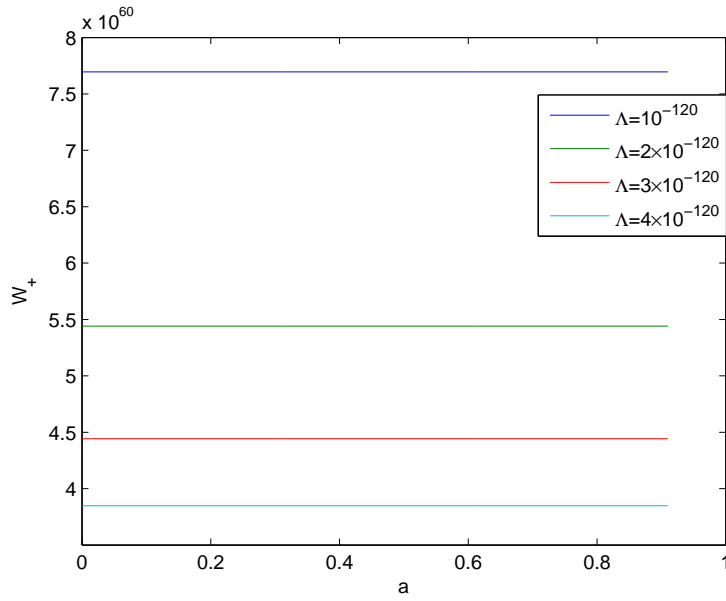
Acknowledgments

Fig. 3 Variation of W_+ with respect to the ultraviolet correction parameter α, for different values of the cosmological constant $\Lambda \sim 10^{-120}$.

References

1. Q.-Q. Jiang and S.-Q. Wu, Hawking radiation of charged particles as tunneling from Reissner-Nordström de Sitter black holes with a global monopole, Phys. Lett. B, 2006, 635(2-3): 151.
2. Yu. P. Goncharov and N. E. Firsova, Hawking radiation for twisted complex scalar fields on the Reissner-Nordstrm black holes and Dirac monopoles, Nucl. Phys. B, 2010, 486(1-2):371.
3. Z. Zhai and W. Liu, Nernst theorem and Hawking radiation from a Reissner-Nordstrom black hole, Astrophys. Space Sci., 2010, 325(1):63.
4. R. Zhao, L.-C. Zhang and H.-F. Li, Hawking radiation of charged particles in Reissner-Nordstrom black hole, Commun. Theor. Phys. 2010, 53(3): 499.
5. K.-X. Jiang, S.-M. Ke and D.-T. Peng, Hawking Radiation as tunneling and the unified first law of thermodynamics for a class of dynamical black holes, Int. J. Mod. Phys. D 2009, 18(11): 1707.
6. H. Pasaoglu and I. Sakalli, Hawking Radiation of Linear Dilaton Black Holes in Various Theories, Int. J. Theor. Phys. 2009, 48(12): 3517.
7. S. W. Hawking, Particle Creation by Black Holes, Commun. Math. Phys. 1975, 43(3): 199.
8. S. W. Hawking, Black hole explosions?, Nature 1974, 248: 30.
9. P. Kraus and F. Wilczek, Selfinteraction correction to black hole radiance, Nucl. Phys. B 1995, 433(2): 403.
10. P. Kraus and F. Wilczek, Effect of selfinteraction on charged black hole radiance, Nucl. Phys. B 1995, 437(1): 231.

11. M. K. Parikh and F. Wilczek, Hawking radiation as tunneling, Phys. Rev. Lett. 2000, 85(24): 5042.
12. T.-M. HE and J.-Y. ZHANG, Tunneling Radiation from a Static Spherically Symmetric Black Hole Surrounded by Quintessence, Commun. Theor. Phys. 2009, 52(4): 619.
13. D. Chen and X. Zu, Massive Particle Tunnels from the Taub-NUT Black Hole, Acta Phys. Pol. B 2008, 39(6): 1329.
14. R. Zhao, L.-C. Zhang and H.-F. Li, Hawking Radiation of Charged Particles in Reissner-Nordstrm Black Hole, Commun. Theor. Phys. 2010, 53(3): 499.
15. Q.-Q. Jiang, Fermions tunnelling from GHS and non-extremal D1D5 black holes, Phys. Lett. B 2008, 666(5): 517.
16. De-You Chen, Q.-Q. Jiang and Xiao-Tao Zu, Hawking radiation of Dirac particles via tunnelling from rotating black holes in de Sitter spaces, Phys. Lett. B 2008, 665(2-3): 106.
17. D.-J. Qi and H.-Q. Ru, Quantum Tunneling of Dirac Particles from the Generalized Spherical Symmetric Evaporating Charged Black Hole, Int. J. Theor. Phys. 2011, 50(1): 269.
18. D.-J. Qi, Dirac Particles' Tunneling Radiation from Dilaton Space-time with Squashed Horizons, Commun. Theor. Phys. 2011, 56(6): 1171.
19. Q.-Q. Jiang and Y. Han, On black hole spectroscopy via adiabatic invariance, Phys. Lett. B 2012, 718(2): 584.
20. C.Z. Liu and G.X. Yu, Entropy spectrum and area spectrum of a modified Schwarzschild black hole via an action invariance, JETP Lett. 2015, 100(10): 615.
21. W.Y. Wen, Gravitational waves from cosmic bubble collisions, Eur. Phys. J. C 2015, 75: 78.
22. S.W. Zhou, G.R. Chen and Y.C. Huang, Entropy Spectrum of a KS Black Hole in IR Modified Hořava-Lifshitz Gravity, Adv. High Energy Phys. 2014, 2014: 396453.
23. Q.Q. Jiang, D.Y. Chen and D. Wen, Remark on massive particle's de Sitter tunneling, JCAP. 2013, 11: 027.
24. H.L. Li and R. Lin, Spectroscopy from the d-dimensional Reissner-Nordström black hole via adiabatic covariant action, Eur. Phys. J. C 2013, 73: 2316.
25. G. González, A. Viceré and L. Wen, Gravitational wave astronomy, Front. Phys. 2013, 8(6):771.
26. S.-N. Zhang, Black hole binaries and microquasars, Front. Phys. 2013, 8(6):630.
27. L. Vanzo, G. Acquaviva and R. Di Criscienzo. Tunnelling Methods and Hawking's radiation: achievements and prospects. Class. Quant. Grav. 2011, 28(18): 183001.
28. H. Ding and W. Liu. Hawking radiation from a Vaidya black hole by Hamilton-Jacobi method. Front. Phys. 2011, 6(1): 106.
29. K. Lin and S.-Z. Yang. Fermions Tunneling from Non-Stationary DilatonMaxwell Black Hole via General Tortoise Coordinate Transformation. Chin. Phys. Lett. 2009, 26(10): 100401.
30. Y.-X. Chen and K.-N. Shao. Invariance of the Hamilton-Jacobi tunneling method for black holes and FRW model. arXiv:1007.4367v2.
31. J.-J. Liu, De-You Chen and S.-Z. Yang. A new method to study the Hawking radiation of the charged black hole with a global monopole. Rom. J. Phys. 2008, 53(5-6): 659.
32. S. Mahamat, T.B. Bouetou and T.C. Kofane. Quasinormal modes and Hawking radiation of a Reissner-Nordstrm black hole surrounded by quintessence, Astrophys. Space Sci. 2011, 333(2): 449.
33. R.-G. Cai, L.-M. Cao and Y.-P. Hu. Hawking radiation of an apparent horizon in a FRW universe. Class. Quantum Grav. 2009, 26(15): 155018.
34. H. Gohar and K. Saifullah. Scalar field radiation from dilatonic black holes. Gen. Relativ. Gravit. 2012, 44(12): 3163.
35. M. Aiello, R. Ferraro and G. Giribet. Exact solutions of Lovelock-Born-Infeld black holes. Phys. Rev. D 2004, 70(10): 104014.
36. J.-H. Chen and Y.-J. Wang. Quasinormal modes of the scalar field in five-dimensional Lovelock black hole spacetime. Chin. Phys. B 2010, 19(6): 060401.
37. M. Agheben, M. Nadalini, L. Vanzo and S. Zerbini, Hawking radiation as tunneling for extremal and rotating black holes, JHEP 2005, 0505: 014.
38. K. Srinivasan and T. Padmanabhan, Particle production and complex path analysis, Phys.Rev. D 1999, 60(2): 24007.
39. S. Shankaranarayanan, K. Srinivasan, and T. Padmanabhan, Method of complex paths and general covariance of Hawking radiation, Mod. Phys. Lett. A 2001, 16(09): 571.

40. S. Shankaranarayanan, T. Padmanabhan, and K. Srinivasan, Hawking radiation in different coordinate settings: Complex paths approach, Class.Quant.Grav. 2002, 19(10): 2671.
41. J. D. Barrow and D. J. Shaw, The Value of the Cosmological Constant, Gen. Relativ. and Gravit. 2011, 43(10): 2555.