## A review of progress in the physics of open quantum systems: theory and experiment

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## Abstract

This Report on Progress explores recent advances in our theoretical and experimental understanding of the physics of open quantum systems (OQSs). The study of such systems represents a core problem in modern physics that has evolved to assume an unprecedented interdisciplinary character. OQSs consist of some localized, microscopic, region that is coupled to an external environment by means of an appropriate interaction. Examples of such systems may be found in numerous areas of physics, including atomic and nuclear physics, photonics, biophysics, and mesoscopic physics. It is the latter area that provides the main focus of this review, an emphasis that is driven by the capacity that exists to subject mesoscopic devices to unprecedented control. We thus provide a detailed discussion of the behavior of mesoscopic devices (and other OQSs) in terms of the projection-operator formalism, according to which the system under study is considered to be comprised of a localized region (Q), embedded into a well-defined environment (P) of scattering wavefunctions (with Q + P = 1). The Q subspace must be treated using the concepts of non-Hermitian physics, and of particular interest here is: the capacity of the environment to mediate a coupling between the different states of Q; the role played by the presence of exceptional points (EPs) in the spectra of OQSs; the influence of EPs on the rigidity of the wavefunction phases, and; the ability of EPs to initiate a dynamical phase transition (DPT). EPs are singular points in the continuum, at which two resonance states coalesce, that is where they exhibit a non-avoided crossing. DPTs occur when the quantum dynamics of the open system causes transitions between non-analytically connected states, as a function of some external control parameter. Much like conventional phase transitions, the behavior of the system on one side of the DPT does not serve as a reliable indicator of that on the other. In addition to discussing experiments on mesoscopic quantum point contacts that provide evidence of the environmentally-mediated coupling of quantum states, we also review manifestations of DPTs in mesoscopic devices and other systems. These experiments include observations of resonance-trapping behavior in microwave cavities and open quantum dots, phase lapses in tunneling through single-electron transistors, and spin swapping in atomic ensembles. Other possible manifestations of this phenomenon are presented, including various superradiant phenomena in low-dimensional semiconductors. From these discussions a generic picture of OQSs emerges in which the environmentally-mediated coupling between different quantum states plays a critical role in governing the system behavior. The ability to control or manipulate this interaction may even lead to new applications in photonics and electronics.

## Introduction

A core problem in modern physics, and one which has evolved to assume an unprecedented interdisciplinary character, is related to the study of open quantum systems (OQSs). In the broadest sense, these systems may be defined as consisting of some localized, microscopic, region that is coupled to an external environment by means of an appropriate interaction. In discussions of the crossover from quantum mechanics to classical physics, the role of such an environment is often invoked to describe the influence of a classical measuring apparatus. Even if one is to remove such an apparatus, however, the description of most open systems may nonetheless be reduced to one in which the properties of some microscopic region are influenced by its coupling to its own, "natural", environment. In contrast to the aforementioned measurement problem, the influence of this environment can never be deleted but exists at all times, independent of any observer. In essence, this environment functions as an "intrinsic" (natural) measuring apparatus, introducing a coupling between different states of the open system. Two very-different cases in which this situation applies are provided by the decay of unstable states in nuclei, and the transport of electrons through mesoscopic quantum dots. In the former case, the environment consists of a continuum of scattering wavefunctions outside of the nucleus, which may mediate the escape of either a neutron or a proton from that structure [1]. Similarly, in discussions of quantum-dot transport, the discrete quantized states of an isolated cavity develop a broadening when the cavity is coupled to macroscopic reservoirs through the addition of appropriate leads. In this latter case, the environment is provided by the states of these reservoirs, and the broadening of the discrete cavity levels is strongly dependent upon the mutual overlap with these states, as mediated through the leads [2].

There are numerous examples of OQSs, from a broad spectrum of disciplines within physics. While obvious examples include nuclei, atoms, and molecules, a much broader range of systems is provided by lasers and optically-active media, biomolecules and molecular networks, nanophotonic structures, and mesoscopic electronic devices. It is the latter systems (namely mesoscopic devices) that we focus on in this review, motivated by two of their important features. The first of these is their capacity to exhibit a variety of rich phenomena arising from their environmental interaction, while the second is the ability to subject them to sophisticated external control.

Quite generally, mesoscopic devices are small metallic or semiconducting structures, in which carrier motion is constrained on a spatial scale that is comparable to, or even smaller than, the fundamental length scales associated with transport. These length scales include the elastic mean free path, the phase-coherence length, and the inelastic scattering length, all of which may exceed the size of sub-micron scale devices at sufficiently low temperatures [3]. In terms of their general structure, these devices consist of some central scattering region, in the form of a quasi-one dimensional wire or a quasi-zero-dimensional quantum dot (QD), whose carriers may be injected into and extracted from in order to allow for transport. Such transport is achieved by connecting the central scattering region of the device to macroscopic charge reservoirs, by means of appropriate lead structures. When compared with systems such as nuclei, the great advantage that mesoscopic devices offer is the capacity to externally control the key parameters of the system. These include the system size, and thus the nature of the quantum states involved in transport, the strength of the coupling between the system and its environment, and the energy of the carriers involved in transport. This control makes these structures ideally suited to the study of forefront issues associated with OQSs. We demonstrate this here for few-electron QDs [3, 4], which are the solid-state analog of scattering billiards, and for quasi-one-dimensional quantum point contacts (QPCs), in which the presence of a strong environmental coupling may modify the very nature of the quantum states responsible for transport [5].

In this Report on Progress, we focus on providing a detailed discussion of the behavior of OQSs in terms of the projection operator formalism of quantum mechanics. In this approach, as indicated in Fig. 1, we consider the system under study to be comprised of a localized region (Q) that is embedded in a well-defined environment (P) of scattering wavefunctions. The complete function space satisfies the requirement Q + P = 1, and is described in terms of a Hermitian Hamiltonian. The Hamiltonian describing the subspace of interest (Q) is non-Hermitian, however, and has complex eigenvalues that define both the energy and broadening of its states [1]. These eigenstates are coherently coupled to each other, via an interaction that is mediated by the environment (P). This projection operator formalism differs from that in which complex scaling is used to analyze non-Hermitian systems, an approach that has been summarized in Ref. [6] and that we will not discuss. We also note that the role of the environment considered here is very different to that which is often invoked in discussions of decoherence, arising from the uncontrolled interaction of a quantum

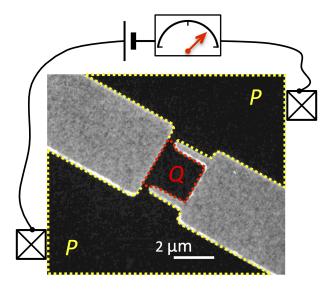


FIG. 1: Electron micrograph showing an example of a mesoscopic device, namely a gated quantum dot. The gray regions in the figure correspond to a pair of metal gates, which are formed on top of a semiconductor substrate (the dark areas). Application of a suitable voltage to the gates creates depletion fields that remove electrons from underneath the gates, forming a square-shaped cavity approximately 2  $\mu$ m in size. To illustrate the connection to the non-Hermitian concept, we have also indicated in the figure how this mesoscopic device may essentially be viewed as consisting of a localized quantum system (denoted here as the region 'Q') that is embedded within an environment of scattering states (denoted as the regions 'P'). The coupling between regions Q and P is regulated by means of the two QPC openings in the cavity, which correspond to the "leads" referred to in the main text. In this figure, we have also indicated the presence of a classical measuring apparatus that is *outside* of the P + Q space. While this measurement apparatus provides an "environment" that can be removed from the system, the influence of the environment P, in contrast, can never be deleted.

system with its environment (such as an electron or phonon bath). This latter problem has been treated in other works (see, for example, the reviews of [7, 8]), where the feedback from the environment onto the quantum system is not explicitly considered. This is in contrast with the situation here, where we are concerned, above all, with understanding the influence of the natural environment (P) on the system (Q). This environmental influence remains even when sources of decoherence have been suppressed (by lowering the temperature, for example). As such, the approach that we outline is expected to be useful in describing the behavior of small systems, in which the number of particles active in transport is small, and in which the number of quantum states they occupy is similarly restricted in number. Under such conditions we will see that, far from acting as a source of decoherence, the coupling to the environment can in fact mediate an extremely robust and coherent interaction.

The objective of this review is to focus on critical implications of non-Hermiticity, namely

the presence of exceptional points (EPs) in the spectrum of OQSs, their influence on the rigidity of the wavefunction phases, and their ability to initiate a dynamical phase transition (DPT). As we discuss in further detail below, these features can be observed because of our ability to vary the strength of the environmental interaction in OQSs, by means of some suitable control parameter. EPs are singular points in the continuum, at which two resonance states coalesce, that is where they exhibit a non-avoided crossing. This behavior should be contrasted with the well-known avoided crossings, exhibited by Hermitian systems at so-called diabolic points. Due to the coalescence of the two resonance states at an EP, the time evolution of the system becomes undefined in its neighborhood. At the same time, the EPs are branch points in the complex plane and are responsible for the generation of double poles of the S matrix, with associated implications for transmission.

An important quantity that indicates the range of influence of an EP is the phase rigidity of its two eigenfunctions. In isolated (Hermitian) systems the phases of the eigenfunctions are said to be rigid, a statement that expresses the well-known orthogonality of these functions. In open (non-Hermitian) systems, however, the presence of the environmentally-mediated coupling between the eigenstates means that they are no longer orthogonal, but rather exhibit the property of biorthogonality. This allows the rigidity of the eigenfunction phases to be reduced, particularly near an EP where the rigidity actually vanishes. This surprising behavior expresses the very strong influence of the environment on the localized system at an EP.

While the physical significance of EPs for the dynamics of OQSs has only recently begun to be studied (see, for example, the review [1]), they are nonetheless understood to be intimately connected to the observation of DPTs in different open systems. The term DPT has been coined to refer to a phenomenon in which the quantum dynamics of the open system undergoes a phase transition between non-analytically connected states as a function of some suitable control parameter. Here, time-dependent approaches to the description of the problem fail [9]. Instead, much like a conventional phase transition in, for example, magnetism or superconductivity, the physical behavior of the system on one side of the transition does not serve as a reliable indicator of that on the other. The connection of DPTs to the EPs is provided by the phenomenon of width bifurcation, which results in a spectroscopic redistribution in the system into long-lived states with narrow linewidths and a much smaller number of short-lived, strongly-broadened, ones. An important objective of

this review is to connect the theoretical concepts of EPs and DPTs to the physical behavior manifested in experimental investigations of OQSs.

One of the earliest works to explicitly invoke the notion of a DPT involved studies of spin swapping in atomic systems [10]. In these experiments, the authors studied Rabi oscillations due to spin flips in the <sup>13</sup>C<sup>-1</sup>H system, and showed a transition to strongly damped motion by increasing the coupling to an environment formed by a spin bath. Although not explicitly discussed as such, evidence of DPTs is apparent, also, in earlier work on resonance trapping in microwave cavities [11], and on einselection [12–14] in mesoscopic quantum dots. In both of these systems, the focus was on understanding the manner in which the states of a quantized cavity are affected by coupling them to an external environment. The common phenomenon revealed in both cases was of width bifurcation, with certain eigenstates actually becoming narrower when the environmental coupling was increased over a specific range. The accompanying short-lived eigenstates appear as a background with which the long-lived ones interfere, a behavior that has been observed experimentally in microwave cavities [11]. Ultimately, the long-lived states may even become discrete, forming so-called bound states in the continuum [15]. With the renewed interest in such problems, it is actually interesting to note that the appearance of long-lived "merkwürdigen" (remarkable) states is actually a problem that dates back to the earliest days of quantum mechanics [16]!

Elsewhere, evidence of a DPT has also been provided in work where a multi-level quantum dot was embedded into one of the arms of an Aharonov-Bohm interferometer, allowing the evolution of the transmission phase to be monitored across a sequence of resonance states [17–19]. These experiments revealed the presence of unexpected regularity in the measured scattering phases (so-called "phase lapses"), when the number of states occupied by electrons in the dot was sufficiently large. While this behavior could not be fully explained within approaches based upon Hermitian quantum theory [20, 21], it has recently been established that the phase lapses can be attributed to the non-Hermitian character of this mesoscopic system, and to a DPT that occurs as the number of electrons in the dot is varied [22]. The observed regularity arises from the overlap of the many long-lived states with the short-lived one, all of which are formed due to the DPT. More recently, studies of transport in ballistic quantum wires may have also revealed a DPT, involving the formation of a protected channel for conduction under strongly nonequilibrium conditions. In these experiments the environment is essentially provided by the phonon system, whose influence

is controlled by means of the applied source bias and by the strong quantum confinement of the carriers induced within the wire [23]. A well-known phenomenon from optics that may also involve a DPT is that of Dicke superradiance [24]. This refers to the effect in which an ensemble of excited atoms within a cavity does not emit radiation randomly, but rather does so in a correlated manner when the atoms experience the same radiation field. Recently, superradiance has been demonstrated for solid-state systems, namely ensembles of self-assembled quantum dots [25] and a dense semiconductor electron-hole plasma [26].

In this review, we will use the various experiments described above to connect the behavior exhibited by OQSs to the key concepts of non-Hermiticity, EPs, and DPTs. We will also introduce the important concept of wavefunction phase rigidity, and the significance of time in open systems, in which we are unable to describe a DPT by means of time-dependent approaches. We emphasize again that the focus of our review will be on discussions of the properties of open systems consisting of a small number of particles. As such, we explore the behavior in a very different limit to that relevant, for example, in heavy nuclei. These contain a much larger number of particles, as well as many closely-neighboring states, and are adequately described by the concepts of random-matrix theory [27]. In fact, with the exception of a few specific examples, we will not address phenomena arising in nuclear systems, preferring to note instead that this topic was recently excellently served by an associated review in this journal [28]. Due to the continued importance of mesoscopic systems for the investigation of fundamental quantum phenomena, we believe that the focus of our review will prove to be a particularly useful one.

The remainder of this paper is organized as follows. In the next section, we introduce the concept of small quantum systems that are coupled to an environment with which they interact. We begin by reviewing some of the well known properties of isolated (Hermitian) quantum systems, emphasizing concepts such as the real nature of their eigenvalues and the rigidity (orthogonality) of their eigenfunctions. Following this, we next introduce the projection-operator formalism for OQSs, in which the total system is described in terms of two coupled subspaces (Q & P). The complex character of the eigenvalues and eigenfunctions of the non-Hermitian operator of the Q subspace is described, and its physical implications are discussed. We introduce also the concept of gain and loss in OQSs. In the last part of this section, some physical examples of the environmentally-mediated coupling of quantum states are presented, focusing on their manifestations in mesoscopic structures.

In Section III, we discuss the importance of EPs to the behavior of OQSs. We formulate this discussion by first of all focusing on the interaction of two resonance states near an EP, following which we study the influence of an EP on the eigenvalues and eigenfunctions of the non-Hermitian Hamiltonian. In Section IIID we discuss the implications of EPs for the S-matrix of the system, a problem with important implications for the analysis of transmission. In Section IV, we consider the role of EPs in giving rise to DPTs in open systems subjected to some form of external control. In the presence of this control, DPTs are found to occur when the range of influence of several different EPs develops a sufficiently strong overlap. The connection of DPTs to the notion of width bifurcation is emphasized, and several experimental demonstrations of such phase transitions are presented. These include demonstrations of resonance trapping in microwave cavities, studies of phase lapses in tunneling through quantum dots, and measurements of spin swapping in atomic systems. Emphasizing our belief that DPTs are, in fact, an inherent feature of OQSs in general, in Section V we discuss further examples of physical problems in which DPTs may occur. In Section VI, we conclude this review by summarizing its main points and by identifying important issues for further study. Two appendices to the paper are provided, in the first of which (Appendix A) we make a few additional remarks on the non-Hermitian Hamitonian (with and without using perturbation theory), and give expressions for the coupling matrix elements (the partial width amplitudes) between the system and the environment. Appendix B provides a list of principal symbols and acronyms used in the paper.

## Conclusions

In this Report on Progress, we have reviewed the key physical attributes of OQSs in the regime of closely-neighboring states and have described a number of different experimental systems in which they are manifested. At the most basic level, these attributes may be understood as arising from the fact that the Hamiltonians needed to describe the dynamics of open systems are non-Hermitian, in marked contrast to their Hermitian counterparts that so accurately describe the behavior of isolated quantum systems. In the non-Hermitian formalism, the open system is viewed as being comprised of a localized region (Q), embedded in a well-defined environment (P) of scattering wavefunctions, and the total system (Q +P=1) is Hermitian. The coupling between the two subspaces is found to mediate an effective interaction among the original states of the localized system. This natural (intrinsic) environment does not serve as a source of decoherence (the role of which is not considered here) and its influence can never be deleted from the system. Instead, it gives rise to complex eigenvalues, whose real component defines the energy of the state and whose imaginary part describes an effective level broadening. The eigenfunctions also exhibit an important difference with those of Hermitian systems, in that they are no longer orthogonal to one another but are instead biorthogonal. Physically, the biorthogonality expresses the fact that there is an environmentally-induced coupling between the different states of the system. This should be contrasted with the case of Hermitian quantum mechanics, which yields orthogonal stationary states that require a perturbation of the system in order to be coupled. The description of open systems in terms of this approach is found to be well suited to the solution of a broad range of problems, providing proper care is taken to accurately identify the subspaces Q and P. Under such conditions, this approach is expected to provide a good description of small systems, in which the number of relevant quantum states, as well as the number of channels, is restricted. In larger systems that do not conform to these constraints, other approaches, such as random-matrix theory, should be more effective. Such approaches have been extensively treated in the literature and have therefore not been considered here.

We have seen in this review how the eigenstates of the non-Hermitian Hamiltonian exhibit fundamentally new behavior not encountered in Hermitian physics. The first example is provided by the appearance of singular points, referred to as exceptional points (EPs). Arising when an appropriate system parameter (such as the coupling strength between the

Q and P subspaces) is varied, the EP involves the coalescence of two eigenvalues of the non-Hermitian operator and (up to a phase) their corresponding eigenfunctions. Although a point of measure zero, the EP exerts a strong influence on the spectral properties of the system, extending over a wider region than the singular point at which the two eigenvalues coalesce. An important property that indicates the range of influence of the EP is the phase rigidity of the eigenfunctions. Essentially, the phase rigidity is a measure of the biorthogonality of different eigenfunctions, and, therefore, of the strength of the environmentally-induced interaction between the different states. In the region of the EP the phase rigidity is significantly reduced, expressing the fact that this interaction via the environment is very strong at the EP. As a result, the wavefunctions of these states are no longer orthogonal with one another but are equal, up to a phase. Over the range where the phase rigidity is reduced, the two eigenstates undergo the phenomenon of width bifurcation, with one state becoming strongly broadened while the other becomes more stable as it decouples from the environment. Under this condition, transport through the system occurs with high efficiency. Simultaneous with this, the eigenvalues may cross in energy. This is in complete contrast to the behavior of Hermitian systems, which may only ever exhibit an avoided level crossing when two of their eigenstates are close in energy.

The phenomenon of width bifurcation, in which the width of one state increases with increasing environmental coupling while that of the other decreases, is connected to a violation of the Fermi golden rule. This rule suggests that increased coupling should always lead to stronger broadening, in contrast to the behavior of the long-lived state produced by width bifurcation.

The other important feature of OQSs that we have discussed is their ability to undergo a dynamical phase transition (DPT), when the ranges of influence of different EPs overlap with one another. The DPT involves a change in the dynamics of the system, which arises when width bifurcation occurs hierarchically for many states. The simplest example of this phenomenon occurs when all states of the system are coupled to a single channel, resulting in the formation of just one strongly-broadened state. The remaining states are then essentially stabilized, through a process in which they strongly decouple from the environment. In the two-channel case relevant for transport, essentially the same picture holds although now two of the original states develop the strong broadening, leaving all but these two states stabilized. Much like conventional phase transitions in thermodynamics, the details of the

dynamics on either side of the DPT are not analytically connected to each other. That is, the behavior on one side of the transition does not serve as a reliable indicator of that on the other. Both the width bifurcation caused by the EPs, and the DPT that these give rise to, result from the fact that the coupling strength to the environment has an imaginary component.

Experimental evidence in support of these various concepts has been provided throughout the course of this review, with a particular emphasis having been placed on their manifestations in mesoscopic systems. At the heart of the non-Hermitian scheme is the idea of environmentally-mediated coupling, which yields an effective interaction among the states of the quantum system. We have seen how this coupling is manifested directly in experiments involving non-locally coupled quantum point contacts (QPCs), in which an intervening continuum serves as the P subsystem and mediates a coupling between remote pairs of QPCs. We have described how this coupling gives rise to novel Fano-resonance phenomenology, and have also observed that the environmentally-mediated interaction may provide a stronger coupling between quantum states than a simple tunneling overlap. This may be attributed to the fact that the environmental interaction is supported by a large number of states that comprise a continuum, resulting in a much stronger effect than would be expected to arise from the direct interaction between the different states of Q.

Several experimental demonstrations of DPTs in physical systems were also presented. These examples included: observations of resonance-trapping and its related phenomena, in open microwave cavities and QDs; a crossover from mesoscopic to universal behavior in the phase lapses found in single-electron tunneling through Coulomb-blockaded QDs; observations of spin-swapping dynamics in ensembles of interacting spins, and; energy-transfer processes in photosynthesis. Similar phenomena are observed also in optics, most notably in the form of the long-studied problem of Dicke superradiance, as well as more recent investigations of systems with gain and loss. Here, however, a better understanding of the equivalence between the optical-wave and Schrödinger equations is needed before convincing conclusions can be drawn. In addition, we have also highlighted several other examples of complex physical phenomena that may well be related to the occurrence of a DPT. These include superradiance in QDs, and the observation of protected-subband formation in the nonequilibrium transport of hot electrons through QPCs. These observations by no means represent an exhaustive list, but rather suggest instead that DPTs are a common property

of OQSs in general.

Having summarized the general status of this field, we can now identify a number of important issues that should be resolved in future studies. While the different experiments described in this review provide several hints that DPTs do indeed occur in OQSs, the presence of the distinct time scales that are expected to accompany this phenomenon has yet to be directly demonstrated. According to the description of width bifurcation given above, the environmentally-induced interaction in the vicinity of an EP is expected to result in the formation of eigenstates with very different lifetimes; the first of these is very short and associated with the strongly-broadened state, while the other is much longer and related to the stabilized state. Experiments that can probe the existence of these different time scales are therefore desired. A problem in attempting to design such experiments, however, comes from the fact that this problem must first be clearly formulated from a theoretical perspective. A simpler task may be to provide a demonstration of the width bifurcation that should serve as the signature of the different time scales, but, to the best of our knowledge, this has not yet been done experimentally, in either mesoscopic or any other physical systems.

Time-resolved investigations of OQSs may also be useful in exploring issues associated with irreversibility in the vicinity of the DPT. The issues here can be discussed by referring to the features of Fig. 9, in which we show the system properties and their variation in the vicinity of two EPs. As either EP is approached from the left or the right of the figure, the width bifurcation begins at the EP and becomes maximal once d = 0. At this point the phase rigidity returns to unity, indicating that the two eigenfunctions are orthogonal. Physically, this can be understood to result from the fact that the two states are now associated with very different time scales, and so no longer interact with one another. While the results of Fig. 9 suggest that, by passing continuously through d = 0 and the point of maximum bifurcation, it should be possible to return to two distinct states, in practice this is not possible. In essence, one of the original states has now been "lost" to the environment, and the remaining stable state is therefore unable to interact directly with it. This is completely consistent with the notion of a DPT; the spectroscopic properties of the system on one side of the transition are not analytically connected to those on the other, and nonlinear terms in the Schrödinger equation play an important role in this transition.

Another important property that is of interest in characterizing the properties of OQSs is the phase rigidity of their eigenfunctions. The value of this parameter becomes vanishingly small near an EP, and it is useful also for establishing the range of influence of such points. While the rigidity itself can probably not be measured directly in a physical system, we have seen here that it is related to a high efficiency of transport (recall Fig. 13). While this connection may not have been widely appreciated to date, it implies that measurements of the conductance of mesoscopic systems can provide a flexible probe of this important system property. Moreover, the enhancement of the conductance around a DPT is, by itself, of interest for possible applications.

Throughout this review, we have provided a number of different examples of DPTs, occurring in a variety of physical systems. From these examples we suggest that the DPT should be a very general characteristic of open systems, with broader consequences than have been appreciated to date. Indeed, the stability properties of classical systems are also governed by non-Hermitian degeneracies (EPs), as has been nicely shown in [116]. Our hope therefore is that this review can stimulate interest in the study of this still relatively unexplored problem, and ultimately lead to a deeper understanding of its implications.

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Note added in proof. Subsequent to the acceptance of our report we became aware of recently published work by Ruderman et al. [118], demonstrating the possibility of a quantum dynamical phase transition in molecular chemical bond formation and dissociation. The occurrence of this transition was connected to the non-Hermitian nature of the Hamiltonian in the molecular system, consistent with the dicussions of dynamical phase transitions given here.

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