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Complete construction of magical, symmetric and homogeneous $\mathcal{N}=2$ supergravities as double copies of gauge theories

M. Chiodaroli,[a] M. Günaydin,[b] H. Johansson,[c,d] and R. Roiban[b]

[a] *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Am Mühlenberg 1, 14476 Potsdam, Germany*
[b] *Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park PA 16802, USA*
[c] *Department of Physics and Astronomy, Uppsala University, 75108 Uppsala, Sweden*
[d] *Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 10691 Stockholm, Sweden*

We show that scattering amplitudes in magical, symmetric or homogeneous $\mathcal{N}=2$ Maxwell-Einstein supergravities can be obtained as double copies of two gauge theories, using the framework of color/kinematics duality. The left-hand-copy is $\mathcal{N}=2$ super-Yang-Mills theory coupled to a hypermultiplet, whereas the right-hand-copy is a non-supersymmetric theory that can be identified as the dimensional reduction of a D-dimensional Yang-Mills theory coupled to P fermions. For generic D and P, the double copy gives homogeneous supergravities. For $P=1$ and $D=7,8,10,14$, it gives the magical supergravities. We compute explicit amplitudes, discuss their soft limit and study the UV-behavior at one loop.

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Perturbative calculations in gravity and gauge theory have long been considered to be on fundamentally different footing. Gravity is characterized by a non-polynomial, non-renormalizable action that produces an infinite number of interaction vertices, whereas renormalizable gauge theories only have cubic and quartic interactions. Despite these obvious differences, modern work has clarified that the perturbative expansion of gravity is directly related to that of a pair of gauge theories through a double-copy structure.

It has long been known that the asymptotic states of gravity can be obtained as tensor products of gauge-theory states. That such a simple relationship can be extended to certain interacting theories was first shown 30 years ago by Kawai, Lewellen and Tye [1] using string theory. Modern understanding of this double-copy structure comes from work by Bern, Carrasco and one of the current authors [2, 3], who found a framework that is applicable to loop-level amplitudes and to a broader range of theories. The key observation is that gauge-theory amplitudes can be organized to expose a kinematic Lie algebra which mirrors the gauge-group color structure. Once gauge-theory amplitudes exhibit this duality between color and kinematics, gravity amplitudes are obtained by substituting the color factors with equivalent kinematic objects. This procedure doubles the kinematic structures and thus expresses spin-2 theories as double copies of spin-1 theories [2].

The double copy construction has proven itself to be a powerful computational tool. It fostered rapid progress in ultraviolet (UV) studies up to four loops in maximal, half-maximal and $\mathcal{N}=5$ supergravities [4–6]. Moreover, a class of black-hole solutions has been shown to exhibit a double-copy structure which relates them to solutions of Maxwell's equations with sources [7–9].

The double copy permits the construction of a broad range of gravitational theories by varying the content of matter (spin $\leq 1/2$) fields and their representations and interactions in the two gauge theories. Pure and matter-coupled gravities, including examples of Maxwell-Einstein and Yang-Mills-Einstein theories, are some of the theories that admit an elegant perturbative formulation in this framework [1–3, 10–16].

A systematic classification of $\mathcal{N}<4$ supergravities that admit double-copy constructions has not yet been obtained. There is a rich space of such theories, and it is not *a priori* obvious that the double copy can reproduce this abundance. Indeed, in this context it is natural to ask whether the double-copy structure can be a general property of gravitational theories.

In this letter we consider $\mathcal{N}=2$ Maxwell-Einstein supergravity (MESG) theories dimensionally reduced from five to four spacetime dimensions. These theories provide a tractable arena in which ideas about scattering amplitudes in generic gravitational theories can be tested. Unlike more supersymmetric theories, they are not uniquely specified by their matter content alone. However, due to their five-dimensional origin, theories in this class can be identified from their three-point interactions [17]. Using this property, we shall provide a double-copy construction for three complete classes of $\mathcal{N}=2$ MESG theories: magical, symmetric, and homogeneous theories (the latter class containing the former).

Homogeneous $\mathcal{N}=2$ MESG theories. While we are ultimately interested in MESG theories in four dimensions, we shall begin our analysis in five dimensions. Unlike $4D$ theories, the full U-duality groups of $5D$, $\mathcal{N}=2$ MESG theories are symmetries of their Lagrangians. Furthermore, $\mathcal{N}=2$ MESG theories that describe low-energy effective theories of compactified M/superstring theory admit uplifts to five dimensions once quantum corrections are neglected [18]. When coupled to n vector multiplets, such five-dimensional theories contain $(n+1)$ abelian vector fields A^I_μ $(I,J=0,\ldots,n)$, n real scalar fields ϕ^x $(x,y=1,\ldots,n)$, and n symplectic-Majorana spinors. Their Lagrangian is [17]:

$$e^{-1}\mathcal{L} \;=\; -\frac{1}{2}R - \frac{1}{4}\overset{\circ}{a}_{IJ}F^I_{\mu\nu}F^{\mu\nu J} - \frac{1}{2}g_{xy}(\partial_\mu\phi^x)(\partial^\mu\phi^y) + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda + \text{fermions}\;, \quad (1)$$

where $F^I_{\mu\nu}$ are abelian field-strengths. A remarkable property of these theories is that the Lagrangian is uniquely determined by the constant symmetric tensor C_{IJK} whose invariance group coincides with the U-duality group. The scalar manifold of $5D$ MESG theories can be interpreted as the hypersurface defined by $\mathcal{V}(\xi)\equiv(2/3)^{3/2}C_{IJK}\xi^I\xi^J\xi^K=1$ in an $(n+1)$-dimensional ambient space with the metric

$$a_{IJ}(\xi)\equiv -\frac{1}{2}\frac{\partial}{\partial\xi^I}\frac{\partial}{\partial\xi^J}\ln\mathcal{V}(\xi)\,. \quad (2)$$

The matrix $\overset{\circ}{a}_{IJ}$ in the kinetic-energy term of the vector fields is the restriction of the ambient-space metric to the constraint surface, while the metric g_{xy} of the scalar manifold is the pullback of the ambient-space metric to the constraint surface.

The given structure is sufficient to calculate the bosonic part of the amplitudes we will discuss in this letter. We refer the reader to [17] for further details and for the fermionic terms. These terms involve a symmetric tensor T_{xyz} which is the pullback of the C-tensor to the constraint surface. The C-tensors of the theories with covariantly-constant T_{xyz} are defined by the cubic norms of Euclidean Jordan algebras of degree three, and their scalar manifolds are symmetric spaces [17]. The four magical MESG theories are defined by simple Jordan algebras of Hermitian 3×3 matrices over reals and complex numbers, quaternions, and octonions and are unified theories [19]. The generic Jordan theories are defined by the infinite family of non-simple Jordan algebras of degree three. These two classes exhaust the list of $5D$ MESG theories with symmetric target spaces such that the full isometry group is a symmetry of the Lagrangian. In $4D$ there exists an additional family of homogeneous theories whose target spaces are the complex projective spaces $SU(n,1)/U(n)$ [20] and which can be obtained from those defined by Jordan algebras of degree 3 by truncation.

The most general form of the C-tensor consistent with unitarity was given in [17] and depends on $n(n^2-1)$ parameters. The cubic norms $\mathcal{V}(\xi)$ of MESG theories with homogeneous scalar manifolds and a transitive group of isometries can be brought to the form [21]

$$\mathcal{V}(\xi)=\sqrt{2}\big(\xi^0(\xi^1)^2-\xi^0(\xi^i)^2\big)+\xi^1(\xi^\alpha)^2+\tilde{\Gamma}^i_{\alpha\beta}\xi^i\xi^\alpha\xi^\beta\;, \quad (3)$$

where $i,j=2,3,\ldots,q+2$ and α,β are indices with range r. $\tilde{\Gamma}^i_{\alpha\beta}$ are symmetric gamma matrices forming a real representation of the Clifford algebra $\mathcal{C}(q+1,0)$. $\mathcal{V}(\xi)$ in eq. (3) are generically labeled by two integers $q\geq-1$ and $P\geq0$, except when $q=0,4$ (mod 8), in which case the extra parameter $\dot{P}\geq0$ is also present.

The corresponding MESG theories give the coupling of $(2+q+r)$ vector multiplets to the gravity multiplet in $5D$, with $r=P\mathcal{D}_q$ or $r=(P+\dot{P})\mathcal{D}_q$. The values for $\mathcal{D}_q$ are listed in table I. The generic Jordan family corresponds to $q=\dot{P}=0$ and P arbitrary and to $P=\dot{P}=0$ and q arbitrary; the magical theories correspond to $P=1$ and $q=1,2,4,8$, while the generic non-Jordan family theories correspond to $q=-1$.

To obtain scattering amplitudes for the theories defined by the C-tensors corresponding to eq. (3) it is convenient to first reduce their Lagrangian (1) to four dimensions. The bosonic spectrum of the resulting 4D MESG theory contains the graviton, $(n+2)$ vectors $A^{-1}_\mu, A^0_\mu,\ldots,A^n_\mu$ and $(n+1)$ complex scalars $z^0,\ldots,z^n$. The $4D$ Lagrangian is associated to the following holomorphic prepotential in a symplectic formulation [22–24],

$$F(Z^A)=-\frac{2}{3\sqrt{3}}\frac{C_{IJK}Z^IZ^JZ^K}{Z^{-1}}\;, \quad (4)$$

where $Z^A(z)$ are holomorphic functions of the scalars z^I.

To carry out perturbation theory it is necessary to expand the Lagrangian around some base point, as well as redefine (and dualize) fields to enlarge the manifest symmetry and obtain canonically-normalized quadratic terms. To this end we follow [16] and:

1. Choose the base point $Z^A=\big(1,\frac{i}{2},\frac{i}{\sqrt{2}},0,\ldots,0\big)$, which corresponds to $\xi^I=\big(\frac{1}{\sqrt{2}},1,0,\ldots,0\big)$ in $5D$.

2. Dualize the graviphoton field, A^{-1}_μ.

3. Take linear combinations of the new vector fields,

$$\begin{aligned}A^{-1}_\mu &\to \frac{1}{4}\big(A^{-1}_\mu - A^0_\mu - \sqrt{2}A^1_\mu\big)\;,\\ A^0_\mu &\to \frac{1}{2}\big(-A^{-1}_\mu + A^0_\mu - \sqrt{2}A^1_\mu\big)\;,\\ A^1_\mu &\to -\frac{1}{\sqrt{2}}\big(A^{-1}_\mu + A^0_\mu\big)\;. \end{aligned} \quad (5)$$

4. Dualize the new A^1_μ field and redefine $z^1\to -iz^1$.

The resulting Lagrangian is used to construct amplitudes which are compared with the ones from the double copy.

Double-copy construction. The m-point amplitudes of YM theories are naturally represented by cubic graphs

q	$\mathcal{D}_q$	$r(q,P)$	conds	flavor group	C
-1	1	P	R	$SO(P)$	$\mathcal{C}_q$
0	1	$P+\dot{P}$	RW	$SO(P)\times SO(\dot{P})$	$\mathcal{C}_q\mathcal{P}_\pm$
1	2	$2P$	R	$SO(P)$	$\mathcal{C}_q$
2	4	$4P$	R/W	$U(P)$	$\mathcal{C}_q$
3	8	$8P$	PR	$USp(2P)$	$\mathcal{C}_q\Omega$
4	8	$8P+8\dot{P}$	PRW	$USp(2P)\times USp(2\dot{P})$	$\mathcal{C}_q\Omega\mathcal{P}_\pm$
5	16	$16P$	PR	$USp(2P)$	$\mathcal{C}_q\Omega$
6	16	$16P$	R/W	$U(P)$	$\mathcal{C}_q$
$k+8$	$16\,\mathcal{D}_k$	$16\,r(k,P)$	as for k	as for k	as for k

TABLE I: Parameters in the construction of homogeneous MESGs as double copies. The second column gives the parameter $\mathcal{D}_q$, the third column gives the number r of $4D$ irreducible spinors in the $\mathcal{N}=0$ gauge theory, which can obey a reality (R), pseudo-reality (PR) or Weyl (W) conditions. The global flavor group and the matrix C in eqs. (13-14) are listed in the last two columns. $\mathcal{P}_\pm$ denote projectors restricting the spinor representations to the desired chiralities.

labeled by their topology, gauge-group representations of internal and external edges, and particle momenta. The i-th graph is associated to the product of the corresponding propagators, to a color factor c_i constructed by dressing each cubic vertex by the Clebsh-Gordan coefficient of the representations of the three fields (structure constants or group generators), and to a kinematic numerator n_i encoding the remaining state dependence. To construct an amplitude which manifestly obeys color/kinematics duality one must find kinematic numerators with the same symmetries and algebraic identities as the color factors. Schematically

$$c_i+c_j+c_k=0 \quad \Leftrightarrow \quad n_i+n_j+n_k=0\ , \qquad (6)$$

where the color factor identities stem from the commutation/Jacobi relations of the gauge group and thus involve three graphs.

The double-copy principle states that once duality-satisfying numerators are found, the L-loop amplitudes of a supergravity theory are given by

$$\mathcal{M}^{(L)}_m = i^{L+1}\left(\frac{\kappa}{2}\right)^{2L+m-2}\sum_{i\in\text{cubic}}\int\frac{d^{LD}\ell}{(2\pi)^{LD}}\frac{1}{S_i}\frac{n_i\tilde{n}_i}{\prod_{\alpha_i}s_{\alpha_i}}, \qquad (7)$$

where κ is the gravity coupling, S_i are symmetry factors, and $1/s_{\alpha_i}$ are propagator denominators. The $n_i,\tilde{n}_i$ may be identical or distinct gauge-theory numerators. The formula is valid if at least one of the two sets of numerators satisfy manifestly the duality [3, 25].

The gauge-theory copies. The first (left) gauge theory entering the construction is an $\mathcal{N}=2$ SYM theory with a single half-hypermultiplet transforming in a pseudo-real representation R, *i.e.* a representation such that there exists a unitary matrix V obeying $VT^{\hat{a}}V^\dagger = -(T^{\hat{a}})^*$, $VV^*=-1$, where $T^{\hat{a}}$ are the representation matrices. This choice allows for double-copy constructions for larger classes of supergravities than with a full hypermultiplet, including in particular all the magical supergravity theories. Amplitudes in this theory can be organized as a superamplitude with manifest $\mathcal{N}=2$ supersymmetry. At three points, the (MHV) superamplitude has the expression

$$\mathcal{A}^{(0)}_3(1\mathcal{V}^{\hat{a}}_-,2\Phi,3\Phi) = i\frac{g}{\langle 23\rangle}\delta^4\Big(\sum\eta^A_i|i\rangle\Big)T^{\hat{a}}\ , \qquad (8)$$

where $|i\rangle$ are spinors and η^A_i are Grassmann parameters in the spinor-helicity notation (see *e.g.* ref. [26]). We have organized the asymptotic states in the following on-shell vector and hypermultiplets superfields:

$$\mathcal{V}^{\hat{a}}_- = \bar{\phi}^{\hat{a}}+\eta^A\psi^{\hat{a}}_{-A}+\eta^1\eta^2A^{\hat{a}}_-\ , \quad \Phi=\chi_++\eta^A\varphi_A+\eta^1\eta^2\chi_-\ .$$

The second (right) gauge theory is a non-supersymmetric YM theory with $(q+2)$ scalars and r fermions. Its Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu}+\frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^a)^{\hat{a}}+\frac{i}{2}\overline{\lambda}^\alpha D_\mu\gamma^\mu\lambda_\alpha + \frac{g}{2}\phi^{a\hat{a}}\Gamma^a_\alpha{}^\beta\overline{\lambda}^\alpha\gamma_5T^{\hat{a}}\lambda_\beta-\frac{g^2}{4}f^{\hat{a}\hat{b}\hat{e}}f^{\hat{c}\hat{d}\hat{a}}\phi^{a\hat{a}}\phi^{b\hat{b}}\phi^{a\hat{c}}\phi^{b\hat{d}}. \qquad (9)$$

The scalars transform in the adjoint representation, while fermions transform in the pseudo-real representation R. D_μ are covariant derivatives. $\alpha,\beta=1,\ldots,r$ and $a,b=1,\ldots,q+2$ are global-symmetry indices, while $\hat{a},\hat{b}$ are adjoint gauge-group indices. Spacetime spinor indices and indices associated to the representation R are not displayed. Imposing color/kinematics duality on the numerators of four-point amplitudes gives the following constraint in the two-scalar-two-fermion case:

$$n_s+n_t+n_u=0 \quad\rightarrow\quad \{\Gamma^a,\Gamma^b\}=2\delta^{ab}\ , \qquad (10)$$

i.e. that the constant matrices Γ^a appearing in the Yukawa couplings form a $(q+2)$-dimensional Clifford algebra. It is convenient to think of the theory above as the dimensional reduction of a $(q+6)$-dimensional YM theory with matter fermions to four dimensions. From a higher-dimensional perspective, the spinor λ_α includes P copies (or flavors) of irreducible $SO(q+5,1)$ spinors, taken to obey reality or pseudo-reality conditions:

$$\text{R}:\ \ \overline{\lambda}=\lambda^t\mathcal{C}_q\mathcal{C}_4V\ , \qquad \text{PR}:\ \ \overline{\lambda}=\lambda^t\mathcal{C}_q\mathcal{C}_4\Omega V\ ,$$

where $\mathcal{C}_q$ and $\mathcal{C}_4$ are the $SO(q+2)$ and $SO(3,1)$ charge-conjugation matrices, Ω is an anti-symmetric real matrix acting on the flavor indices, and V is the matrix in the pseudo-reality condition for the gauge representation matrices. R conditions are appropriate for $q=0,1,2,6,7$ (mod 8) and generically yield a $SO(P)$ manifest flavor symmetry. PR conditions are imposed for $q=3,4,5$ (mod 8) and yield a $USp(2P)$ flavor symmetry.

For even q, we can impose Weyl conditions of the form $\Gamma_*\lambda = \pm\lambda$, where Γ_* is the chirality matrix. For $q = 0, 4$ (mod 8), Weyl conditions are compatible with R and PR conditions, and the representations with different chiralities are inequivalent. Hence the corresponding theories are parameterized by two distinct integers P and $\dot{P}$ counting the number of representations of each kind. Finally, for $q = 2, 6$ (mod 8) one can rewrite the Lagrangian in terms of Weyl spinors, enhancing the manifest flavor symmetry to $U(P)$. From a double-copy perspective, the resulting $4D$ supergravity has one vector multiplet for each $4D$ fermion in the non-supersymmetric gauge theory. The various possibilities are listed in table I, which provides a novel perspective on the results of ref. [21]. In particular, the parameter $\mathcal{D}_q$ equals the minimal number of $4D$ fermions in the $\mathcal{N} = 0$ gauge theory. The full U-duality Lie algebras of $4D$ homogeneous supergravity theories decompose as $\mathcal{G} = \mathcal{G}_0 \oplus \mathcal{G}_1 \oplus \mathcal{G}_2$ with

$$\begin{aligned} &\mathcal{G}_0 = so(1,1) \oplus so(q+2,2) \oplus \mathcal{S}_q(P,\dot{P}) \ , \\ &\mathcal{G}_1 = (1, \text{spinor}, \text{vector}), \quad \mathcal{G}_2 = (2,1,1) \ , \end{aligned} \tag{11}$$

where $\mathcal{S}_q(P,\dot{P})$ is the flavor group, and the grade 1 and 2 generators are labeled by their grade zero representations. The $4D$ supergravity theories with symmetric target spaces have additional symmetry generators corresponding to the grade -1 and -2 subspaces of the isometry Lie algebras [17].

Amplitudes from the double copy. For the construction given here, the map between the double-copy $(\mathcal{N} = 2) \otimes (\mathcal{N} = 0)$ and Lagrangian fields is

$$\begin{aligned} A_-^{-1} &= \bar{\phi} \otimes A_- \ , & h_- &= A_- \otimes A_- \ , \\ A_-^0 &= \phi \otimes A_- \ , & i\bar{z}^0 &= A_+ \otimes A_- \ , \\ A_-^a &= A_- \otimes \phi^a \ , & i\bar{z}^a &= \bar{\phi} \otimes \phi^a \ , \\ A_{\alpha -} &= \chi_- \otimes (U\lambda_-)_\alpha \ , & i\bar{z}_\alpha &= \chi_+ \otimes (U\lambda_-)_\alpha \ , \end{aligned} \tag{12}$$

with similar relations for the CPT-conjugate states. U is a unitary matrix, which – in order to compare to eq. (3) – is written as $U = \frac{e^{i\theta} + e^{i\theta'}\Gamma^1 C}{\sqrt{2}}$, with appropriately chosen phases $e^{i\theta}, e^{i\theta'}$ to guarantee unitarity. C is the matrix listed in table I. With this identification, the three-point amplitudes given by the double-copy construction (7) are

$$\mathcal{M}_3^{(0)}\big(1A_-^a, 2A_{\alpha -}, 3\bar{z}_\beta\big) = -\frac{\kappa}{2\sqrt{2}}\langle 12\rangle^2 (U^t C \Gamma^a U)^{\alpha\beta}. \tag{13}$$

Comparing them with the three-point amplitudes computed from the supergravity Lagrangian implies the identity

$$(U^t C \Gamma^a U) = \big(\mathbf{1} \ , \ -i\tilde{\Gamma}^i\big) \ , \tag{14}$$

where $\mathbf{1}$ is the identity matrix and $\tilde{\Gamma}$ are the real gamma matrices in the cubic form (3). This identity can be verified for all values of q with an appropriate choice for θ, θ'. The particular case $P = 0$ is in agreement with ref. [15].

Even without comparing the double-copy three-point amplitudes with their Lagrangian counterparts, it is possible to confirm that our construction yields supergravities with scalar manifolds that are locally-homogeneous close to the base point. Indeed, a generalization of the arguments of ref. [27] implies that if the scalar fields parametrize a homogeneous manifold, then all single soft-scalar limits vanish. Symmetry considerations imply that this is indeed the case if the soft particle is a scalar that transforms under a manifest symmetry. All the double-copy scalars except the dilaton-axion pair z^0 transform under the manifest $SO(q+2)$ global symmetry.

Thus, only the soft dilaton limit requires a detailed analysis; its vanishing implies that the double-copy theory is invariant under the $U(1)$ transformations with charge given by the difference of the helicities of the left and right gauge-theory fields. We have verified that this is indeed the case and that the tree-level amplitudes of a field configuration with a total non-zero $U(1)$ charge vanish identically at four and five points. From a double-copy perspective these amplitudes are constructed from gauge-theory amplitudes with four, two or no fields in the representation R. The latter amplitudes are the same as in $\mathcal{N} = 8$ supergravity and thus these $U(1)$ transformations are indeed a symmetry. For fixed fields in the adjoint representation, the kinematic factors in the former two types of amplitudes differ from those of $\mathcal{N} = 8$ amplitudes only by numerical factors due to the Yukawa couplings. However, the cancellation of the contributions to the corresponding supergravity amplitude occurs before the summation over the permutation of external legs and thus it is insensitive to these numerical factors.

Our construction carries over to loop-level amplitudes. As an example, we give the one-loop divergence for amplitudes between four identical matter vectors:

$$\begin{aligned} \mathcal{M}_4^{(1)}\big(1A_-^0, 2A_-^0, 3A_+^0, 4A_+^0\big)\Big|_{\text{div}} &= \frac{b}{\epsilon}\Big(\frac{10}{3} - \frac{q}{6} + \frac{r}{3}\Big) \ , \\ \mathcal{M}_4^{(1)}\big(1A_-^a, 2A_-^a, 3A_+^a, 4A_+^a\big)\Big|_{\text{div}} &= \frac{b}{\epsilon}\Big(\frac{10}{3} + \frac{q}{3} + \frac{r}{12}\Big) \ , \end{aligned}$$

with $b = -2i/(4\pi)^2(\kappa/2)^4\langle 12\rangle^2[34]^2$. Interestingly, the two amplitudes have the same divergence when $r = 2q$. This condition is satisfied only by the four magical theories, which are unified, and by the so-called STU model $(q = r = 0)$ [28].

In conclusion, we have shown that scattering amplitudes in homogeneous $\mathcal{N} = 2$ supergravities – including magical and symmetric theories – can be obtained as double copies of two simple gauge theories using the framework of color/kinematics duality. To date, this is the largest known family of double-copy-constructible theories. Color/kinematics duality naturally requires the Clifford algebra structure that has been instrumental in the classification of homogeneous theories and provides an alternative perspective on these theories; in particular, the homogeneity of their target spaces manifests itself in the amplitudes' vanishing soft limits. The double-copy approach is particularly well-suited for carrying out loop-level computations. The existence of a double-copy construction for such a large family of theories suggests that the double-copy can play a fundamental role in general gravity theories; generalizing our construction to accommodate even larger classes of theories, including supergravities with a lower number of isometries and hypermultiplets, appears to be within reach.

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