RESISTIVE BALLOONING STABILITY OF ASDEX EQUILIBRIA

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1. Introduction – Studies of ideal ballooning modes of MHD equilibria reconstructed from ASDEX experimental data have shown that in this plasma configuration with relatively small aspect-ratio ($A \approx 4$) toroidal effects play an important role. The large differential displacement of the magnetic surfaces at high β_p and the effects of increased q and shear in the neighbourhood of the separatrix require highly accurate, genuinely toroidal reference equilibria and a correspondingly careful treatment of the stability equations. Detailed stability analyses under this aspect [1] prove ASDEX equilibria to be close to the β-limit as well as to the marginal ballooning stability limit.
In this paper we extend these investigations to resistive ballooning modes. We present, in coordinate-invariant form, a closed system of four equations describing the resistive evolution of velocity and magnetic fields in the high-m stability limit. Subsequently the Fourier approximation of this set, leading to the resistive ballooning equations of reference [2], is considered. We formulate a variational approach to this boundary value problem in four dependent variables with real and imaginary parts of the growth rate as parameters. Stationary values of the corresponding Lagrangian L are associated with resistive modes. The resulting growth rates will be represented as functions of the poloidal flux Ψ, the toroidal mode number n and pressure scaling dp_M/dp (p being the equilibrium and p_M the marginally-scaled pressure).

2. Theory – For sufficiently large values of the poloidal mode number m the resistive MHD equations predict the following linear evolution of the velocity field $\tilde{\mathbf{v}}$ and the magnetic induction $\tilde{\mathbf{B}}$ from their values in the equilibrium state:

$$\tilde{\mathbf{v}} = \frac{1}{B^2}(\tilde{\mathbf{v}}\cdot\mathbf{B}\mathbf{B} + \mathbf{B}\times(\mathbf{B}\times(\nabla\tilde{\Phi}\times\nabla\sigma))) \qquad (1)$$

$$\tilde{\mathbf{B}} = \frac{\mu_0}{B^2}(-\tilde{p}\mathbf{B} + \mathbf{B}\times(\mathbf{B}\times(\nabla\tilde{A}\times\nabla\sigma))) \qquad (2)$$

Here σ is the coordinate along the equilibrium magnetic field $\mathbf{B}$ with $d\sigma = ds/B$, s being the field line arc length. The evolution of the four field scalars $\tilde{\mathbf{v}}\cdot\mathbf{B}$, $\tilde{\Phi}$, $\tilde{A}$ and $\tilde{p}$ is determined by solution of the following closed set of equations:

$$\frac{1+\beta}{(c_p/c_v)p}\frac{\partial\tilde{p}}{\partial t} = (2\mathbf{k} - \frac{1+\beta}{(c_p/c_v)p}\nabla p)\cdot(\nabla\sigma\times\nabla\tilde{\Phi}) - \mathbf{B}\cdot\nabla(\frac{\tilde{\mathbf{v}}\cdot\mathbf{B}}{B^2}) + \frac{1}{B^2}\eta\Delta\tilde{p} \qquad (3)$$

$$\rho\frac{\partial(\tilde{\mathbf{v}}\cdot\mathbf{B})}{\partial t} = -\mu_0(\nabla\sigma\times\nabla\tilde{A})\cdot\nabla p - \mathbf{B}\cdot\nabla\tilde{p} \qquad (4)$$

$$\mu_0\frac{\partial\tilde{A}}{\partial t} = \mathbf{B}\cdot\nabla\tilde{\Phi} + \eta\Delta\tilde{A} \qquad (5)$$

$$\frac{\rho}{B^2}\Delta(\frac{\partial\tilde{\Phi}}{\partial t}) = \mathbf{B}\cdot\nabla(\frac{1}{B^2}\Delta\tilde{A}) + 2\mathbf{k}\cdot(\nabla\tilde{p}\times\nabla\sigma) \qquad (6)$$

β is a local beta value and $\mathbf{k}$ the curvature vector:

$$\beta = \frac{\mu_0(c_p/c_v)p}{B^2}\,, \qquad \mathbf{k} = (\mathbf{B}\cdot\nabla(\mathbf{B}/B))/B; \tag{7}$$

Δ is the Laplacian $\Delta = \nabla\cdot\nabla$, c_p/c_v the ratio of the specific heats, η the resistivity and ρ the mass density. Assuming a time-dependence $\sim \exp\{\gamma t\}$ of the perturbed quantities with complex growth rate γ, and Fourier representing them in the neighbourhood of a localization field line leads to the stability criterion formulated in [2]: The plasma is unstable with respect to resistive ballooning modes if there are square-integrable solutions u and v of

$$\mathbf{B}\cdot\nabla\Big\{\frac{(1+S^2)}{D|\nabla\Psi|^2}\mathbf{B}\cdot\nabla u\Big\} + \Big(2\mu_0\frac{dp}{d\Psi}\frac{k_gS-k_n}{|\nabla\Psi|} - \mu_0\rho\gamma^2\frac{1+S^2}{|\nabla\Psi|^2}\Big)u + 2\mu_0\frac{dp}{d\Psi}v = 0 \tag{8}$$

$$\mathbf{B}\cdot\nabla\Big\{\frac{1}{B^2}\mathbf{B}\cdot\nabla v\Big\} - 2\mu_0\frac{dp}{d\Psi}\Big(\frac{n^2\eta}{\mu_0\gamma} + \frac{\rho\gamma^2}{\mu_0(dp/d\Psi)^2}\Big)\frac{k_gS-k_n}{|\nabla\Psi|}u -$$

$$- \Big\{\frac{n^2\eta}{\mu_0\gamma}\Big(2\mu_0\frac{dp}{d\Psi}\frac{k_gS-k_n}{|\nabla\Psi|} + \mu_0\rho\gamma^2\frac{1+S^2}{|\nabla\Psi|^2}\Big) + \frac{\mu_0\rho\gamma^2}{B^2}\frac{1+\beta}{\beta}\Big\}v = 0 \tag{9}$$

on that line with $\mathrm{Real}\{\gamma\} > 0$. Here $\mathbf{k}_g$ and $\mathbf{k}_n$ are the obvious decompositions of $\mathbf{k}$ in geodesic and normal components, respectively;

$$D \equiv 1 + \frac{n^2\eta}{\mu_0\gamma}\frac{B^2(1+S^2)}{|\nabla\Psi|^2} \tag{10}$$

and S is the local shear, a secular quantity given by

$$S(\Psi,\Theta,\Theta_0) \equiv \frac{|\nabla\Psi|^2}{B}\int_{\Theta_0}^{\Theta}\frac{1}{|\nabla\Psi|^4}(\mathbf{B}\times\nabla\Psi)\cdot\mathbf{rot}(\mathbf{B}\times\nabla\Psi)\frac{d\Theta'}{\mathbf{B}\cdot\nabla\Theta'} \tag{11}$$

where Θ is any angle-coordinate along $\mathbf{B}$. The equations (8–9) can be seen to be equivalent to the stationarity conditions with respect to $\mathbf{u}$ of the quadratic functional

$$L(\gamma,\Psi,\Theta_0) = \int_{-\infty}^{+\infty}\mathcal{L}(\gamma,\Psi,\Theta,\Theta_0,\mathbf{u}(\Theta),\dot{\mathbf{u}}(\Theta))d\Theta \tag{12}$$

with the Lagrange density

$$\mathcal{L} = \frac{1}{2}(\dot{\mathbf{u}}^T\cdot\mathbf{P}\cdot\dot{\mathbf{u}} - \mathbf{u}^T\cdot\mathbf{Q}\cdot\mathbf{u}) \tag{13}$$

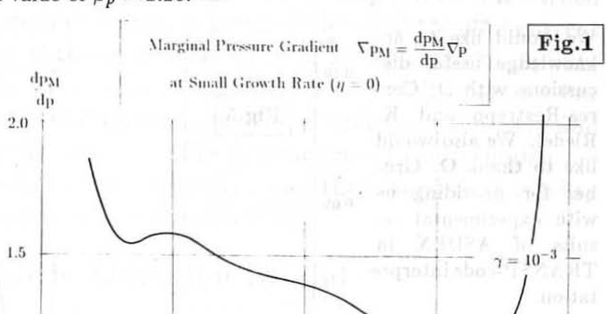
$\mathbf{u} = (u^1,u^2,u^3,u^4)$ comprises real and imaginary parts of u and v in equations (8) and (9) and $\dot{\mathbf{u}} = d\mathbf{u}/d\Theta$ the components $du^k/d\Theta$, $k = 1,..,4$. $\mathbf{Q}$ and $\mathbf{P}$ are equilibrium determined symmetric matrices with nonlinear dependence on the the complex growth rate γ. Thus unstable resistive ballooning modes $\mathbf{u}$ are those stationary points of L with $\mathrm{Real}\{\gamma\} > 0$.

3. Computational Procedure – TRANSP code interpreted radial profile data are used for the calculation of distributions of toroidal current I and pressure p over the magnetic surfaces. Corresponding two-dimensional MHD equilibria are determined by iterative solution of the equilibrium partial differential equation

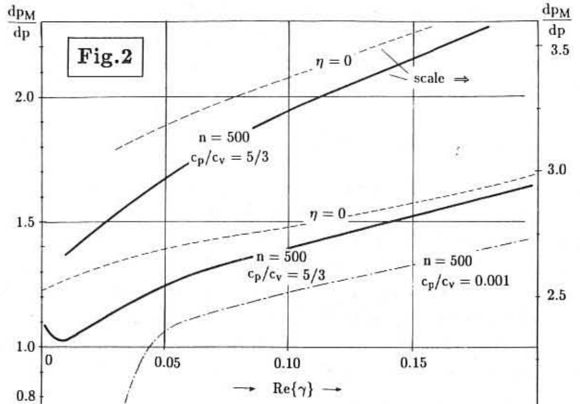
$$\mathrm{div}(\nabla\Psi/R^2) + 4\pi^2\mu_0(1/R^2\langle 1/R^2\rangle^{-1} + \beta_p(1 - 1/R^2\langle 1/R^2\rangle^{-1})\frac{dI}{dV} = 0 \quad (14)$$

applying the Garching equilibrium code NIVA. R is the distance from the axis of symmetry, V the volume of the considered magnetic surface, $\langle..\rangle$ the usual flux surface average and $\beta_p \equiv (dp/d\Psi)(dV/dI)$. Resistivity profiles are calculated using experiment data on Z_{eff}, n_e and T_e [3]. Discretization of (1) on a sufficiently large interval of the localization line in finite elements [4] and appropriate numbering of function values lead to a finite-dimensional quadratic form $L = u^T \cdot S \cdot u$ with a growth-rate dependent system matrix $S(\gamma)$. For the investigation of resistive ballooning modes we perform a detailed singular-value analysis of S, the methods will be described elsewhere [5].

4. Results – Ideal ballooning stability results for a typical high-β_p ASDEX discharge are shown in Fig.1, where dp_M/dp is plotted as a function of $(V/V_B)^{1/2}$ at the maximum attained, code-calculated value of $\beta_p \simeq 2.26$.



It can be seen that the plasma configuration is ballooning stable on all flux surfaces as $dp_M/dp > 1$ with a minimum value of 1.15. Fig.2 demonstrates the destabilizing effect of resistivity for a particular magnetic surface with a value of $r/a = (V/V_B)^{1/2} \simeq 0.65$ with $q \simeq 2.04$, where the right-hand scale refers to the second stability region. The η-value of $3.32 \cdot 10^{-8}$ Vm/A for this surface and the relatively high ideal ballooning stability of ASDEX – for comparison we have represented the results for $\eta = 0$ by the dashed curves – require an anomalous large toroidal mode number (n = 500) to make this effect plain. It is interesting to note that at small growth rates a minimum of dp_M/dp is observed which moves to smaller growth rates with decreasing resistivity. This effect must be ascribed to the stabilizing influence of the compressibility which is separately demonstrated plotting the undermost curve which is valid for a very small value of c_p/c_v. Finally, Fig.3 shows the radial dependence of the growth rate for different values of n, where γ was normalized by multiplication with the Alfven transit time $\tau_A = (\mu_0\rho)^{1/2}R/B_T$ (where ρ is taken on the considered flux surface and R/B_T on the magnetic axis). As in the case of ideal ballooning modes (Fig.1) the most unfavourable flux surfaces with respect to stability are those with a $r/a-$ value of about 0.8. Fig.3 illustrates that in the considered range of growth rates $\mathrm{Real}\{\gamma\} \in (0.005, 0.1)$ and resistivities only modes with very high values of n are of remarkable effect.



Marginal Pressure Gradient $\nabla p_M = \frac{dp_M}{dp}\nabla p$ and Growth Rate

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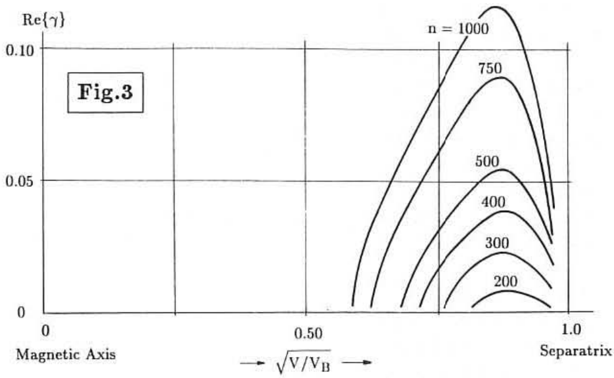

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