Ultraviolet cutoffs and the photon mass

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Abstract The momentum UV cutoff in Quantum Field Theory is usually treated as an auxiliary device allowing to obtain finite amplitudes satisfying all physical requirements. It is even absent (not explicit) in the most popular approach - the dimensional regularization. We point out that the momentum cutoff treated as a bona fide physical scale, presumably equal or related to the Planck scale, would lead to unacceptable predictions. One of the dangers is a non-zero mass of the photon. In the naive approach, even with the cutoff equal to the Planck scale, this mass would grossly exceed the existing experimental bounds. We present the actual calculation using a concrete realization of the physical cutoff and speculate about the way to restore gauge symmetry order by order in the inverse powers of the cutoff scale.

In usual applications of Quantum Field Theory (QFT) the momentum cutoff (explicit or, as in the Dimensional Regularization, implicit) is treated as an auxiliary parameter and sent to infinity at the end of the renormalization procedure. However in the context of a quest for a fundamental theory unifying elementary particle interactions with gravity, QFT models should be viewed as only effective theories with a real momentum cutoff which, as in QFT applications to statistical physics problems, should have a concrete physical interpretation, most probably of the intrinsic scale Λ of the underlying fundamental theory. In this short note, based on the previous work [1] and on the accompanying paper [2] (where all the relevant references can be found) we would like to point out some important aspects of treating the cutoff scale as a *bona fide* physical scale Λ (the problem was partly analyzed in connection with quadratic divergences in QFT [3–5]).

The most spectacular danger of keeping Λ finite is, unless the effective field theory is of very special form, generation of the photon mass proportional to inverse powers of Λ. This is because the gauge symmetry ensuring the vanishing of the photon mass for $\Lambda \to \infty$, for finite Λ remains generically broken. Since the bounds are extremely stringent, even the natural assumption $\Lambda \approx M_{Pl}$ (M_{Pl} being the Planck scale) could lead to unacceptably large photon mass, bigger than the experimental limit. In this note we illustrate this on a simple example and speculate how the problem could possibly be avoided in the context of an underlying more fundamental finite theory.

To define the framework we consider first renormalization of a general YM theory choosing (out of many other possibilities) the momentum cutoff regularization which consists of modifying *every* derivative in the Lagrangian (including the recursively generated counterterms - see below) according to the rule

$$\partial_\mu \to \exp(\partial^2/2\Lambda^2)\partial_\mu \,. \tag{1}$$

In the momentum space this prescription corresponds to the replacement

$$k_\mu \to \mathcal{R}_\mu(k) \equiv \exp(-k^2/2\Lambda^2)\,k_\mu \,. \tag{2}$$

For instance, the regularized ghost contribution to the vacuum polarization tensor (diagram C in Fig. 1) reads

$$\widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l) = -\mathrm{tr}(e_\alpha e_\beta)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,i\,\frac{\mathcal{R}^\mu(k)\mathcal{R}^\nu(k+l)}{\mathcal{R}^2(k)\mathcal{R}^2(k+l)} \,. \tag{3}$$

where e_α are the antihermitian generators of the adjoint representation with included coupling constants (i.e. $e_\alpha = g\,T^{ADJ}_\alpha$ for a simple gauge group).

With the replacement (2) the Wick rotation is, strictly speaking, not justified and neglecting the integral over the contour at infinity must be regarded a part of the regularization prescription (alternatively, the prescription can be formulated directly in the Euclidean version of the theory).

As the standard analysis carried out in [2] shows, in the regularization (1) all diagrams of a renormalizable theory are convergent with the exception of one-loop vacuum graphs (which anyway cannot appear in physically interesting amplitudes as divergent subdiagrams). Computation of diagrams regularized in this way is more complicated than in the Dimensional Regularization but still manageable. For example, each one-loop diagram can be expressed in terms of the confluent hypergeometric function

$$U(a,b,z) = \frac{1}{\Gamma(a)}\int\limits_0^\infty \mathrm{d}t\, t^{a-1}(1+t)^{b-a-1}\exp(-zt)\,,$$

after applying the expansion

$$\frac{1}{\mathcal{R}^2(k)-m^2} = \frac{e^{k^2/\Lambda^2}}{k^2-m^2}\sum_{n=0}^\infty\left[\frac{m^2}{m^2-k^2}\left(1-e^{k^2/\Lambda^2}\right)\right]^n,$$

to all regularized propagators. The first term in this expansion bears a very close resemblance to the propagator used in the context of Wilson's exact renormalization group equations [6]. The advantage of our expression is that it is better suited for theories with spontaneous symmetry breaking, in which m^2 in general depends on background scalar fields (which can keep track of vacuum expectation values of dynamical scalar fields). In the Euclidean space, for $k^2 \to -k_E^2$, the above series is absolutely convergent. In particular, owing to the growing powers of $m^2 - k^2$ in successive terms, only a finite number of terms of a given one-loop diagram yield integrals which are divergent when the factors $e^{k^2/\Lambda^2}(1 - e^{k^2/\Lambda^2})^n$ are omitted. The remaining terms would be integrable without these factors which implies that their contributions vanish in the limit $\Lambda \to \infty$.

Since the regularization (1) breaks the gauge (more precisely, the BRST) invariance, a special subtraction scheme in necessary in order to arrive at a finite (renormalized) one-particle irreducible effective action Γ (the generator of the strongly connected Green's functions) satisfying *in the limit* $\Lambda \to \infty$ the requirements of the BRST invariance (embodied in the appropriate functional identity). In the accompanying paper [2] such a subtraction scheme (called Λ-$\overline{\text{MS}}$), belonging to the class of mass independent schemes and adapted to the regularization (1), is proposed. It relies on the Quantum Action Principle [7] and is defined recursively in the following way. Having a local action I_n^Λ (with all counterterms up to the order $\hbar^n$ included and with the replacement (1) made in all derivatives), one considers Γ_n - the asymptotic part of $\Gamma_n^\Lambda \equiv \Gamma[I_n^\Lambda]$, obtained from it by omitting all terms that would vanish for $\Lambda \to \infty$ and constructs order $\hbar^{n+1}$ "minimal" counterterms $-\Gamma_n^{(n+1)\text{div}}$ which subtract (appropriately defined) "pure", order $\hbar^{n+1}$, divergences of Γ_n. In the next step one constructs finite non-minimal counterterms $\delta_\flat\Gamma_n^{(n+1)}$ of the restricted schematic form (A^μ, ϕ and ψ stand respectively for generic gauge, scalar and fermion fields)

$$\delta_\flat\Gamma_n^{(n+1)} \in \int (\partial^\mu A_\mu)(\partial^\nu A_\nu) \oplus A_\mu A^\mu \oplus A_\mu \overline{\psi}\gamma^\mu P_L \psi$$
$$\oplus A_\mu \overline{\psi}\gamma^\mu P_R \psi \oplus \phi\phi A_\mu A^\mu \oplus A_\mu \partial^\mu\phi \oplus \phi A_\mu \partial^\mu\phi$$
$$\oplus \phi A_\mu A^\mu \oplus AA\,\partial A \oplus AAAA\,, \tag{4}$$

so that $I_{n+1} \equiv I_n - \Gamma_n^{(n+1)\text{div}} + \delta_\flat\Gamma_n^{(n+1)}$ leads to Γ_{n+1} - the asymptotic part of $\Gamma_{n+1}^\Lambda \equiv \Gamma[I_{n+1}^\Lambda]$ - which up to terms of order $\hbar^{n+1}$ is finite and satisfies the Slavnov-Taylor identities (STIds) following from the required BRST invariance. In [2] it is shown that the choice (4), which is particularly natural (no non-minimal counterterms are generated for terms of the action without gauge fields), satisfies all the requirements and is equivalent to the usual specification of the renormalization conditions. As a result of the procedure sketched above one constructs

the action I_∞

$$I_\infty = I_0 + \sum_{n=0}^\infty (-\Gamma_n^{(n+1)\text{div}} + \delta_\flat\Gamma_n^{(n+1)}) \tag{5}$$

expressed in terms of renormalized parameters and couplings, depending explicitly on Λ (through the counterterms $-\Gamma_n^{(n+1)\text{div}}$) and such that Green's functions obtained from $\Gamma[I_\infty^\Lambda]$ satisfy STIds *in the strict limit* $\Lambda \to \infty$.

Being mass independent, the proposed scheme introduces, similarly as the ordinary $\overline{\text{MS}}$ scheme, an auxiliary mass scale μ. We have verified by explicit one-loop calculations in a general renormalizable model (with a non-anomalous fermionic representation) that the proposed subtraction scheme is equivalent to the standard $\overline{\text{MS}}$ scheme based on the dimensional regularization (DimReg) with the anticommuting γ^5 matrix (the so-called naive prescription): the one-loop 1PI effective action in Λ-$\overline{\text{MS}}$ can be obtained from its $\overline{\text{MS}}$ counterpart via a reparametrization (a "finite renormalization") of fields and couplings. Furthermore, we have proved recursively, that the finite effective action $\Gamma[I_\infty^\Lambda]$ and the action I_∞^Λ itself satisfy the Renormalization Group Equations (RGE) which ensure independence of physical result of the auxiliary mass scale μ. (The one-loop equivalence of the $\overline{\text{MS}}$ and Λ-$\overline{\text{MS}}$ schemes allowed us to obtain in [2] the two-loop RGE for the Λ-$\overline{\text{MS}}$ scheme parameters). Finally, we have performed some two-loop consistency checks as well.

Established RG invariance of I_∞^Λ consisting of the regularized original action I_0^Λ and the constructed counterterms, in which the replacement (1) is made (as pointed out in [2], this is necessary for consistency of the entire scheme), allowed to show that, despite not having the same functional form as I_0 (e.g. each interaction term depending on the gauge fields A^μ is multiplied by a different series of renormalized couplings with coefficients divergent as $\Lambda \to \infty$), it does wind up into a bare action I_B which depends on Λ only through the appropriately defined bare parameters and through the regularizing exponential factors (1) accompanying derivatives. This (technically nontrivial in the case of a gauge symmetry violating regularization) result opens the possibility to view I_∞^Λ (after expanding the exponentials, so that they give rise to infinite sum of higher and higher dimension operators) as a part of the complete Lagrangian density of an effective field theory which in the perturbative expansion reproduces results of some *finite* fundamental theory of all interactions. The scale Λ should be then identified with an intrinsic physical scale of the putative fundamental theory rather than with the scale introduced by the Wilsonian procedure of integrating out some high energy degrees of freedom. For this interpretation to be possible it is, however, indispensable to address the problem of the residual breaking of the gauge (BRST) invariance by terms suppressed by inverse powers of Λ of which one of the consequences is the photon mass generation.

To illustrate the problem we consider here, using the regularization (2), the one-loop contribution to the standard gauge field self energy (vacuum polarization) tensor $\widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l)$ contracted with the four-momentum l_μ. Before making subtractions (as indicated by the superscript $1B$) we find (in the Landau gauge, using a developed Mathematica package described in [2]):

$$l_\mu \widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l)^{(1B)} = -3g_s^2\, \delta_{\alpha\beta}\, \frac{l^\nu}{(4\pi)^2}\Big\{ -\Lambda^2 - \frac{5}{24}l^2$$
$$- \frac{7}{384}\frac{l^4}{\Lambda^2} + \frac{1}{1536}\frac{l^6}{\Lambda^4} + \mathcal{O}(\Lambda^{-6})\Big\}\,,$$

in "QCD without quarks" (diagrams A, B and C in Fig. 1), and

$$l_\mu \widetilde{\Gamma}^{\mu\nu}(l)^{(1B)} = e^2\, \frac{l^\nu}{(4\pi)^2}\left\{ -\frac{2}{3}(l^2 - 3m^2 + 3\Lambda^2) \right.$$
$$\left. + \frac{1}{\Lambda^2}\left[-\frac{11}{96}l^4 + l^2 m^2 + 6m^4 \ln\frac{m^2}{(0.37\Lambda)^2} \right] + \mathcal{O}(\Lambda^{-4})\right\}\,,$$

in QED with a single charged lepton of mass m (diagram G). These contributions clearly give non-vanishing correction to the gluon and photon masses, respectively.

Terms that survive in the limit $\Lambda \to \infty$ clearly show that the gauge symmetry is badly broken by the regularization prescription (1). As explained above, they are removed by local counterterms (i.e. minimal subtraction of the divergent part followed by the addition of the first two terms in (4) with appropriate coefficients). On the other hand, the terms of the above expressions suppressed by the inverse powers of Λ also break the gauge invariance but are not subtracted by adding non-minimal counterterms which are (in the construction following from the QAP principle) independent of Λ. If present, they would imply a contribution to the photon mass m_γ of the order of $(\alpha_{\rm EM}/4\pi)^{1/2}M_{\rm top}^2/\Lambda$. In the usual approach ($\Lambda \to \infty$) such terms would be absent but if Λ is a physical cutoff scale we have to consider them as possible genuine corrections. However, because of the experimental limit on the photon mass ($m_\gamma < 1.7 \times 10^{-22}$ GeV from the dispersion relations from pulsar emissions [8] and $m_\gamma < 10^{-27}$ GeV from combination of all data [9]) this kind of breaking is excluded even for Λ as high as the Planck scale which would give

$$|m_\gamma| \approx \left(\frac{\alpha_{\rm EM}}{4\pi}\right)^{1/2} \frac{M_{top}^2}{M_{Pl}} \approx 10^{-18}\ \text{GeV}.$$

In fact, the situation is even worse since m_γ^2 generated in the above example not only grossly exceeds the experimental limits but has also the wrong sign.

Therefore, if the cutoff Λ is to be treated as a *finite* physical scale of an underlying fundamental theory, one has to assume that the *complete* bare action $I_{\rm B}^{complete}$ of the effective QFT, which reproduces all results (including those depending on the gravitational sector) of the latter theory has also additional, as compared to the local action $I_{\rm B} = I_\infty^\Lambda$ (counter)terms suppressed by inverse powers of Λ which conspire to restore exact BRST invariance of the amplitudes. The structure of the residual gauge symmetry breaking revealed by the above two examples suggests that such a solution may be viable: the Λ suppressed terms which must be subtracted do not involve logarithms of momenta so that $I_{\rm B}^{complete}$ can still be analytic. Additional terms with higher derivatives which must be present in $I_{\rm B}^{complete}$ would complement those which implementing the regularization (2) reflect finiteness of the underlying theory - the assumption that higher derivative terms of $I_{\rm B}^{complete}$ combine solely to exponential factors (1) of an otherwise *renormalizable* action is certainly too simplistic for the action of an effective field theory corresponding to the fundamental theory of all interactions. In turn the presence of logarithms of masses (which in general dependent on the background fields), simply indicates that in order to restore BRST-invariance of all amplitudes for finite Λ, the $I_{\rm B}^{complete}$ should depend on non-polynomial functions of fields (similarly as the action of the field theory describing the Ising lattice model depends on $\cosh(\phi)$, where ϕ is the order parameter field), which after expansion around the background (vacuum expectation values) give rise to vertices with arbitrary numbers of scalar fields.

The ultimate structure of $I_{\rm B}^{complete}$ would therefore be such as can naturally be expected on the basis of the general principle of constructing effective theories. In the case considered here it is tempting to assume that the limit $\Lambda \to \infty$ corresponds in the effective theory to complete neglect of a gravitational sector, which for finite Λ is entangled with the other sectors and is indispensable for consistency.

Summarizing, we conjecture that even if the cutoff scale Λ is a real physical scale it is possible to introduce local counterterms to the bare action that restore the requisite identities and render the vanishing of the photon mass order by order in the inverse powers of the scale Λ.

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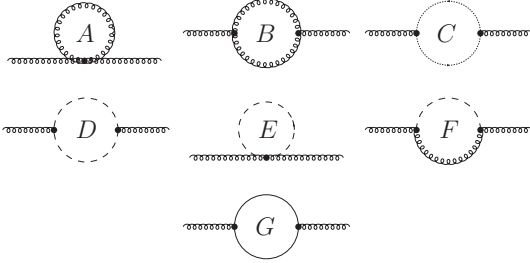[9] K.A. Olive et al. (Particle Data Group), Chin. Phys. C**38**, 090001 (2014) and 2015 update.

FIG. 1. One-loop contributions to the vacuum polarization.