Minkowski flux vacua of type II supergravities

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We study flux compactifications of 10d type II supergravities to 4d Minkowski space-time, supported by parallel orientifold O_p-planes with $3 \leq p \leq 8$. Upon few geometric restrictions, the 4d Ricci scalar can be written as a negative sum of squares involving BPS-like conditions. Setting all squares to zero provides automatically a solution to 10d equations of motion. This way, we characterise a broad class, if not the complete set, of Minkowski flux vacua. We also conjecture an extension to include non-geometric fluxes. None of our results rely on supersymmetry.

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I. CONTEXT AND RESULTS

We are interested in vacua of ten-dimensional (10d) type II supergravities on a 4d maximally symmetric space-time times a 6d compact manifold, with non-trivial fluxes. These vacua are a major framework for string phenomenology, given the role of fluxes in moduli stabilisation. Having a classification of these vacua would thus be an important achievement. In this paper, with only few requirements, we reveal and characterise a broad class of Minkowski flux vacua.

The best understood flux compactifications of type II supergravities are those with just fluxes and no sources. Well-known no-go theorems [1] imply that such reductions only lead to anti-de Sitter (AdS) vacua. Even more, those AdS vacua are no genuine compactifications since they lack a tunable hierarchy between the AdS length scale and the Kaluza-Klein scale [2]. Space-time filling orientifold planes are a natural way out of this problem and allow to find Minkowski compactifications [3, 4] (see [5] for a review) or AdS compactifications with scale separation (see e.g. [6]). However such reductions come with an extra level of complication; the backreaction of the orientifolds themselves. In the case of non-intersecting O_p-planes (and possibly D_p-branes) the backreaction is rather well-understood: it introduces a warp factor e^A in the 10d metric

$$\mathrm{d}s^2_{10} = e^{2A}(\mathrm{d}\tilde{s}^2_4 + \mathrm{d}\tilde{s}^2_{6||}) + e^{-2A}\mathrm{d}\tilde{s}^2_{6\perp} \ , \qquad (1)$$

with inverse powers along parallel and transverse directions to the sources, a non-trivial profile for the sourced Ramond-Ramond (RR) gauge potential, and a dependence on the internal coordinates for the dilaton ϕ when $p \neq 3$. So we work here in this context: type II supergravities supplemented by space-time filling, parallel (i.e. non-intersecting), backreacted O_p/D_p sources, with a fixed value $3 \leq p \leq 8$, and study Minkowski flux vacua. We follow conventions of a companion paper [7].

A. O_3-planes

Let us recall known results on O_3 compactifications in IIB, following [4, 7, 8]. The sourced flux is the 5-form F_5, the other RR fluxes being the 1- and 3-forms F_1, F_3, and the NSNS 3-form flux is denoted H. For our purposes, we need two equations: the first one is a combination of the dilaton equation of motion (e.o.m.), the 10d and the 4d traces of the Einstein equation, giving an expression for the Ricci scalar $\tilde{\mathcal{R}}_4$ of $\mathrm{d}\tilde{s}^2_4$; the second equation is the F_5 Bianchi identity (BI)

$$\begin{aligned} & e^{-2A}\tilde{\mathcal{R}}_4 = -\tfrac{1}{2}|H|^2 - \tfrac{e^{2\phi}}{2}|F_3|^2 - e^{2\phi}|F_5|^2 + \tfrac{e^{\phi}}{4}T_{10} + \dots \ , \\ & \mathrm{d}F_5 = H \wedge F_3 + \tfrac{\varepsilon_3}{4}\, T_{10}\, \mathrm{vol}_6 \ , \end{aligned} \qquad (2)$$

where the dots stand for terms explicitly dependent on $\partial\phi$ and ∂A, and we leave the definitions of the square of forms, the sign ε_p, and the sources contributions T_{10} to [15]. We see from above that, at least in the smeared limit where $\partial\phi = \partial A = 0$, the flux contributions cannot be canceled to find a Minkowski vacuum without the O_3 contribution in T_{10}. We now combine both equations of (2) and rewrite the result as in [7]

$$\begin{aligned} e^{-2A}\tilde{\mathcal{R}}_4 = & -\left|e^{4A}{*_6}\mathrm{d}e^{-4A} - \varepsilon_3 e^{\phi}F_5\right|^2 \\ & - \tfrac{1}{2}\left|{*_6}H + \varepsilon_3 e^{\phi}F_3\right|^2 + e^{-2A}\partial(\dots) \ , \end{aligned} \qquad (3)$$

where $\partial(\dots)$ denotes a total derivative over the (unwarped) compact manifold. Upon integration, we deduce that a Minkowski vacuum requires both squares to vanish. One leads to the well-known ISD condition [4]:

$H = \varepsilon_3 e^{\phi} *_6 F_3$. The combination in the other square is also present in the total derivative, which thus vanishes. This also fixes F_5, and relates eventually $\mathrm{d}F_5$ to $\tilde{\Delta}_6 e^{-4A}$ as expected from the BI. With further combinations of e.o.m., one can show that $F_1 = 0$ and ϕ is constant (see below). Using the H and F_3 BI (i.e. that those fluxes are closed), one then shows that *all e.o.m. are solved*, provided the (unwarped) compact manifold is Ricci flat. In other words, given this geometric requirement, and assuming satisfied BI, one finds all Minkowski flux vacua, and those are characterised through the expression (3) by setting the squares of BPS-like conditions to zero.

B. O_p-planes with $p > 3$

The essence of the present paper is to derive analogous results for O_p-planes with $p > 3$. First, up to little additional structure, $\tilde{\mathcal{R}}_4$ can again be written in terms of squares of BPS-like conditions; with few geometric restrictions, asking for a Minkowski vacuum then requires to set these conditions to zero, fixing the sourced flux F_{8-p}, $3 \leq p \leq 8$, and relating H to F_{6-p}. We further show that these three fluxes are the only non-trivial ones. Finally, assuming their BI, we show that *all e.o.m. are satisfied*, giving us a broad class of Minkowski flux vacua.

Let us point out the main differences when $p > 3$. For $p = 3$, all internal directions are transverse to the sources, while for $p > 3$, some are parallel, as in (1). A is then only dependent on transverse directions, and $*_6$ gets replaced by $*_\perp$, $\tilde{\Delta}_6$ by $\tilde{\Delta}_\perp$, etc. Fluxes may now have different components: $F_l = F_l^{(0)} + F_l^{(1)} + \dots$ where n in $F_l^{(n)}$ denotes the number of parallel indices of the component. We will as well introduce the projection on the transverse subspace $F_l|_\perp = F_l^{(0)}$. A second difference is the possibility of RR fluxes F_l with $l > 8 - p$. Taking these features into account, a lengthy computation [7] leads to an expression for $\tilde{\mathcal{R}}_4$, analogous to (3), given by

$$\begin{aligned} e^{-2A}\tilde{\mathcal{R}}_4 = &- \left| e^{4A} *_\perp \mathrm{d}e^{-4A} - \varepsilon_p e^{\phi} F_{8-p}^{(0)} \right|^2 \\ &- \tfrac{1}{2} \left| *_\perp H|_\perp + \varepsilon_p e^{\phi} F_{6-p}|_\perp \right|^2 \\ &- \tfrac{1}{2} \sum_{a_{||}} \left| *_\perp (\mathrm{d}e^{a_{||}})|_\perp - \varepsilon_p e^{\phi} (\iota_{\partial_{a_{||}}} F_{8-p}^{(1)}) \right|^2 \\ &- (\mathrm{flux})^2 - \mathrm{curvature} + e^{-2A}\partial(\dots) \ , \end{aligned} \tag{4}$$

for $0 \leq 8 - p \leq 5$, where we use the internal one-form basis $e^a = e^a{}_m \mathrm{d}y^m$ constructed with Vielbeins, and the contraction ι_V by a vector V such that $\iota_{\partial_a} e^b = \delta_a^b$. We use from now on the shorthand $\iota_a \equiv \iota_{\partial_a}$, and the 6d metric in flat indices δ_{ab}. One has $\mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c$, where the anholonomicity symbol (from now on "geometric flux") f does not need to be constant. Thus by definition, $(\mathrm{d}e^{a_{||}})|_\perp = -\frac{1}{2} f^{a_{||}}{}_{b_\perp c_\perp} e^{b_\perp} \wedge e^{c_\perp}$. This expression (4) is derived in [7] where on top the analogue of (2), one uses the trace of the Einstein equation along internal parallel directions (an equation trivial for $p = 3$). Differences between (4) and (3) are related to those mentioned above: there is a third BPS-like condition involving $F_{8-p}^{(1)}$, the $(\mathrm{flux})^2$ are squares of $F_{8-p}^{(n>1)}$ and of $F_{l>8-p}$, and finally the "curvature" terms involve parallel directions.

These curvature terms are the only ones with indefinite sign. We thus set them here to zero, and will do the same for purely transverse directions. For practical convenience, we set the corresponding (unwarped) geometric fluxes to zero

$$\tilde{f}^{a_{||}}{}_{b_{||} c_{||}} = \tilde{f}^{a_{||}}{}_{b_\perp c_{||}} = \tilde{f}^{a_\perp}{}_{b_{||} c_{||}} = \tilde{f}^{a_\perp}{}_{b_\perp c_{||}} = 0 \ , \tag{5}$$

$$\tilde{f}^{a_\perp}{}_{b_\perp c_\perp} = 0 \ , \tag{6}$$

where (5) makes the curvature terms in (4) vanish. These geometric requirements are the only ones made on the vacua, and can be viewed as the analogue of the Ricci flatness for $p = 3$. Asking for the transverse subspace to have no boundary amounts to $\tilde{f}^{a_\perp}{}_{a_\perp c_\perp} = 0$, implied by (6); we then integrate (4) over this subspace, giving finally $\tilde{\mathcal{R}}_4$ only in terms of BPS-like conditions and squares of fluxes. A Minkowski vacuum requires to set them to zero, making the total derivative vanish, as for $p = 3$. These conditions are enough to characterise a broad class of Minkowski flux vacua, as we show in the next sections.

II. FLUXES

Without the curvature terms, asking for a Minkowski vacuum sets the BPS-like conditions of (4) and the squares of fluxes to zero. This, together with the trace of the Einstein equation along internal parallel directions [7], implies that fluxes (and components) within the BPS-like conditions are the only ones allowed to be non-zero, i.e. $H = H|_\perp$, $F_{6-p} = F_{6-p}|_\perp$, $F_{8-p} = F_{8-p}^{(0)} + F_{8-p}^{(1)}$, with

$$\begin{aligned} F_{6-p} &= -e^{-\phi} \varepsilon_p *_\perp H \ , \\ F_{8-p}^{(0)} &= e^{-\phi} \varepsilon_p e^{4A} *_\perp \mathrm{d}e^{-4A} \ , \\ F_{8-p}^{(1)} &= e^{-\phi} \varepsilon_p \, \delta_{ab} \, e^{a_{||}} \wedge *_\perp (\mathrm{d}e^{b_{||}})|_\perp \ . \end{aligned} \tag{7}$$

Indeed, we get that $F_{l>8-p} = 0$, and that the lowest RR flux F_{4-p}, meaning F_0 for $p = 4$ and F_1 for $p = 3$, vanishes [16]. The only possible flux content is then (7)!

The BI for these fluxes are given by

$$\mathrm{d}H = \mathrm{d}F_{6-p} = 0 \ , \ \mathrm{d}F_{8-p} - H \wedge F_{6-p} = \tfrac{\varepsilon_p T_{10}}{p+1} \mathrm{vol}_\perp \ , \tag{8}$$

and projecting the F_{8-p} BI on the transverse directions, together with (7), gives

$$e^{\phi} \frac{T_{10}}{p+1} = e^{6A} \tilde{\Delta}_\perp e^{-4A} + e^{2\phi} |F_{6-p}|^2 + e^{2\phi} |F_{8-p}^{(1)}|^2 \tag{9}$$

$$= e^{6A} \tilde{\Delta}_\perp e^{-4A} + |H|^2 + |f^{a_{||}}{}_{b_\perp c_\perp}|^2 \ , \tag{10}$$

the last term being defined in [17]. Upon integration of the last equation one recovers the RR tadpole condition that expresses the net charge in terms of the fluxes. Clearly, both the H-flux and the geometric flux thus contribute to cancel the O_p charge, and the two contributions can be viewed as T-dual [8, 9]. While $F^{(1)}_{8-p}$ is related to the geometric flux, $F^{(0)}_{8-p}$ leads to $\tilde{\Delta}_\perp e^{-4A}$ generating the δ-functions, and can thus be interpreted as related to the sources backreaction. Explicit vacua with both pieces are given e.g. in [10, 11], while a vacuum with all fluxes (7) turned on is given in (6.42) of [10].

We now show that the fluxes (7) solve their e.o.m. [7], given here by

$$\begin{aligned} & e^{-4A}\mathrm{d}(e^{4A} *_6 F_{8-p}) = 0 \ (4 \leq p \leq 7) \ , \qquad (11)\\ & e^{-4A}\mathrm{d}(e^{4A} *_6 F_{6-p}) + H \wedge *_6 F_{8-p} = 0 \ (3 \leq p \leq 5) \ ,\\ & e^{-4A}\mathrm{d}(e^{4A-2\phi} *_6 H) - F_{6-p} \wedge *_6 F_{8-p} = 0 \ , \end{aligned}$$

provided relevant BI are satisfied. To that end, it is convenient to rewrite F_{8-p} as

$$F_{8-p} = (-1)^p \varepsilon_p \, e^{-4A} *_6 \mathrm{d}\left(e^{4A-\phi}\mathrm{vol}_{||}\right) \ , \qquad (12)$$

with the internal parallel volume form (see details in [18]), and use for an l-form A_l that $*_6 A_l|_\perp = (-1)^{l(p+1)}\mathrm{vol}_{||} \wedge *_\perp A_l|_\perp$. It is then straightforward to see that the fluxes (7) solve the three above e.o.m., given the H and F_{6-p} BI. We now turn to the remaining e.o.m.; the F_{8-p} BI will help to prove they are also satisfied.

III. SATISFYING ALL E.O.M.

The remaining e.o.m. are the dilaton and the Einstein equation; we split the latter into the 4d one, the transverse internal one, and for $p \neq 3$ the internal parallel and off-diagonal ones. Verifying that they are satisfied is purely technical; we give here and in the appendix only the main insights. For all these e.o.m., we use the 10d Einstein trace [7], leading us to consider in the following the trace reversed Einstein equation. We also use (9) and the dilaton and warp factor formulas of appendix B.1 of [7]. The dilaton e.o.m. and the 4d Einstein equation are then straightforward to verify. For instance, the latter boils down to

$$\begin{aligned} & \mathcal{R}|_{\mu\nu} + 2\nabla|_\mu \partial|_\nu \phi - \tfrac{g_{\mu\nu}}{4}(2|\partial\phi|^2 - \Delta\phi) \qquad (13)\\ & = \tfrac{g_{\mu\nu}}{8}\Big(\tfrac{e^\phi}{2}\tfrac{7-p}{p+1}\ T_{10} - |H|^2 + \tfrac{e^{2\phi}}{2}\sum_{q=0}^{6}(1-q)|F_q|^2\Big) \ , \end{aligned}$$

with even/odd RR fluxes for IIA/IIB, and each side can be shown to be equal to (A.1). We have introduced the shorthand $|_\mu \equiv {}_{M=\mu}$, and similarly $|_a \equiv {}_{A=a}$.

Internal Einstein equations are treated using flat indices. The Ricci tensor is computed using formulas of appendix B.2 of [7] and $f^{a_\perp}{}_{BC} = \delta^b_B \delta^c_C f^{a_\perp}{}_{bc}$. Starting from the general tensor expression, one should pay attention going from 10d to 6d indices, parallel and transverse, and further to warped versus unwarped quantities. We also use the source contribution $T_{ab} = \delta^{a_{||}}_a \delta^{b_{||}}_b \delta_{a_{||}b_{||}} T_{10}/(p+1)$. The internal parallel Einstein equation requires the dilaton derivatives (A.2). One should then prove that the Ricci tensor is given by (A.3), which is achieved using assumption (5). We also use (5) for the internal off-diagonal Einstein equation and compute for the dilaton (A.4); this equation eventually becomes

$$\mathcal{R}|_{a_{||}b_\perp} = (p-2)\delta^{e_\perp d_\perp}\delta_{a_{||}c_{||}} f^{c_{||}}{}_{e_\perp b_\perp} e^{-A}\partial_{d_\perp} e^A \qquad (14)$$

The computed Ricci tensor (A.5) is however different. Interestingly, the match is achieved using the F_{8-p} BI along non-transverse directions, i.e. $\mathrm{d}F_{8-p} - (\mathrm{d}F_{8-p})|_\perp = 0$, which has two components due to $F^{(1)}_{8-p}$. Setting the one along $e^{a_{||}} \wedge e^{b_{||}} \wedge e^\perp$, resp. along $e^{a_{||}} \wedge e^\perp$, to zero gives identities (A.6), resp. (A.7), using (5) and $\tilde{f}^{a_\perp}{}_{b_\perp a_\perp} = 0$. Identity (A.7) allows to solve the off-diagonal Einstein equation. Finally, using the expression (A.8) for the dilaton, the internal transverse Einstein equation is given by

$$\begin{aligned} \mathcal{R}|_{a_\perp b_\perp} = & -\tfrac{1}{2}\delta_{g_{||}h_{||}}\delta^{c_\perp e_\perp} f^{g_{||}}{}_{c_\perp a_\perp} f^{h_{||}}{}_{e_\perp b_\perp} \qquad (15)\\ & + \delta_{a_\perp b_\perp}(2p-7)|\widetilde{\mathrm{d}e^A}|^2 + \delta_{a_\perp b_\perp} e^A \tilde{\Delta}_\perp e^A\\ & - 2(p-3)\partial_{a_\perp}\partial_{\tilde{b}_\perp} e^A - 2(p+1)\partial_{\tilde{a}_\perp} e^A \partial_{\tilde{b}_\perp} e^A\\ & + 2\omega_{a_\perp}{}^{c_\perp}{}_{b_\perp}|_{(\partial A=0)}\partial_{c_\perp}\phi\\ & + \tfrac{1}{2}\iota_{a_\perp}H \cdot \iota_{b_\perp}H + \tfrac{e^{2\phi}}{2}\iota_{a_\perp}F_{6-p} \cdot \iota_{b_\perp}F_{6-p}\\ & - \delta_{a_\perp b_\perp}\tfrac{e^{2\phi}}{2}|F_{6-p}|^2 \ . \end{aligned}$$

This holds as well for $p=3$ where $F_5 = F^{(0)}_5$, using (7). The last three rows of (15) vanish using (6) and (7). Computing the Ricci tensor with (5) and (6), one then obtains a match, up to $\frac{1}{2}\partial_{g_{||}} f^{g_{||}}{}_{a_\perp b_\perp}$, which is zero due to the identity (A.6). All e.o.m. are thus satisfied.

IV. NON-GEOMETRIC FLUXES EXTENSION

A natural extension of (4) (without curvature terms) including non-geometric NSNS fluxes would be

$$\begin{aligned} 2e^{-2A}\tilde{\mathcal{R}}_4 = & -2\left|e^{4A} *_\perp \mathrm{d}e^{-4A} - \varepsilon_p e^\phi F^{(0)}_{8-p}\right|^2 \qquad (16)\\ & - \left|H_{a_\perp b_\perp c_\perp} *_\perp e^{a_\perp b_\perp c_\perp}/3 + \varepsilon_p e^\phi F^{(0)}_{6-p}\right|^2\\ & - \sum_{a_{||}}\left|f^{a_{||}}{}_{b_\perp c_\perp} *_\perp e^{b_\perp c_\perp}/2 + \varepsilon_p e^\phi(\iota_{a_{||}} F^{(1)}_{8-p})\right|^2\\ & - \sum_{a_{||}b_{||}}\left|Q_{c_\perp}{}^{a_{||}b_{||}} *_\perp e^{c_\perp} + \varepsilon_p e^\phi(\iota_{b_{||}}\iota_{a_{||}} F^{(2)}_{10-p})\right|^2\\ & - \sum_{a_{||}b_{||}c_{||}}\left|R^{a_{||}b_{||}c_{||}} *_\perp 1 + \varepsilon_p e^\phi(\iota_{c_{||}}\iota_{b_{||}}\iota_{a_{||}} F^{(3)}_{12-p})\right|^2\\ & - (\mathrm{flux})^2 + e^{-2A}\partial(\dots) \ . \end{aligned}$$

This combination of BPS-like conditions is T-duality invariant: in 4d one lifts or lowers T-dualized indices of NSNS fluxes [12], while parallel and transverse directions get exchanged when T-dualized. Setting the curvature terms, or even conditions (5) and (6), to zero may also have analogues with non-geometric fluxes. Finally, one could extend the F_{8-p} BI on transverse directions (9) by adding $+|Q_{c_\perp}{}^{a_{||}b_{||}}|^2 + |R^{a_{||}b_{||}c_{||}}|^2$. In these extended expressions, all terms are however not present for any p, due to the number of parallel and transverse directions. All fluxes are allowed for $p=6$, but not for the others: on top of the first BPS-like condition, only the one in H is present for $p=3$, H, f for $p=4$, H, f, Q for $p=5$, f, Q, R for $p=7$ and Q, R for $p=8$. This restriction can also be understood applying T-duality to toroidal orientifolds, starting with an O_3 vacuum on T^6 with H-flux.

A first setting to derive these conjectured expressions would be 4d gauged supergravities, replacing $\tilde{\mathcal{R}}_4$ by the vacuum value of the scalar potential. Another option would be the 10d formalism of β-supergravity [13], or extensions thereof, including [14]. But this goes beyond the scope of this paper.

V. DISCUSSION

In this paper, we have characterised a broad class of Minkowski flux vacua with parallel localized O_p/D_p sources; this is achieved thanks to the general rewriting (4) of the 4d Ricci scalar as a negative sum of squares, extending a well-known result for O_3-planes [4] to general O_p reductions. The fluxes are (7) (their BI are assumed to be satisfied), the metric is (1) with restrictions (5) and (6), and the dilaton is given in [16]. Interestingly, requiring only the geometric restrictions (5) and (6), we have shown that these are *the only Minkowski flux vacua*!

Our analysis does not rely on supersymmetry (SUSY) so our vacua can capture both SUSY and non-SUSY solutions. To get SUSY O_3 vacua, the Ricci flat condition should be supplemented by Calabi-Yau, and the ISD 3-form flux has to be (1,2) and primitive [4]. Here, it would be interesting to compare our geometric restrictions to the generalized Calabi-Yau condition and our fluxes to the SUSY ones [10, 11]. Already, our sourced flux takes the value (12) for a calibrated brane, which would be the SUSY value in presence of an SU(3)×SU(3) structure (see references in [7]).

A natural question is whether there exists other Minkowski flux vacua with parallel sources than the class found here, thus meaning some with the geometric restrictions not satisfied (see a related discussion in [7]). We may capture all no-scale vacua, but still wonder whether other vacua exist, that do not arise from the BPS-like conditions. We can for instance look at the set of SUSY Minkowski flux vacua on twisted tori: all known solutions are reviewed in [11]. All listed vacua with parallel sources verify (5) and (6) (in particular those of [10]), except for the new ones found in [11]. For the latter however, the Ricci tensor still vanishes, indicating a suspected possible refinement of our geometric conditions. Then, up to this detail, we do not find vacua beyond the class presented here.

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Appendix: Ricci tensor and dilaton derivatives

Each side of the 4d Einstein equation (13) is equal to

$$\frac{g_{\mu\nu}}{16}(7-p)\left(e^{6A}\tilde{\Delta}_\perp e^{-4A} - e^{8A}|\mathrm{d}e^{-4A}|^2\right) \ . \tag{A.1}$$

For the internal parallel Einstein equation, we use

$$\nabla|_{a_{||}}\partial|_{b_{||}}\phi = -\frac{\delta_{a_{||}b_{||}}}{4}e^{2\phi-2A}\delta^{c_\perp d_\perp}\partial_{c_\perp}e^{2A}\partial_{d_\perp}e^{-2\phi} \tag{A.2}$$

$$\begin{aligned}\mathcal{R}|_{a_{||}b_{||}} &= \frac{\delta_{a_{||}b_{||}}}{8}\left(2e^{6A}\tilde{\Delta}_\perp e^{-4A} - (p-1)e^{8A}|\mathrm{d}e^{-4A}|^2\right) \\ &\quad + \frac{\delta_{a_{||}c_{||}}\delta_{b_{||}d_{||}}}{4}\delta^{e_\perp f_\perp}\delta^{g_\perp h_\perp}f^{c_{||}}{}_{e_\perp g_\perp}f^{d_{||}}{}_{f_\perp h_\perp} \ ,\end{aligned} \tag{A.3}$$

where the warp factor terms of (A.3) can be rewritten as $-\delta_{a_{||}b_{||}}\delta^{c_\perp d_\perp}\left(e^A\partial_{\tilde{c}_\perp}\partial_{\tilde{d}_\perp}e^A + (2p-7)\partial_{\tilde{c}_\perp}e^A\partial_{\tilde{d}_\perp}e^A\right)$. For the internal off-diagonal Einstein equation, we compute with (5) and $2\partial_{[a}\partial_{b]} = f^c{}_{ab}\partial_c$

$$\nabla|_{a_{||}}\partial|_{b_\perp}\phi = \nabla|_{b_\perp}\partial|_{a_{||}}\phi = -\frac{\delta_{a_{||}e_{||}}\delta^{c_\perp d_\perp}}{2}f^{e_{||}}{}_{d_\perp b_\perp}\partial_{c_\perp}\phi \tag{A.4}$$

$$\begin{aligned}\mathcal{R}|_{c_{||}d_\perp} &= (p-3)\delta^{b_\perp e_\perp}\delta_{c_{||}h_{||}}f^{h_{||}}{}_{e_\perp d_\perp}\partial_{\tilde{b}_\perp}e^A \\ &\quad + \tfrac{1}{2}\delta^{b_\perp e_\perp}\delta_{c_{||}h_{||}}\partial_{b_\perp}f^{h_{||}}{}_{e_\perp d_\perp} \\ &\quad + \tfrac{1}{4}\delta_{c_{||}i_{||}}\delta_{d_\perp g_\perp}\delta^{e_\perp h_\perp}\delta^{b_\perp j_\perp}f^{i_{||}}{}_{e_\perp j_\perp}f^{g_\perp}{}_{h_\perp b_\perp} \ ,\end{aligned} \tag{A.5}$$

and the F_{8-p} BI along non-transverse directions gives

$$\partial_{a_{||}}f^{c_{||}}{}_{d_\perp e_\perp} = 0 \ , \tag{A.6}$$

$$\begin{aligned}&-2\delta^{l_\perp d_\perp}f^{b_{||}}{}_{c_\perp l_\perp}\partial_{\tilde{d}_\perp}e^A + \delta^{d_\perp l_\perp}\partial_{d_\perp}f^{b_{||}}{}_{c_\perp l_\perp} \\ &-\tfrac{1}{2}\delta_{k_\perp c_\perp}\delta^{e_\perp h_\perp}\delta^{f_\perp g_\perp}f^{b_{||}}{}_{h_\perp g_\perp}f^{k_\perp}{}_{e_\perp f_\perp} = 0 \ .\end{aligned} \tag{A.7}$$

For the transverse Einstein equation, we compute

$$\begin{aligned}\nabla|_{a_\perp}\partial|_{b_\perp}\phi &= \partial_{a_\perp}\partial_{b_\perp}\phi - \omega_{a_\perp}{}^{c_\perp}{}_{b_\perp}|_{(\partial A=0)}\partial_{c_\perp}\phi \\ &\quad + \partial_{a_\perp}\phi\partial_{\tilde{b}_\perp}e^A - \delta_{a_\perp b_\perp}\delta^{c_\perp d_\perp}\partial_{c_\perp}\phi\partial_{\tilde{d}_\perp}e^A \ .\end{aligned} \tag{A.8}$$

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[15] We define the square of the q-form A_q as $|A_q|^2 = A_{m_1\dots m_q}A^{m_1\dots m_p}/q!$, the sign $\varepsilon_p = (-1)^{[\frac{9-p}{2}]+p+1}$, and sources are localized in transverse directions through $T_{10} = \frac{2\kappa_{10}^2}{\sqrt{|g_\perp|}}(p+1)T_p\Big(\sum_{O_p}2^{p-5}\delta(y_\perp) - \sum_{D_p}\delta(y_\perp)\Big)$. We consider BPS sources where tension and charge are related as $T_p = \mu_p$, and without world-volume flux.

[16] Proving that $F_{4-p} = 0$ actually requires another combination of e.o.m. worked-out in [7], namely the one that allows to conclude on the no-go for $p = 7, 8$. That equation also led to the standard value $e^\phi = g_s e^{A(p-3)}$ with a constant g_s, that we use in the present paper.

[17] $|f^{a_{||}}{}_{b_\perp c_\perp}|^2 = \frac{\delta_{ab}}{2}\delta^{ef}\delta^{gh}f^{a_{||}}{}_{e_\perp g_\perp}f^{b_{||}}{}_{f_\perp h_\perp} = \sum_{a_{||}}|(\mathrm{d}e^{a_{||}})|_\perp|^2$

[18] The internal manifold and transverse subspace having no boundary, one has $\tilde{f}^{a_{||}}{}_{a_{||}d_\perp} = 0$. One then shows that $\widetilde{\mathrm{dvol}}_{||} = e^{-A(p-3)}\sum_{a_{||}} *_6 (e^{a_{||}} \wedge *_\perp(\mathrm{d}e^{a_{||}})|_\perp)$, allowing to prove that F_{8-p} in (7) can be rewritten as (12).